MUON ACCELERATION IN FFAG RINGS

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Abstract

Compared to muon acceleration in re-circulating linear accelerators, considered earlier, muon acceleration in FFAG rings holds the promise of having one arc instead of several, and a smaller number of RF cavities; i.e., using more than a few turns for acceleration from about 6 to about 20 GeV/c. We consider non-scaling FFAG machines (but with limited tune variation) that are essentially strong-focusing rings with a dispersion small enough to keep muons over the full momentum range inside the same magnet aperture. We compare several scenarios: (i) Rings with straight sections, long enough - a few metres - to house super-conducting RF cavities at about 200 MHz (and with enough space for decay of the magnetic fields in neighbouring components to a level of about 10 mGauss; (ii) Rings with shorter straight sections, just long enough about a metre to house normal-conducting RF cavities also operating at about 200 MHz; (iii) Racetrack-shaped rings with compact arcs without RF cavities, joined to long straight sections with super-conducting RF cavities by adiabatic transition sections that match between them over the wide momentum range needed; (iv) Rings with a number of superperiods (about 10) so that there are only a number of sc RF sections. In all four scenarios we consider the travel time of the muons around the ring which depends on the muon momentum and, hence, produces variable phase at the RF cavities during the acceleration process.

INTRODUCTION

Neutrino factory studies [1, 2, 3], based on accelerating and storing muons, assumed that the muons are accelerated in a linear accelerator up to a few GeV, and in one or two recirculating linear accelerators RLA similar to CEBAF to the energy of the storage ring, 20 to 50 GeV. In an RLA, the muons travel through two linear accelerators several times, and through half-circular arcs, one for each recirculation, at either end. Between the linear accelerators and the arcs are spreaders, which distribute the muons of different energy from the linear accelerators into the arcs, and combiners, which do the opposite. Spreaders and combiners become rapidly more complicated and costly when the number of recirculations increases. It is 4 or 5 in [1, 2, 3].

Non-Scaling FFAG Rings

In FFAG rings, muons are accelerated by a factor ≈ 3 in a lattice with a single aperture. There are no spreaders and combiners. Such lattices hold the promise of a larger number of recirculations, and hence of reduced cost. We discuss how muons are accelerated in an FFAG ring, and present several designs of non-scaling FFAG rings. The phase advances μ_x and μ_y in a cell vary approximately in inverse proportion to the particle momentum. They should remain below the half-integral resonance at the lower edge of the operating range in $\delta p/p$. The tunes Q_x and Q_y in the whole ring cover many integral and half-integral values.

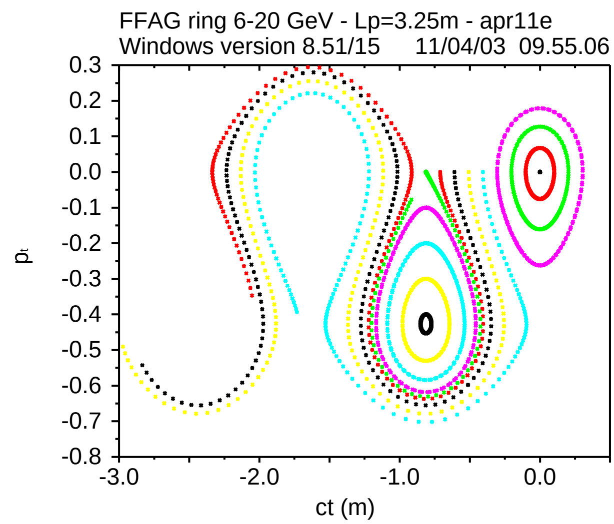

Figure 1: Longitudinal phase space trajectories of 13 particles, tracked for 30 turns, cf. Section .

