LINEAR MODEL FOR NON-ISOSCELES ABSORBERS

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Abstract

Previous analyses have assumed that wedge absorbers are triangularly shaped with equal angles for the two faces. In this case, to linear order, the energy loss depends only on the position in the direction of the face tilt, and is independent of the incoming angle. One can instead construct an absorber with entrance and exit faces facing rather general directions. In this case, the energy loss can depend on both the position and the angle of the particle in question. This paper demonstrates that and computes the effect to linear order.

INTRODUCTION

Ionization cooling can be achieved in the transverse direction with a straight cooling channel. However, in the longitudinal direction, one generally gets heating. Energy straggling leads to further heating in the longitudinal plane. To achieve 6-D cooling, one must couple the transverse motion with longitudinal motion. One method to achieve this is to use a triangular cross-section absorber in a location with dispersion. Particles with higher energy then go through a larger length of absorber and lose more energy, thus reducing the energy spread. Unfortunately, this occurs at the cost of an increase in transverse beam size [1]. This process is often referred to as "emittance exchange."

Existing computations have only considered triangular wedges with equal face tilts. The entrance and exit faces of the absorber can be tilted rather generally. This will give an energy loss dependence on transverse coordinates which is different from what occurs when the face tilts are equal and in the same plane. This paper calculates the linear transfer matrix for such a wedge absorber.

First, the path length in the absorber is calculated for general face angles. The computation is first done in the case where the faces are tilted in the same plane, to give a more intuitive picture of what is going on, followed by formulas for more general face tilts. This calculation is then used to find the transfer matrix through the absorber. Finally, possible uses of more general face angles are discussed.

GEOMETRIC LENGTH CALCULATION

The energy loss (ignoring stochastic effects) in the absorber is proportional to the distance that the particle travels through the absorber. Thus, to calculate the effect of the absorber, we will calculate the length of the particle trajectory that is inside the absorber. We further assume that the particle trajectory is straight: i.e., there are no electromagnetic fields acting on the particle. First, we compute a case where the plane in which the absorber faces are tilted from vertical is the same for both cases. This becomes a one-dimensional problem, and helps give an understanding of what is going on. We then do the computation for a more general case.

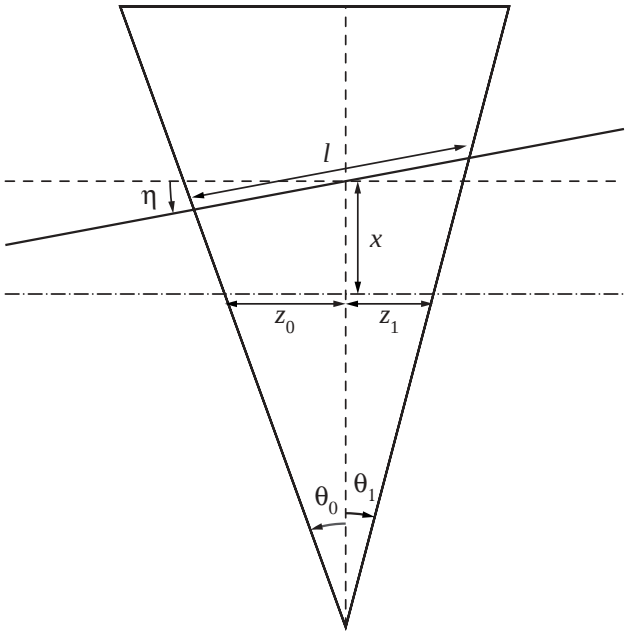

Figure 1: Planar absorber geometry.

Fig. 1 gives the parameters for the one-dimensional problem. The length of the path inside the absorber is

$$l = \left(\frac{L}{\tan\theta_0 + \tan\theta_1} + x\right) \frac{\cos\eta \sin(\theta_0 + \theta_1)}{\cos(\eta - \theta_0)\cos(\eta + \theta_1)}, \quad (1)$$

where $L = z_1 + z_2$. To linear order in x and η, this is

$$l = L + x(\tan\theta_0 + \tan\theta_1) + \eta L(\tan\theta_1 - \tan\theta_0) \quad (2)$$

If $\theta_0 = \theta_1$ (i.e., the absorber cross-section is an isosceles triangle), to linear order, the path length (and thus the energy loss) does not depend on the incoming particle angle, but does depend on the transverse position. This is the situation that has been analyzed in the past. On the other hand, if $\theta_0 = -\theta_1$ (i.e., the faces are parallel but the absorber is tilted), the path length does not depend on the incoming particle position, but it does depend on the incoming particle angle.

