# TRANSMISSION OF MUONS IN AN ALTERNATING GRADIENT FUNNELING SYSTEM

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*Abstract*

One important issue in a neutrino factory is the target system for pion production. The A.G. funneling system addressed here consists of four horns, with 1 MW target per horn, followed by a recombination A.G. channel and a FODO decay channel. The transmission of this scheme is analyzed in detail. It is compared with that of a simple solenoid for several sets of optics parameters. This study makes it possible to gain in transmission efficiency in comparison with earlier proposals.

## INTRODUCTION

This paper reports on optimization studies concerning a muon production channel based on alternating gradient focusing. The system has been proposed some years ago [1] and underwent several design changes to improve the collection efficiency.

The latter is defined for a given beam emittance as the ratio between the number of muons that make it to the end of the channel in that emittance and the number of protons on the target.

We first describe the channel which is an evolution of the scheme proposed in [1]. Then the efficiency optimisation is described and compared with standard schemes using high field solenoids.

## AG CHANNEL OPTICS

The system features an ensemble of four horns placed close to each other and an alternating gradient matching section which steers the four beams into the FODO channel. The advantages of this system have been described in [2] : the life of the horn is lengthened and the power on the target is reduced by four, which makes the target feasible according to studies done for spallation sources.

The optics functions in the matching section, computed with MAD [3], are shown on fig. 1. The curve labelled $x$ represents the trajectory of the beam centroid. At the exit of the horn (top left in fig. 1), this trajectory makes an angle of about 0.1 rad with the axis of the system. This axis is common both to the matching section and the decay channel. Consequently the primary beam with an energy much higher than that of the pions goes practically straight and can be extracted in a dump placed at the entrance of the FODO channel.

The optics functions in the latter are a periodic continuation of the last oscillations shown on the right of fig. 1.

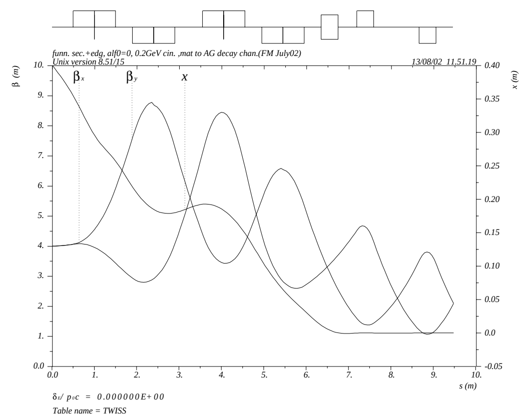


Figure 1: Optics functions of in the funneling section.

An important feature of the matching section is that the dispersion (not shown on fig. 1) is matched to zero at its end, i.e. it constitutes a first order achromat. The maximum amplitude of the dispersion is of the order of 10 cm.

The edge fields of the quadrupoles have been taken into account in the matching. The angle $\theta$ the beam centroid trajectory makes with the ends of the quadrupoles at position $x$ in the horizontal plane introduces a thin quadrupole of focal length $\theta x k$ in the vertical plane, $k$ being the normalised gradient of the quadrupole. Consequently the MAD matching procedure has to be iterative as a change of the gradients causes a change of the trajectory.

It has been checked with a numerical integration of the trajectory, using the code ZGOUBI [4], that this approach is accurate enough. The maximum residual amplitude of the centroid trajectory in the FODO channel obtained this way, is of the order of 1mm (instead of zero as on fig. 1), i.e. negligible compared with the apertures of the system.

The efficiency of muon transmission has been estimated by tracking trajectories also with ZGOUBI (detailed examples can be found in ref. [2]).

Aperture limitations along the FODO decay channel cause only weak pion losses if the aperture radius is larger than about 0.3 m. Muons losses concern predominantly low energy particles [5] ; because of the emittance induced by the pion decay, the aperture radius has to be at least 0.4 m in order to minimize the muon losses. In the matching section the aperture has to be 0.6 m at entrance and can be reduced at exit.

It is important to recall that these aperture radii are associated with an emittance of 0.01 $\pi$ m. Should a larger emittance be transmitted, apertures would have to scale with the square root of the emittance. This shows clearly the limits of the system which cannot be used for a muon collider in its present status.

## PION DISTRIBUTION

The efficiency of the channel has been assessed by tracking the trajectories of 40000 pions pertaining to a realistic distribution [6]. The coordinates of these pions were obtained by tracking $10^6$ protons on the target with the simulation program MARS [7].

