A TECHNIQUE TO MEASURE CHROMATICITY BASED ON THE HARMONIC ANALYSIS OF A LONGITUDINALLY KICKED BEAM

G. Rumolo, GSI, R. Tomás, Uni. of Valencia

Abstract

Recently a method to measure chromaticity has been proposed by applying a transverse and a longitudinal kick to the beam. Assuming a Gaussian bunch in the 6-dimensional phase space, analytical expressions were derived, which relate the synchrotron sidebands to the machine chromaticity. To assess limitations and extensions of this technique, a more realistic accelerator model is considered including dispersion, transverse non–linearities and second order chromaticity, which modify the Fourier spectrum. Tracking and analytical studies are performed to describe these effects.

INTRODUCTION

The standard procedure for measuring the machine chromaticity is based on changing the beam energy and central orbit via frequency modulations of the radio frequency system. The chromaticities are then inferred by the variation of the machine tunes. The above procedure tends to be lengthy and perturbs the normal machine operation. Nevertheless a recent study has shown that by using a phase locked loop tune meter the performance of this method can be largely improved [1]. Various alternative techniques to measure chromaticity have been proposed during the last years. Two remarkable ones are described in [2], [3] and [4]. The first one uses the phase difference between the oscillations of the head and the tail of the bunch. The second one consists in modulating the radio frequency phase and measuring the induced tune modulation.

In [5] yet another method to determine the chromaticity is proposed by measuring the amplitude and phase of the synchrotron sidebands of the transverse motion after simultaneously applying a longitudinal and a transverse kick. The analytical expression obtained in this paper for the amplitude of the synchrotron sideband of order q, i.e. with frequency given by $Q_x + qQ_s$, is expressed as

$$AMP(q) \quad = \quad e^{-\varsigma^2} |I_q(\varsigma^2 - i\varsigma k)| \,, \qquad (1)$$

where $\varsigma = Q'_x \sigma_\delta / Q_s$, Q'_x is the chromaticity, σ_δ is the momentum spread and k is the longitudinal kick in sigma units. This expression holds for a Gaussian bunch matched to the bucket, i.e. $\sigma_\delta = \sigma_z \frac{Q_s}{\eta R}$, η being the slippage factor and R the machine radius. To obtain this expression a linear motion was assumed both in the longitudinal and the transverse planes and only first order chromaticity was considered. The above equation shows that by applying the longitudinal kick the amplitudes of the synchrotron sidebands are linear (first non vanishing order) in the chromaticity. The sign of the chromaticity is contained in the phase of the sidebands.

To assess limitations and extensions of this technique a more realistic accelerator model is considered in this article including dispersion, transverse non–linearities and second order chromaticity. A Gaussian bunch matched to the bucket will be assumed for the derivations. Macroparticle simulations using the HEADTAIL [6] code are also performed.

CONSIDERING DISPERSION AND NON–LINEARITIES

The most relevant effect of non-linearities is the fact that they produce amplitude detuning and, as a consequence, the decoherence of the beam oscillations after a transverse kick. In this paper we assume that the amplitude detuning is low enough to ensure the resolution of the synchrotron sidebands in the Fourier spectrum of the transverse motion. This condition is normally desirable in the normal operation of an accelerator.

The transverse dynamics of a particle in presence of chromaticity, dispersion and non-linearities is approximated by assuming that the energy oscillations are much slower than the transverse oscillations. The equation describing the transverse motion is obtained by directly introducing the time dependence of the energy in the expressions derived for a constant energy. Therefore the most relevant contributions to the dynamics are those produced by the feed down from the non–linear fields due to the dispersion offset. In general, the contribution of all the feed down fields to the transverse motion can be taken into account by introducing chromatic Twiss parameters in the following way,

$$\begin{aligned} Q &= Q_0 + Q'\delta + \frac{1}{2}Q''\delta^2 + \ldots \\ \beta &= \beta_0 + \Delta\beta_1\delta + \ldots \\ D &= D_0 + D_1\delta + \ldots \,, \end{aligned} \qquad (2)$$

where β represents the betatronic function and D the dispersion function. In the first place we describe the Fourier spectrum of the motion considering only the first chromatic order of Twiss parameters. In the following section the effect of Q'' will be studied. The single particle position as function of the turn number is given by taking into account all the former quantities in the following way,