In the more general situation, we describe the absorber by its entrance and exit planes. We describe these planes as passing through a point $\boldsymbol{p}_i$ and having a unit normal $\boldsymbol{u}_i$, where $i = 0$ for the entrance plane and $i = 1$ for the exit

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plane. The particle trajectory is described as line passing through a point $\boldsymbol{x}_0$ with a unit tangent vector $\boldsymbol{t}$. We can calculate the path length within the absorber in terms of these coordinates and vectors as

$$l = \frac{(\boldsymbol{p}_1 - \boldsymbol{x}_0)\cdot\boldsymbol{u}_1}{\boldsymbol{t}\cdot\boldsymbol{u}_1} - \frac{(\boldsymbol{p}_0 - \boldsymbol{x}_0)\cdot\boldsymbol{u}_0}{\boldsymbol{t}\cdot\boldsymbol{u}_0} \tag{3}$$

We can compute this length to linear order. The coordinates of these vectors are (see Fig. 1):

$$\boldsymbol{x}_0 = (x, y, 0) \tag{4}$$

$$\boldsymbol{t} = \left(\frac{p_x}{p}, \frac{p_y}{p}, \frac{\sqrt{p^2 - p_x^2 - p_y^2}}{p}\right) \tag{5}$$

$$\boldsymbol{p}_0 = (0, 0, -z_0) \tag{6}$$

$$\boldsymbol{p}_1 = (0, 0, z_1) \tag{7}$$

$$\boldsymbol{u}_0 = (\sin\theta_0\cos\phi_0, \sin\theta_0\sin\phi_0, \cos\theta_0) \tag{8}$$

$$\boldsymbol{u}_1 = (-\sin\theta_1\cos\phi_1, -\sin\theta_1\sin\phi_1, \cos\theta_1). \tag{9}$$

TRANSFER MATRIX

One can easily compute the transfer matrix to lowest order in the relative energy loss in the absorber. In this case, only the path length in the absorber matters. First, compute the evolution of the transverse momenta, as well as the evolution of the energy deviation ignoring the face angles. The equations of motion are

$$\frac{d\boldsymbol{p}_\perp}{ds} = -\kappa_\perp \boldsymbol{p}_\perp \qquad \frac{d\delta}{ds} = -\kappa_\parallel \delta \tag{10}$$

$$\kappa_\perp = \frac{1}{\beta pc}\frac{dE}{dx} \qquad \kappa_\parallel = \frac{d}{dE}\left(\frac{dE}{dx}\right), \tag{11}$$

and their solution is

$$\boldsymbol{p}_\perp(s) = \boldsymbol{p}_\perp(s_0)e^{-\kappa_\perp(s-s_0)} \tag{12}$$

$$\delta(s) = \delta(s_0)e^{-\kappa_\parallel(s-s_0)}. \tag{13}$$

Imagine a sequence of planes with normal $\boldsymbol{u}(z)$ and passing through the point $(0, 0, z)$. Using Eq. (3), the integrated path length to linear order in $\boldsymbol{x} = (x, y)$ and $\boldsymbol{p}_\perp/p$ (where the $\perp$ subscript refers to the transverse coordinates) from an arbitrary point to the plane at z is a constant plus

$$z - \boldsymbol{x}\cdot\frac{\boldsymbol{u}_\perp}{u_z} - z\frac{\boldsymbol{p}_\perp}{p}\cdot\frac{\boldsymbol{u}_\perp}{u_z}. \tag{14}$$

We will parameterize $\boldsymbol{u}$ by η according to

$$\boldsymbol{u} = \frac{\boldsymbol{u}_0\sin(\xi-\eta) + \boldsymbol{u}_1\sin\eta}{\sin\xi} \quad \boldsymbol{u}_0\cdot\boldsymbol{u}_1 = \cos\xi. \tag{15}$$

