EFFECT OF SOLENOID FIELD ERRORS ON ELECTRON BEAM TEMPERATURES IN THE RHIC ELECTRON COOLER

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Abstract

As part of a future upgrade to the Relativistic Heavy Ion Collider (RHIC), electron cooling is foreseen to decrease ion beam emittances. Within the electron cooling section, the “hot” ion beam is immersed in a “cold” electron beam. The cooling effect is further enhanced by a solenoid field in the cooling section, which forces the electrons to spiral around the field lines with a (Larmor) radius of 10 micrometers, reducing the effective transverse temperature by orders of magnitude. Studies of the effect of solenoid field errors on electron beam temperatures are reported.

INTRODUCTION

To improve the luminosity of the Relativistic Heavy Ion Collider (RHIC) as well as as essential part of the electron-ion collider eRHIC currently under study, electron cooling at high beam energies of $\gamma \approx 100$ is being foreseen [1]. The RHIC electron cooler consists of a superconducting energy-recovery linac to accelerate electrons to energies up to some 55 MeV, thus matching the Lorentz factor of the stored ion beam. To enhance the cooling effect, a magnetized beam and a solenoidal field in the cooling section is foreseen [2]. Figure 1 shows a schematic drawing of the RHIC electron cooler.

Table 1 shows some key parameters of the RHIC electron cooler.

However, to preserve low electron emittances from the gun through the linac and transfer line and through the solenoid, exact optics matching as well as high magnetic field quality is required. Simulation studies were performed to identify the effect of transverse magnetic field components in the RHIC electron cooler on electron beam temperatures. The goal was to determine the required field quality as a function of the wavelength of the transverse field distortion, which in turn defines the optimum spacing of correction coils.

SIMULATION RESULTS

To study the effect of transverse field errors on electron beam temperatures, an ensemble of particles was tracked through a 30 m long 1 T solenoid with an additional transverse field component of

$$B_x(z) = \hat{B}_\perp \sin(k \cdot z), \quad (1)$$

$$\hat{B}_\perp = a \cdot B_\parallel, \quad (2)$$

where $B_\parallel = 1$ T is the main solenoid field, $z$ is the longitudinal coordinate along the solenoid, and $k = 2\pi/\lambda$ is the wave number for an error field wavelength $\lambda$. The coefficient $a$ has been scanned to determine the magnetic field error tolerances for a given temperature increase.

The electrons are assumed to have been created at a cathode with temperature $T = 20000$ K. This beam is accelerated to $\gamma = 10^7$ to match the storage energy of gold ions in RHIC. The longitudinal momentum spread is assumed as $\Delta p/p = 1 \cdot 10^{-4}$, the design value of the RHIC electron cooler.

Tracking within the solenoid is performed by application of the Lorentz force equation,

$$\vec{F} = \frac{d\vec{p}}{dt} = m \frac{d\vec{v}}{dt} = e \cdot (\vec{v} \times \vec{B}), \quad (3)$$

which is integrated using a fourth-order Runge-Kutta method.

To calculate temperatures, all velocities are transformed to the moving beam frame according to

$$v_\parallel = \frac{v_\parallel^* - v_0}{1 - v_\parallel^* \cdot \frac{v_0}{c^2}} \quad (4)$$

and

$$v_\perp = \frac{v_\perp^*}{\gamma_0 \cdot \left(1 - v_\parallel^* \cdot \frac{v_0}{c^2}\right)} \quad (5)$$

Here, $v_\parallel$ and $v_\perp$ are the longitudinal and transverse velocity of one particular electron, respectively, while $v_0$ and $\gamma_0$ are the velocity and Lorentz factor of the bunch center-of-mass. An asterisk denotes the laboratory frame. Using the
velocities in the moving frame, temperatures are calculated from the respective rms values as

\[ T_\parallel = \frac{m \cdot \langle v_\parallel^2 \rangle}{2k} \]  

(6)

and

\[ T_\perp = \frac{m \cdot \langle v_\perp^2 \rangle}{2k} , \]  

