LINEAR COUPLING CORRECTION WITH N-TURN MAPS*

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Abstract

The linear one-turn map of a storage ring contains coupling information on which a correction algorithm can be based. In principal, the one-turn matrix can be fitted from turn-by-turn data of beam position monitors after a kick was applied. However, the so obtained coupling information often sinks into the noise floor. The signal-to-noise ratio of the coupling information can be greatly enhanced by fitting maps for larger turn numbers N, equal to half the beat period. With the so obtained N-turn map an automated global coupling correction is possible without the need for a tune change. This is demonstrated for the Relativistic Heavy Ion Collider where the algorithm is implemented for operational use at injection.

1 INTRODUCTION

Linear coupling [1–3] can make it impossible to set tunes to values close to the coupling resonance $Q_x = Q_y$. These tunes are desirable since the resonance density in this area is low. A widely used method to measure global linear coupling, is to move the tunes until the minimum tune separation $\Delta Q_{min} = |Q_x - Q_y|_{min}$ is reached [1]. A coupling correction is then performed by scanning skew quadrupole corrector settings to minimize ΔQ_{min}. This approach is slow, can lead to beam losses, and cannot be practically applied during an energy ramp. The decoupling method presented here overcomes all these shortcomings. It is based on N-turn maps, fitted from turn-by-turn data after a transverse kick was applied. Based on fitted N-turn maps the minimum tune approach ΔQ_{min} and skew corrector settings to minimize this quantity can be obtained without a tune change. The algorithm lends itself to full automation and allows a coupling correction within seconds. An implementation at RHIC is shown.

2 MATRIX DESCRIPTION

We denote by $\vec{z} = (x, x', y, y')^T$ the 4-vector with the positions and slopes at a certain observation point in the ring (see Fig. 1). The linear one-turn map $\mathbf{M}$ transforms the 4-vector $\vec{z}^i$ at turn i into the 4-vector $\vec{z}^{i+1}$ at turn $i+1$ via

$$\vec{z}^{i+1} = \mathbf{M}\vec{z}^i. \tag{1}$$

The 4×4 matrix $\mathbf{M}$ can be written in terms of 2×2 matrices as

$$\mathbf{M} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix}. \tag{2}$$

The machine is said to be globally decoupled if $\mathbf{B} = \mathbf{0}$, which implies $\mathbf{C} = \mathbf{0}$. If the linear coupling is caused by a number of small sources, rather than a few large ones, global decoupling at any observation point will usually lead

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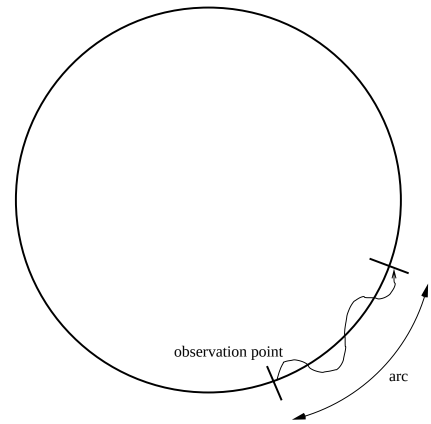

Figure 1: Observation point at the beginning of an arc and beam trajectory in the arc.

to a machine that is almost globally decoupled at any other observation point [3].

We denote by (Q_A, Q_D) the eigentunes of the map (2), and also use the quantities $\mu_{A,D} = 2\pi Q_{A,D}$. The eigentunes can be determined with good precision from turn-by-turn data by filtering, Fourier transformation, and interpolation [4]. The minimum tune approach is then [3]

$$\Delta Q_{min} = \frac{\sqrt{\det|\mathbf{C} + \overline{\mathbf{B}}|}}{\pi(\sin\mu_A + \sin\mu_D)} \tag{3}$$

where $\overline{\mathbf{B}} = -\mathbf{S}\mathbf{B}^T\mathbf{S}$, and $\mathbf{S}$ is the symplectic form. Global decoupling amounts to manipulations that result in $\det|\mathbf{C} + \overline{\mathbf{B}}| = 0$. Note that the sign of $\det|\mathbf{C} + \overline{\mathbf{B}}|$ is negative on sum resonances and positive on difference resonances [3].