η will vary from 0 to ξ while z varies from $-z_0$ to z_1. The definition of the exact relationship between η and z will be left to later. We then have

$$\frac{dl}{dz} = 1 - \frac{\boldsymbol{p}_\perp}{p}\cdot\frac{\boldsymbol{u}_\perp}{u_z} - \left(\boldsymbol{x} + z\frac{\boldsymbol{p}_\perp}{p}\right)\cdot\frac{u_{z0}\boldsymbol{u}_{\perp1} - u_{z1}\boldsymbol{u}_{\perp0}}{u_z^2\sin\xi}\frac{d\eta}{dz}. \tag{16}$$

Now define

$$\frac{d\eta}{dz} = ku_z^2 \tag{17}$$

for some constant k. Then

$$\frac{d}{dz}\left(\frac{\boldsymbol{u}_\perp}{u_z}\right) = k\frac{u_{z0}\boldsymbol{u}_{\perp1} - u_{z1}\boldsymbol{u}_{\perp0}}{\sin\xi} \tag{18}$$

meaning that

$$\frac{\boldsymbol{u}_\perp}{u_z} = k\frac{u_{z0}\boldsymbol{u}_{\perp1} - u_{z1}\boldsymbol{u}_{\perp0}}{\sin\xi}z + \boldsymbol{c} \tag{19}$$

for a constant vector $\boldsymbol{c}$. Applying the known boundary conditions,

$$\frac{\boldsymbol{u}_\perp}{u_z} = \frac{\boldsymbol{u}_{\perp0}}{u_{z0}}\frac{z_1 - z}{L} + \frac{\boldsymbol{u}_{\perp1}}{u_{z1}}\frac{z + z_0}{L}. \tag{20}$$

We can then write

$$\frac{dl}{dz} = 1 - \frac{\boldsymbol{x}}{L}\cdot\left(\frac{\boldsymbol{u}_{\perp1}}{u_{z1}} - \frac{\boldsymbol{u}_{\perp0}}{u_{z0}}\right) - \frac{\boldsymbol{p}_\perp}{p}\cdot\left[\frac{z_1\boldsymbol{u}_{\perp0}}{Lu_{z0}} + \frac{z_0\boldsymbol{u}_{\perp1}}{Lu_{z1}} + \frac{2z}{L}\left(\frac{\boldsymbol{u}_{\perp1}}{u_{z1}} - \frac{\boldsymbol{u}_{\perp0}}{u_{z0}}\right)\right] \tag{21}$$

Now, use Eq. (12), giving

$$l = L - \boldsymbol{x}\cdot\left(\frac{\boldsymbol{u}_{\perp1}}{u_{z1}} - \frac{\boldsymbol{u}_{\perp0}}{u_{z0}}\right) - \frac{\boldsymbol{p}_{\perp0}}{p\kappa_\perp L}\cdot\Bigg[2\left(\frac{1 - e^{-\kappa_\perp L}}{\kappa_\perp} - z_0 - z_1e^{-\kappa_\perp L}\right)\left(\frac{\boldsymbol{u}_{\perp1}}{u_{z1}} - \frac{\boldsymbol{u}_{\perp0}}{u_{z0}}\right) + \left(\frac{z_1\boldsymbol{u}_{\perp0}}{u_{z0}} + \frac{z_0\boldsymbol{u}_{\perp1}}{u_{z1}}\right)(1 - e^{-\kappa_\perp L})\Bigg]. \tag{22}$$

The change in δ is simply $\kappa_\perp l$, and one can then directly read off the matrix elements.

DISCUSSION

First of all, adjusting the absorber geometry simply to keep the sum of the tangents of the face angles constant will not leave the performance of a cooling channel invariant, unless there happens to be no angular dispersion at the location of the absorber. In fact, at a point where there is both angular and positional dispersion, one may get improved performance by adjusting the face angles separately.

One could even consider a lattice with only angular dispersion and no positional dispersion at the absorbers. This could not be easily done in a ring (if one bends in the same direction all the time, one tends to have nonzero positional dispersion), but could be done in a "snaking" configuration where subsequent cells bend in opposite directions, and thus the lattice is straight over larger scales.