(7)

where \( k = 1.3806 \cdot 10^{-23} \text{ J/K} \) is the Boltzmann constant. Figure 2 shows the ratio of longitudinal electron beam temperatures at the entrance, \( T_\parallel(z = 0 \text{ m}) \), and at the exit of the solenoid, \( T_\parallel(z = 30 \text{ m}) \), as a function of the transverse field wavelength for different values of the coefficient \( a \), corresponding to relative transverse field errors in the range of \( \hat{B}_\perp/B_\parallel = 1 \cdot 10^{-6} \ldots 5 \cdot 10^{-5} \). The longitudinal electron beam temperature increase shows a significant resonant behavior when the wavelength of the transverse field component is in the vicinity of the Larmor wavelength of the electrons,

\[ \lambda_{\text{Larmor}} = \frac{2\pi\gamma mc}{eB_\parallel} \approx 1.14 \text{ m}, \]  

(8)

where \( m \) and \( e \) are the electron rest mass and charge, while \( c \) and \( \gamma \) denote the speed of light and the Lorentz factor, respectively.

In contrast to this, the transverse temperature \( T_\perp \) stays practically constant, independent of the wavelength of the electron beam.

Figure 2: Longitudinal electron beam temperature increase \( T_\parallel(z = 30 \text{ m})/T_\parallel(z = 0 \text{ m}) \) at the end of the 30 m long cooler solenoid, as function of the wavelength \( \lambda \) of the transverse field component. The lowest, red line corresponds to a transverse field amplitude of \( \hat{B}_\perp = 1 \cdot 10^{-6} \cdot B_\parallel \), while the top line (black) shows the relative temperature increase for the case \( \hat{B}_\perp = 5 \cdot 10^{-5} \cdot B_\parallel \). Lines in-between correspond to \( \hat{B}_\perp = 2 \cdot 10^{-6} \cdot B_\parallel \) (green), \( \hat{B}_\perp = 5 \cdot 10^{-6} \cdot B_\parallel \) (blue), \( \hat{B}_\perp = 1 \cdot 10^{-5} \cdot B_\parallel \) (purple), and \( \hat{B}_\perp = 2 \cdot 10^{-5} \cdot B_\parallel \) (turquoise), respectively.
Figure 3: Relative longitudinal temperature increase $T_\parallel(z)/T_\parallel(z = 0 \text{ m})$ in the resonant case with $\lambda = \lambda_{\text{Larmor}} \approx 1.14 \text{ m}$ as function of the longitudinal position along the solenoid. The lowest, red line shows a transverse field amplitude of $\hat{B}_\perp = 1 \cdot 10^{-6} \cdot B_\parallel$, while the top line (black) corresponds to the relative temperature increase for the case $\hat{B}_\perp = 5 \cdot 10^{-5} \cdot B_\parallel$. Lines in-between correspond to $\hat{B}_\perp = 2 \cdot 10^{-6} \cdot B_\parallel$, $\hat{B}_\perp = 5 \cdot 10^{-6} \cdot B_\parallel$, $\hat{B}_\perp = 1 \cdot 10^{-5} \cdot B_\parallel$, and $\hat{B}_\perp = 2 \cdot 10^{-5} \cdot B_\parallel$, respectively.

transverse magnetic field. The increase in longitudinal electron beam temperature $T_\parallel$ can therefore be explained as coupling of transverse electron motion into the longitudinal direction, thus effectively increasing the longitudinal energy spread $\Delta p/p$ and hence the longitudinal temperature $T_\parallel$.

The relative longitudinal temperature increase as function of the longitudinal position within the cooler solenoid in the resonant case with $\lambda = \lambda_{\text{Larmor}} \approx 1.14 \text{ m}$ is depicted in Figure 3. As this picture reveals, the longitudinal electron beam temperature increases approximately linearly along the cooler solenoid.

**CONCLUSION**

The effect of a transverse sinusoidal field component on electron beam temperatures inside the solenoid of the RHIC electron cooler has been studied. The longitudinal temperature shows a significant resonant effect when the wavelength $\lambda$ of the transverse magnetic field equals the Larmor wavelength $\lambda_{\text{Larmor}} = 1.14 \text{ m}$, while the transverse temperature remains unchanged over a large range of transverse field amplitudes $\hat{B}_\perp$. To preserve the small longitudinal electron beam temperature, transverse magnetic field components must not exceed $\hat{B}_\perp = 1 \cdot 10^{-6} \cdot B_\parallel$ for wavelengths close to the Larmor wavelength $\lambda_{\text{Larmor}}$ of the electrons inside the solenoidal magnetic field. Therefore, dipole correctors should be placed at distances of $\lambda_{\text{Larmor}}/4$ to enable effective suppression of transverse magnetic fields with these wavelengths.

**REFERENCES**