$$\begin{aligned} x(s,N) \quad = \quad & |a|\sqrt{1 + \frac{\Delta\beta_1}{\beta_0}(s)\delta(N)} \\ & \times \cos(2\pi(Q_x + \Delta Q_x(N))N + \phi_x) \\ & + D_0(s)\delta(N) + D_1(s)\delta^2(N) \,, \end{aligned} \qquad (3)$$

where $\delta(N)$ is the relative momentum deviation as function of the turn number,

$$\delta(N) = \delta\cos(2\pi Q_s N) + z\frac{Q_s}{\eta R}\sin(2\pi Q_s N) \,, \qquad (4)$$

δ and z being the initial longitudinal coordinates and $\Delta Q_x(N)$ is the integral of $Q'_x\delta(N)$ from the turn 0 to the turn N divided by N. It can be deduced from the above equations that the chromatic beta beating modifies the synchrotron sidebands and the orbit oscillates with the frequencies Q_s and $2Q_s$ due to the dispersion and the chromatic dispersion. The contribution of sextupoles to the chromatic beta beating is produced by the quadrupolar feed down due to the dispersion at the sextupole and is expressed as

$$\frac{\Delta\beta_1}{\beta}(s) = \int_s^{s+C} ds' \frac{K_2(s')D_0(s')\beta_0(s')}{2\sin(2\pi Q_x)} \times \cos(2\pi Q_x - 2|\phi_x(s') - \phi_x(s)|) \ , \tag{5}$$

where K_2 is the sextupole strength. The contribution of sextupoles to the chromatic dispersion is produced by the dipolar feed down due to the dispersion at the sextupole and is expressed as

$$D_1(s) = \frac{\sqrt{\beta_0(s)}}{4\sin(\pi Q_x)} \int_s^{s+C} ds' \sqrt{\beta_0(t)} K_2(s') \times D_0^2(s') \cos(\phi(s') - \phi(s) - \pi Q_x) \ . \tag{6}$$

Contributions from other multipoles can be similarly computed. The motion of the centroid is obtained by averaging the single particle motion over the initial bunch density, i.e.

$$\overline{x}(s,N) = \int_{-\infty}^{\infty} d\delta \int_{-\infty}^{\infty} dz \rho(\delta, z) x(s,N) \ , \tag{7}$$

where $\rho(\delta, z)$ is the longitudinal density of the Gaussian bunch displaced in the z axis by k sigmas. To solve this integral the chromatic beta beating is assumed to be much smaller than one and the square root of Eq. (3) is expanded up to first order. To obtain the centroid motion the following integrals are needed,

$$\int_{-\infty}^{\infty} d\delta \int_{-\infty}^{\infty} dz \rho(\delta, z)\delta(N) e^{i2\pi\Delta Q_x(N))N} = \sigma_\delta(k + i\varsigma) \times \sin(2\pi Q_s N) e^{(\varsigma^2 - i\varsigma k)(\cos(2\pi Q_s N)-1)} \ ,$$

$$\int_{-\infty}^{\infty} d\delta \int_{-\infty}^{\infty} dz \rho(\delta, z)\delta(N) = \sigma_\delta k \sin(2\pi Q_s N) \ , \tag{8}$$

$$\int_{-\infty}^{\infty} d\delta \int_{-\infty}^{\infty} dz \rho(\delta, z)\delta^2(N) = \sigma_\delta^2 [1 + k^2 \sin^2(2\pi Q_s N)] \ .$$

The frequencies and amplitudes of the different spectral lines arising in the Fourier spectrum of $\overline{x}(s,N)$ are given in table 1. It is particularly interesting to note that the fundamental spectral line Q_x is not affected by the chromatic Twiss parameters and that the amplitude of the sideband $Q + |q|Q_s$ is different from that of $Q_x - |q|Q_s$. This difference is linear in the chromatic beta beating, the longitudinal kick and the chromaticity. Therefore this difference can be used to determine the sign of the chromaticity, provided that the chromatic beta beating is known or a calibration is done using another method for determining the chromaticity. Likewise, if the chromaticity is known the chromatic beta beating could be measured around the ring.