For weak coupling, the largest elements of the matrices $\mathbf{B}$ and $\mathbf{C}$ can be more than an order of magnitude smaller than those of the matrices $\mathbf{A}$ and $\mathbf{D}$. We now consider a N-turn map and chose the turn number N such that the values of the sub-matrices $\mathbf{B}_N$ and $\mathbf{C}_N$ in the multi-turn map

$$\mathbf{M}^N = \begin{pmatrix} \mathbf{A}_N & \mathbf{B}_N \\ \mathbf{C}_N & \mathbf{D}_N \end{pmatrix}, \tag{4}$$

are maximized. With coupled motion energy is exchanged between the transverse planes with the beat frequency. To observe the maximum effect of the energy transfer from one plane to the other, one has to wait for half the beat period. This can be seen in Fig. 2 (a) and (b). The optimum N is thus approximately

$$N \approx \frac{1}{2|Q_A - Q_D|} = \frac{\pi}{|\mu_A - \mu_D|}. \tag{5}$$

The coupling information can also be obtained from the N-turn map since [5]

$$\mathbf{C} + \overline{\mathbf{B}} = (\mathbf{C}_N + \overline{\mathbf{B}}_N) \times \frac{\cos\mu_A - \cos\mu_D}{\cos(N\mu_A) - \cos(N\mu_D)}. \tag{6}$$

ΔQ_{min} can then be computed with Eq. (3). Based on the obtained value for $\mathbf{C} + \overline{\mathbf{B}}$, skew correctors can be set so that $\det|\mathbf{C} + \overline{\mathbf{B}}| = 0$ when the correctors are included. This will be shown in Sec. 4.

3 CONSTRUCTION OF A N-TURN MAP

We assume that m consecutive turns of a 4-vector $\vec{z} = (x, x', y, y')^T$ were fitted from turn-by-turn orbit data after a transverse kick. For weak coupling this can be done in a robust way, using a number of beam position monitors in an arc [5]. For the N-turn map one has

$$\vec{z}^{k+N} = \mathbf{M}^N \vec{z}^k. \quad (7)$$

To fit the matrix elements of $\mathbf{M}^N$ the function

$$\chi^2(\mathbf{M}^N) = \sum_{k=1}^{m-N} \sum_{i=1}^{4} \left(z_i^{k+N} - \sum_{j=1}^{4} M_{ij}^N z_j^k \right)^2 \quad (8)$$

is minimized. Introducing the two 4×4 matrices $\mathbf{S}^a$ and $\mathbf{S}^b$ with elements

$$S_{ij}^a = \sum_{k=1}^{m-N} z_i^{k+N} z_j^k \quad \text{and} \quad S_{ij}^b = \sum_{k=1}^{m-N} z_i^k z_j^k, \quad (9)$$

the minimization of $\chi^2(\mathbf{M}^N)$ leads to the direct solution

$$\mathbf{M}^N = \mathbf{S}^a (\mathbf{S}^b)^{-1} \quad (10)$$

In an implementation the condition $\det \mathbf{S}^b \neq 0$ needs to be checked, and the direct solution of Eq. (10) may not the best way to solve the problem numerically [6].

4 LINEAR COUPLING CORRECTION

We assume that the eigentunes $Q_{A,D}$ were obtained from a Fourier transform of turn-by-turn data, and the matrix

$$\mathbf{K} = \mathbf{C} + \overline{\mathbf{B}}, \quad (11)$$

from a N-turn map with Eq. (6).

For a correction algorithm we work in a coordinate system, in which the linear motion is represented by circles in phase space. The transformation into the new coordinate system

$$\tilde{\vec{z}} = \mathfrak{B} \vec{z} \quad (12)$$

is provided by the matrix

$$\mathfrak{B} = \begin{pmatrix} \mathfrak{B}_x & 0 \\ 0 & \mathfrak{B}_y \end{pmatrix} \text{ with } \mathfrak{B}_x = \begin{pmatrix} \beta_x^{-1/2} & 0 \\ \alpha_x \beta_x^{-1/2} & \beta_x^{1/2} \end{pmatrix} \quad (13)$$

and similar for $\mathfrak{B}_y$ [3]. The matrix $\mathfrak{B}$ is computed at the observation point. The matrix $\tilde{\mathbf{K}}$ can be written as

$$\tilde{\mathbf{K}} = \mathfrak{B}_y \mathbf{C} \mathfrak{B}_x^{-1} + \overline{\mathfrak{B}_x \mathbf{B} \mathfrak{B}_y^{-1}}. \quad (14)$$

We denote by μ_x^i the horizontal phase advance from the observation point to the skew quadrupole i, and use

$$\begin{aligned} S_x^i &= \sin \mu_x^i \quad , \quad C_x^i = \cos \mu_x^i, \\ S_x^{\bar{i}} &= \sin(\mu_x - \mu_x^i), \quad C_x^{\bar{i}} = \cos(\mu_x - \mu_x^i). \end{aligned} \quad (15)$$