The pion distribution in energy at the exit of the horn is shown on fig. 2. The pions are considered to start all at the same time as the time dilation of the muon beam is mainly due to the speed distribution and the length of the decay channel. The distribution in the transverse horizontal phase space (for the vertical plane it is similar) is shown on fig. 3. This beam has been tracked through both the AG channel and a solenoid channel for the sake of comparison. Both systems have the same 0.4 m radius aperture restriction (apart from the AG channel matching section that must have a larger aperture as stressed earlier).

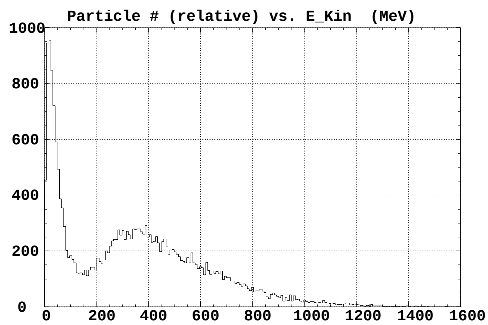


Figure 2: Energy distribution of the pions at the exit of the horn (beginning of the decay channel).

## EFFICIENCY

The collection efficiency of the AG channel has to be defined for a given 6D emittance, because the muons have to be accelerated and stored in machines with a given acceptance. The efficiency is defined as the ratio between the number of muons counted at the exit of the channel and the total number of muons at its entrance, i.e. 40000 in our case. In fact this concept is less clear than it seems because of the large energy spread in the beam. For this reason it cannot be admitted that the optics of the subsequent machine vary little with the momentum deviation in the beam. Nevertheless the particles are counted in ellipses of a given surface, referred to as emittance, in each plane, which implies that there is a sort of "global matching" such that all particles in the ellipse will stay in its transforms when the beam continues downstream whatever their energy.

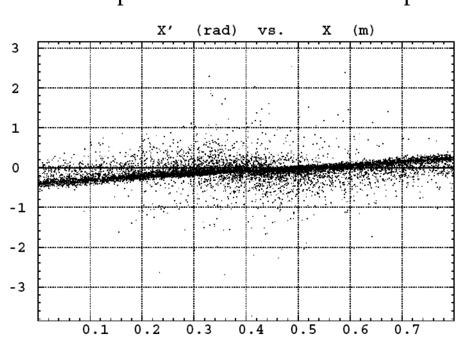


Figure 3: Distribution of the pions in the transverse phase space at the exit of the horn (beginning of the decay channel). The distribution is tilted ($\alpha \neq 0$).

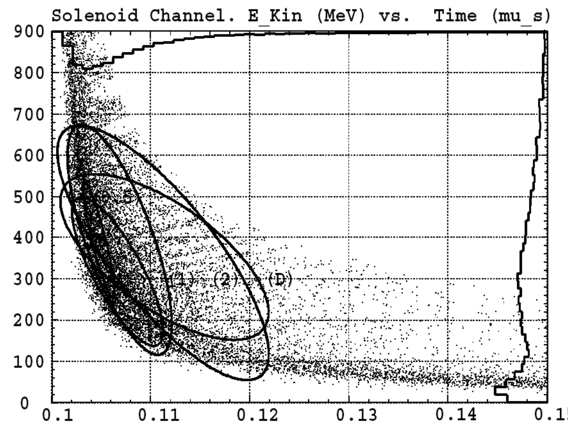


Figure 4: Matching optimum ellipses with surfaces $0.1,\ 0.5,\ 1$ and 2 $\pi$ eV.s for the case of the solenoid channel. (D) is the from the correlation matrix.

In the longitudinal plane there is some difficulty to find the optimum ellipse. If the optics functions are computed with the correlation matrix, it does not provide the optimum because of the shape of the longitudinal distribution which is far from elliptical. This is particularly true for the case of the solenoid where large distribution tails extend along each coordinate axis. Consequently the strategy consists of finding, for a given acceptance $\epsilon_l/\pi$ , the position and form that yield the largest number of particles within the frontier. This is performed with a matching procedure, with variables the ellipse parameters and its position, and with sole constraint the maximum counting within the ellipse. Depending on the surface, the so determined optimum ellipse will have a different shape, position and orientation as shown on fig. 4.

A similar procedure is applied to match the transverse admittance at channel exit, given a limit acceptance $\epsilon_{x,z}/\pi$.