Frequency	Amplitude
$Q_x + qQ_s$	$e^{-\varsigma^2}\lvert\overline{a}\rvert \Big\lvert I_q(\varsigma^2 - i\varsigma k) + \frac{\Delta\beta_1}{4i\beta}\sigma_\delta(k + i\varsigma) \times [I_{q-1}(\varsigma^2 - i\varsigma k) - I_{q+1}(\varsigma^2 - i\varsigma k)] \Big\rvert$
Q_s	$\frac{1}{2}D_0(s)\sigma_\delta k$
$2Q_s$	$\frac{1}{4}D_1(s)\sigma_\delta^2 k^2$

Table 1: Frequencies and amplitudes of the different spectral lines produced by dispersion and sextupolar fields.

In order to verify the achieved expressions a macroparticle simulation has been performed using a model of the CERN SPS. One sextupolar kick at a dispersion region has been introduced in the HEADTAIL code. The value of the dispersion was chosen to be the average dispersion at the SPS sextupoles, $D = 2.24\ m$. The strength of the sextupolar kick was chosen to reproduce the chromatic beta beating at the start of SPS, $K_2L = -0.254564\ m^{-2}$. Second order chromaticity was well corrected by means of a phase space rotation and chromaticity is $Q' = -0.279$. The simulation was performed for a longitudinal kick of one sigma, $\sigma_\delta = 0.00248$ and the synchrotron tune is $Q_s = 0.005$. In Fig. 1 a comparison of the Fourier spectrum obtained from the simulation and the prediction from the model is shown. Although a relevant discrepancy is found for the second order sidebands the first order sidebands are well described by the model. This means that second order chromatic Twiss parameters should be considered to properly describe second and higher order synchrotron sidebands. Simulation and model agree with remarkable precision concerning D_1, as appears from the spectral line with frequency $2Q_s$. This is a very relevant quantity that contains the local non–linear information in a similar way as the resonance driving terms do. It is evident from the figure how important higher orders are, since data were acquired at a dispersion free region but yet lines with Q_s and $3Q_s$ are observed. We can conclude that using the first order synchrotron sidebands of all the BPMs around the ring this technique can provide not only chromaticity but also chromatic beta beating and local non–linear information.

Second order chromaticity

This section studies the effect of the second order chromaticity in the Fourier spectrum of the centroid motion. An analytical expression for the turn-by-turn centroid position is derived considering first and second order chromaticities but not any chromatic Twiss function. The tune shift $\Delta Q_x(N)$ is given by the integral of $Q'\delta(N) + Q''\delta^2(N)/2$ from the turn 0 to the turn N divided by N, giving the fol-

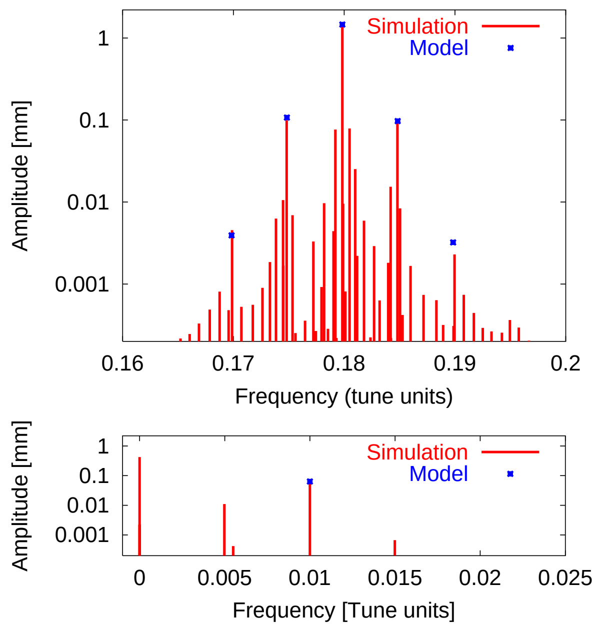

Figure 1: Fourier spectrum of the simulated turn-by-turn motion together with the prediction from the model. Top: Tune and synchrotron sidebands. Bottom: Offset and multiples of Q_s.