In the new coordinate system we have for a number of weak skew quadrupoles [3]

$$\tilde{C} = \begin{pmatrix} \sum k_i \sqrt{\beta_x^i \beta_y^i}\, S_y^{\bar{i}} C_x^i & \sum k_i \sqrt{\beta_x^i \beta_y^i}\, S_y^{\bar{i}} S_x^i \\ \sum k_i \sqrt{\beta_x^i \beta_y^i}\, C_y^{\bar{i}} C_x^i & \sum k_i \sqrt{\beta_x^i \beta_y^i}\, C_y^{\bar{i}} S_x^i \end{pmatrix} \quad (16)$$

and a similar expression for $\tilde{B}$. The strength k_i is the inverse focal length f_i of skew quadrupole i. Assume a corrector family has the same skew corrector strength k_1 in all correctors and results in

$$\tilde{\mathbf{K}}^1 = k_1(\tilde{\mathbf{C}}_1 + \overline{\tilde{\mathbf{B}}}_1). \quad (17)$$

where the elements in $\tilde{\mathbf{C}}_1$ and $\tilde{\mathbf{B}}_1$ were divided by k_1. For a global coupling correction we want to minimize the quantity

$$\chi(k_1) = \det|\tilde{\mathbf{K}} + k_1 \tilde{\mathbf{K}}^1|. \quad (18)$$

It follows

$$k_1 = -\frac{\mathfrak{K}_1}{2 \det \tilde{\mathbf{K}}^1} \quad (19)$$

with

$$\mathfrak{K}_1 = \tilde{K}_{11}\tilde{K}_{22}^1 - \tilde{K}_{12}\tilde{K}_{21}^1 + \tilde{K}_{11}^1\tilde{K}_{22} - \tilde{K}_{12}^1\tilde{K}_{21}. \quad (20)$$

In principal two correctors or families are sufficient to correct linear coupling globally (unless their matrices $\tilde{\mathbf{K}}^1$ and $\tilde{\mathbf{K}}^2$ are linear dependent). After the first strength k_1 has been found, the second strength k_2 can be found with Eqs. (19) and (20), by replacing k_1 by k_2, $\tilde{\mathbf{K}}^1$ by $\tilde{\mathbf{K}}^2$, and $\tilde{\mathbf{K}}$ by $\tilde{\mathbf{K}} + k_1\tilde{\mathbf{K}}^1$.

5 APPLICATION AT RHIC

The above described algorithm for global linear coupling correction has been implemented within the RHIC injection optimization application. After beam is injected, turn-by-turn data are automatically acquired from 12 beam position monitors in the horizontal and 12 monitors in the vertical plane. 1024 turns are recorded in each of the beam position monitors while the beam exhibits injection oscillations. The application has access to an online machine optics model, and can read and set skew corrector strengths [7].

In a test the ΔQ_{min} computed from the N-turn maps was compared with the ΔQ_{min} obtained by bringing the tunes together [5]. Good agreement could be demonstrated over a ΔQ range sufficiently large to cover operation (both RHIC tunes are kept between 0.2 and 0.25).

As an example for a coupling measurement and correction we show the first test of the algorithm in operation, performed in the RHIC Blue ring with deuteron beam. In Fig. 2 (a) and (b) the beam oscillation in the horizontal and vertical plane following the injection are shown. The beating is clearly visible.

From the measured eigentunes, the optimum turn number N is determined with Eq. (5). We get $N = 67$ and the fitted N-turn map is

$$\mathbf{M}^{\mathbf{67}}_{\mathbf{before}} = \begin{pmatrix} -0.01 & 2.75 & -2.28 & -13.18 \\ -0.02 & 0.49 & -0.17 & -1.40 \\ -1.02 & 22.29 & 0.13 & 1.84 \\ 0.02 & -1.31 & -0.04 & -0.23 \end{pmatrix}. \quad (21)$$

from which $\Delta Q_{min} = 0.0064$ is obtained.

RHIC has three families for global decoupling, due to the six-fold symmetry of the machine. Two of those families are selected to minimize the coupling. The predicted ΔQ_{min} after correction is 0.0003 (see Fig. 2 (a)). A nonzero prediction is a sign of measurement errors in the N-turn map, or a mismatch between the optics model and the machine. The correction can be implemented by pressing a single button in the application.