The variation of the efficiencies of both the AG and the solenoid channel with the longitudinal acceptance are shown on fig. 5 for the case where the funneling section is matched for an upright ellipse ($\alpha$ =0 in both planes at the entrance of the section). Two ranges of field values are considered in the AG magnets : either regular values as available from conventional warm technology, or twice that field integral which has the merit of giving optimised transmission. The AG channel shows in a general manner better transmission than the solenoid (the field value in latter has also been optimised for maximum transmission). We notice that the efficiency varies only slowly with the longitudinal emittance when the latter is larger than 1 eV.s. This is a practical optimum longitudinal emittance which has to be considered for subsequent acceleration, whether there is cooling or not.

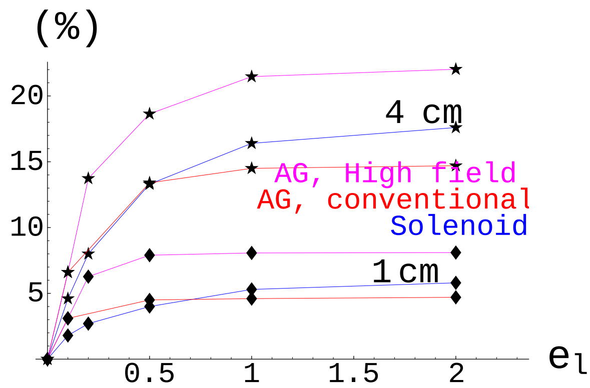


Figure 5: Efficiency of the AG channel versus longitudinal emittance $\epsilon_l/\pi$ (eV.s) for values of transverse emittance $\epsilon_{x,z}/\pi = 1$ cm and $4$ cm. The solenoid case is also shown for comparison.

## PARAMETRIC STUDIES

The variation of two parameters has been investigated, namely the value of $\alpha$ at the entrance of the funneling section and the value of the central energy of the particles.

It is clear on fig. 3 that the phase space ellipse is not upright. From an ellipse fit, we can infer that $\alpha$ is about -2. The funneling section has been re-matched accordingly for a value of $\alpha$ of -1. The transmission is then worse. Then it has been re-matched for a value of $\alpha$ of +0.5 and the transmission is better. In order to understand this it is important to keep in mind that the distribution shown on fig. 3 concerns particles with a large momentum spread. Therefore it is not obvious to know from this distribution to which value of $\alpha$ the section has to be matched.

A similar remark applies to the value of the reference momentum of the distribution for which the matching is done. The average momentum of the particles varies along the line as shown on fig. 6. There seems to be a tendency that the transmission efficiency is improved when

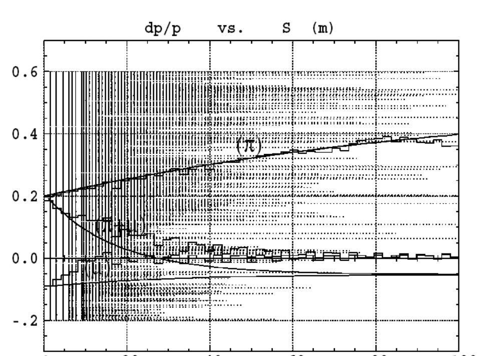


Figure 6: Average momentum of the $\pi$, $\mu$ and $\pi+\mu$ beams as a function of their position along the decay channel, from Monte Carlo. Solid lines are theoretical, in the loss-free case [5].

the quadrupole gradients follow the average muon momentum, so that the beam experiences a constant focusing. This is the subject of studies being carried on presently.

## CONCLUSION

The transmission efficiency of the AG funneling system has been evaluated for a realistic pion distribution and their subsequent decay muons. For small transverse emittance of about 0.01$\pi$rad.m, the AG funneling channel is more efficient than the pure solenoid decay channel with a transmission of about 5% for a longitudinal emittance of about 1 eV.s. As there is about 0.04 pion per p.o.t., this system produces $2.2\times10^{13}$ muons per second for 1 MW on each of the four targets (a total of $1.1\times10^{16}$p/s). This is below the nominal parameters of the neutrino factory only by a factor of four.

Maximised transmission in the AG is obtained with about twice higher magnet field integrals, which may raise such concern as larger betatron functions detrimental to decay induced transverse emittance increase, or as the feasibility of higher field magnets, issues still to be investigated further.

This system can then be considered as a doable proposal if the subsequent acceleration system can accommodate the emittances.

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