lowing expression,

$$\frac{Q'_x}{2\pi Q_s N}\left[\delta\sin(2\pi Q_s N) - z\frac{Q_s}{\eta R}(\cos(2\pi Q_s N)-1)\right] + \frac{Q''_x}{4\pi N}\left[\frac{\delta^2}{2}\left(N+\frac{\sin(4\pi Q_s N)}{4\pi Q_s}\right)+\frac{\delta z}{\eta R 4\pi}(1-\cos(4\pi Q_s N)) + \frac{z^2 Q_s^2}{2\eta^2 R^2}\left(N - \frac{\sin(4\pi Q_s N)}{4\pi Q_s}\right)\right].$$

The centroid position $\overline{x}(s,N)$ is computed as in Eq. (7) with the above tune shift and it is obtained by taking the real part of the following expression (up to a proportionality constant):

$$\frac{e^{\frac{-\varsigma(k+i\varsigma)\left[(i+\varsigma_2 N)(\cos(2\pi Q_s N)-1)+\frac{\varsigma_2}{2\pi Q_s}S(N)\right]+F(N)}{1-2i\varsigma_2 N+\varsigma_2^2\left[-N^2+\frac{\sin^2(2\pi Q_s N)}{4\pi^2 Q_s^2}\right]}+i2\pi Q_x N}}{\sqrt{1-2i\varsigma_2 N+\varsigma_2^2\left[-N^2+\frac{\sin^2(2\pi Q_s N)}{4\pi^2 Q_s^2}\right]}}$$

$S(N)$ and $F(N)$ being

$$\begin{aligned} S(N) &= \sin(2\pi Q_s N) - \sin(4\pi Q_s N)/2 \qquad (9) \\ F(N) &= -ik^2\left(i + \varsigma_2 N + \varsigma_2\frac{\sin(4\pi Q_s N)}{4\pi Q_s}\right)/2\,, \end{aligned}$$

where $\varsigma_2 = \pi Q''\sigma_\delta^2$. This equation shows that for large N the oscillation amplitude is damped proportionally to $1/(\varsigma_2 N)$. This effect increases the width of the synchrotron sidebands. The analytical description of the Fourier spectrum is too complicated to be usable. To illustrate this a

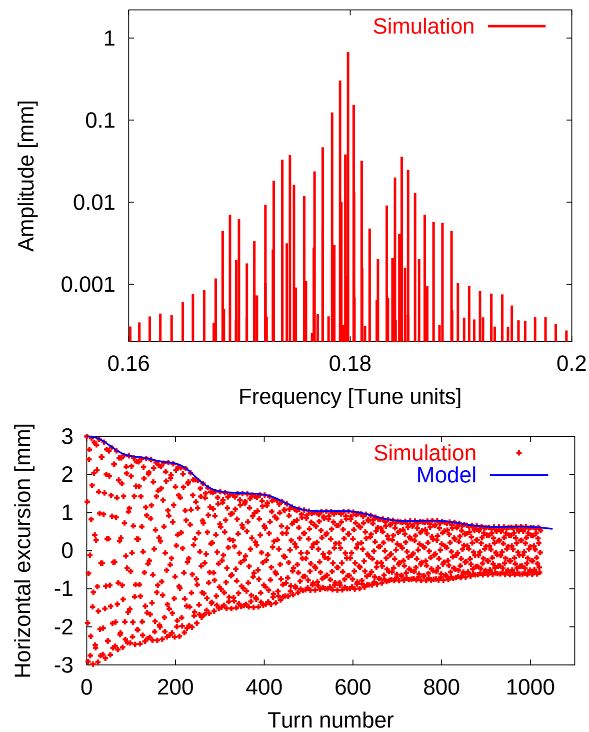

Figure 2: Fourier spectrum and centroid position in presence of first and second order chromaticity from the simulation and from the analytical formula.

simulation has been performed with the same SPS model as above but with $Q' = -0.279$, $Q'' = -150$ and without the sextupolar kick. The turn-by-turn centroid position and the Fourier spectrum obtained from the simulation and the analytical formula are shown in Fig. 2. The first synchrotron sidebands can still be used to estimate Q' although a larger Q'' would completely distort the Fourier spectrum.

CONCLUSIONS

It has been demonstrated that for small amplitude detuning and second order chromaticity the proposed technique can be used to measure chromaticity and to obtain information concerning chromatic Twiss functions and nonlinearities. It probably remains important to study the effect of the non-linearity of the longitudinal phase space motion and collective effects.

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