The result of the coupling correction is shown in Figs. 2 (c) and (d). The recoherence after 650 turns is due to synchrotron motion and nonzero chromaticity. 650 turns are a synchrotron period. After correction, we have

$$\mathbf{M}^{\mathbf{122}}_{\mathbf{after}} = \begin{pmatrix} -0.26 & -11.77 & 3.62 & 4.49 \\ 0.01 & -0.40 & 0.06 & 0.29 \\ 0.19 & -5.62 & -0.17 & -1.23 \\ 0.05 & 2.65 & 0.22 & 0.30 \end{pmatrix} \quad (22)$$

and ΔQ_{min} is reduced to 0.0023. Note the reduction in the matrix elements M_{14} and M_{32}. The ΔQ_{min} reached by the correction was confirmed by a tune measurement after moving the tunes together. In operation it was found that the coupling correction cannot significantly improved beyond $\Delta Q_{min} = 0.002$. This is consistent with the $\Delta Q_{min} = 0.0011$ predicted for the next correction.

6 SUMMARY

A method for global coupling measurement and correction is presented that is based on N-turn maps fitted from turn-by-turn beam position data. N is chosen so as to maximize the signal-to-noise ratio of the coupling information. By using more than two monitors per plane the robustness is increased and the effect of random errors in beam position monitors is ameliorated. No tune change is needed for either the measurement or the correction. The method is implemented for operation as part of the injection optimization application at RHIC. It allows a global coupling correction within seconds after the turn-by-turn data are acquired. The method may be used in situations other than injection, when turn-by-turn data of free beam oscillations can be acquired.

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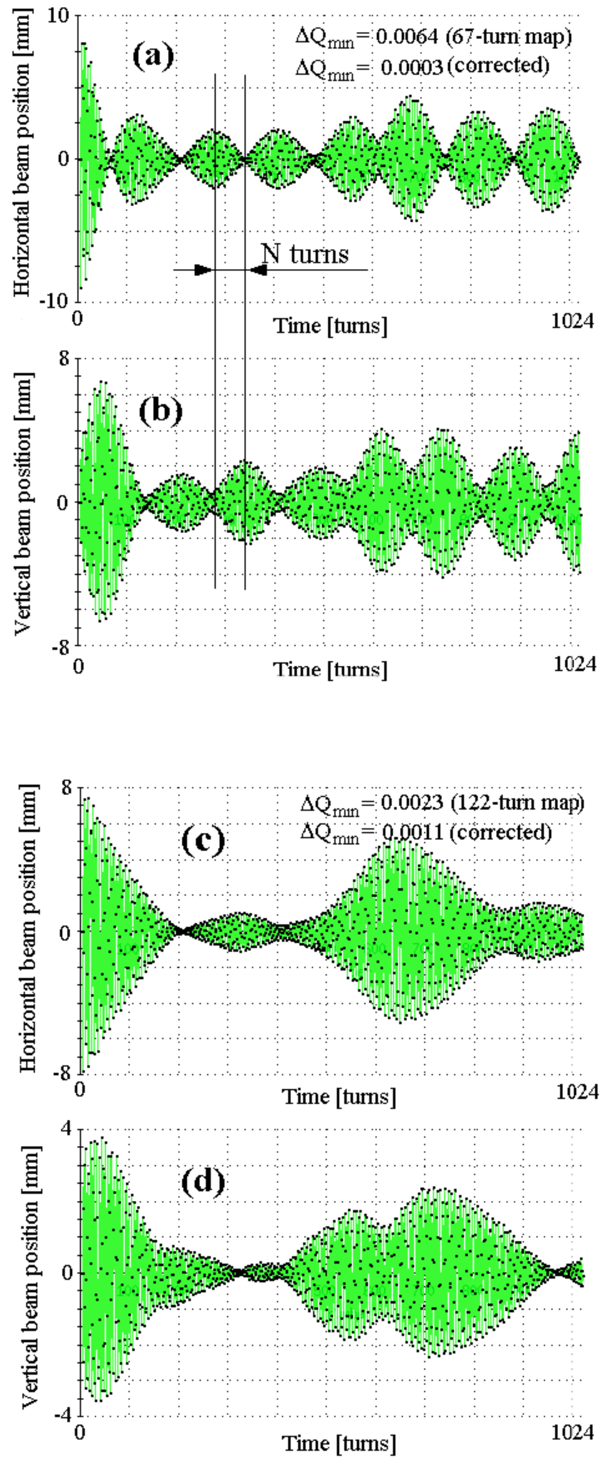

Figure 2: Turn-by-turn signals before and after a coupling correction at injection. In part (a) and (b) the horizontal and vertical injection oscillations are shown before a coupling correction. Part (a) also shows the computed ΔQ_{min} and the predicted ΔQ_{min} after a coupling correction. Parts (c) and (d) show the situation after the computed corrector values were applied.