There are several reasons one might want to do this. First of all, a lattice that does not form a ring allows one to adiabatically vary lattice parameters, thus maximizing the cooling performance as the beam changes. One may be especially interested in doing this for a collider to maximize the

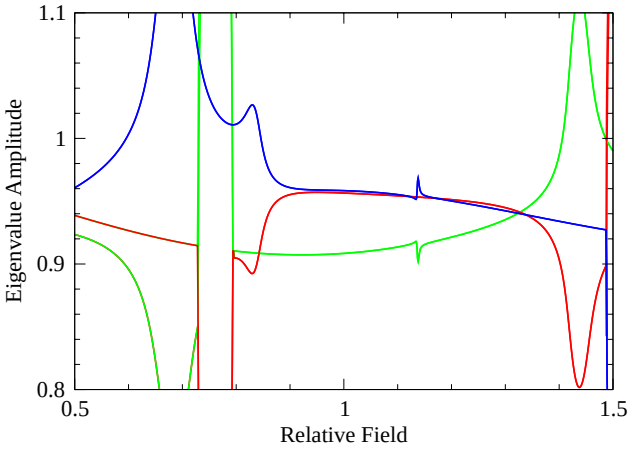

Figure 2: Super-FOFO lattice with bends all in same direction and isosceles wedges. Absolute value of eigenvalues plotted versus field relative to a reference field.

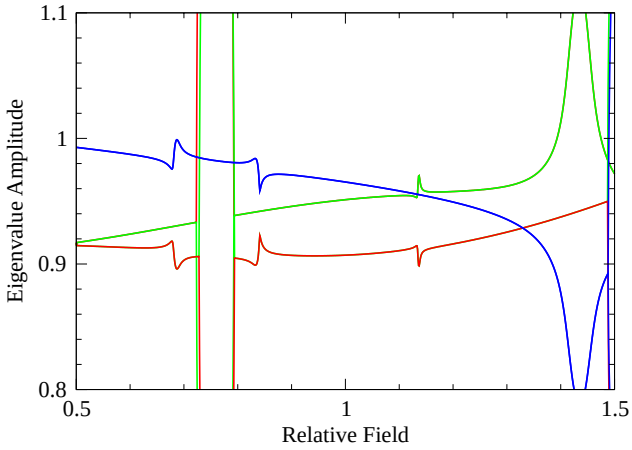

Figure 3: Super-FOFO lattice where bending in one cell is opposite to the bending in the next, and the absorbers are tilted slabs. Plots are as in Fig. 2.

luminosity one achieves. In addition, one avoids the difficulties with injection and extraction.

These considerations apply to any lattice that does not form a ring. The advantage of having only angular dispersion versus positional dispersion at the absorber may lie in the effect of energy straggling. When the energy changes in energy straggling, the betatron amplitude will change since the closed orbit changes. Since the beta function at the absorber is small (whereas the dispersion is not necessarily), energy straggling with positional dispersion will lead to large betatron amplitude changes relative to the beam size. If instead there is angular dispersion at the absorber, energy straggling leads to smaller relative betatron amplitude changes due to the large angular spread at that point. This has the potential to substantially improve the performance of these cooling lattices. This has not been tested in real lattices at this point.

Example

As an example, consider a "Super-FOFO" lattice [2], modified by adding bending as in [3]. A similar lattice has been proposed for achieving 6-D cooling, and shows excellent performance [4, 5]. Fig. 2 shows the absolute value of the eigenvalues as a function of the field strengths for standard isosceles wedges and a lattice where all bends bend in the same direction (giving dispersion at the absorber). This is equivalent to considering the dependence of the eigenvalues on the reference momentum. If the absolute value of all the eigenvalue is less than 1, then the beam will be cooled in all planes. One is able to achieve 6-D cooling over a rather large range of reference momenta.

Figure 3 shows the eigenvalues for a tilted slab in a lattice that has angular dispersion at the absorber. Note that in this case as well, one is able to achieve 6-D cooling over a rather large range of reference momenta. Also note that the wedge angles are steeper than those required for the case with conventional wedge absorbers.

CONCLUSIONS

The energy loss in an absorber with generally placed planar faces has been calculated to linear order in the transverse coordinates. This allows one to calculate eigenvalues for a cooling channel with these rather general wedges. An example was constructed where a cooling channel was constructed with angular dispersion at the absorbers, and parallel-face tilted absorbers were used. Linear performance (without multiple scattering) of that cooling channel was shown to be comparable to that of a channel constructed with more conventional wedges. Such a channel may have multiple scattering performance that is better than a wedge-based 6-D cooling channel.

Further work should incorporate general face orientations into simulation codes such as ICOOL [6]. One can then simulate and optimize proposed 6-D cooling lattices by orienting absorber faces more generally.

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