FFAG Acceleration

Fig. 1 shows trajectories in longitudinal phase space $(ct, p_t = \Delta p/p_0)$ for the lattice in Section . Nine particles are launched at regular intervals between the fixed points at $ct = 0$ m and $p_t = 0$, and $ct = -3.25/4$ m and $p_t = 0$. Four of them execute synchrotron oscillations in buckets around the stable fixed point. Another four particles follow wavy open trajectories around the two stable fixed points. Acceleration happens close to these trajectories, which cover the whole operating range from $p_t \approx -0.6$ to $p_t \approx 0.2$. The green particle starts very close to the unstable fixed point and almost returns to it. Four particles are launched below the unstable fixed point at $ct = -3.25/4$ m with $p_t = -0.1, -0.2, -0.3, -0.4$. Their initial transverse coordinates coincide with their respective closed orbit. These particles execute stable synchrotron oscillations in buckets around a second stable fixed point at $ct = -3.25/4$ m and $p_t \approx -0.4$. This stable fixed point is related to the variation of the travel time $\Delta t(p_t)$ of second and higher order [4] in p_t which results in $\Delta t(p_t) \approx 0$ s at

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$p_t \approx -0.4$. Phase space trajectories with the shape shown in Fig. 1 only occur when V the maximum energy gain per turn of a muon that travels on the crest of the RF wave, exceeds the threshold in Tab. 1. To get the peak RF voltage divide by the transit time factor < 1. Our typical RF voltage is about twice the threshold value.

SCENARIO 1: S.C. RF

The lattice [5] has 3 m long straight sections, long enough to house a single-cell super-conducting RF cavity at about 200 MHz, including the distances needed to protect it from stray field of neighbouring magnetic elements. We have re-evaluated the parameters shown in Tab. 1.

SCENARIO 2: N.C. RF

Two lattice styles have straight sections long enough for a single-cell room-temperature RF cavity at ≈ 200 MHz.

Modified FODO Lattice

The modified FODO lattice is derived from [5] by halving the length of the lattice cell. Horizontal focusing happens in the quadrupole, while bending and vertical focusing happen in a combined-function dipole magnet. The 1.4 m long drift space is still long enough for normal-conducting RF cavities operating at about 200 MHz. Tab. 1 shows parameters, Fig. 2 layout and optical functions.

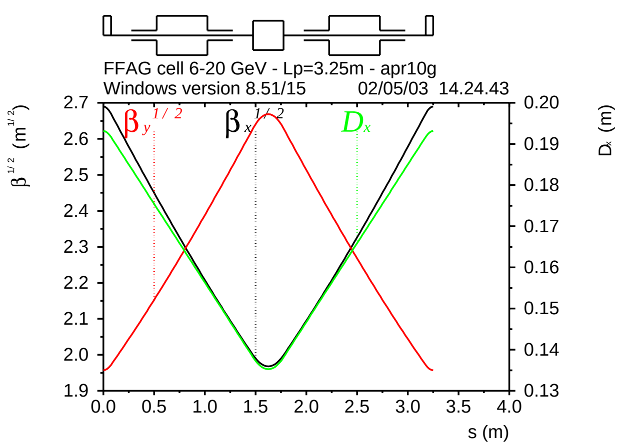

Figure 2: Layout and optical functions of cell apr10g

Modified Achromat

Achromats, used in the study of ELFE [6] and the RLA design at CERN, consist of 2 FODO cells for focusing, and have dipoles in the half cells surrounding every second F quadrupole. The horizontal dispersion D_x vanishes in every second F quadrupole. Our modified achromats for FFAG acceleration have the opposite quadrupole polarities, and $D_x > 0$ everywhere. The half cells without dipoles provide space for normal-conducting RF cavities. Fig. 3 shows layout and optical functions, Tab. 1 parameters. The second zero of ct is at $p_t \approx -0.55$. The half-integral stopband is at $p_t \approx -0.52$. Hence, the modified achromat accelerates from $p_t \approx -0.5$ to ≈ 0.2 inside buckets, when RF voltage and bucket height are large enough. Changing parameters should move the stopband below the second zero, and permit acceleration over a larger momentum range.

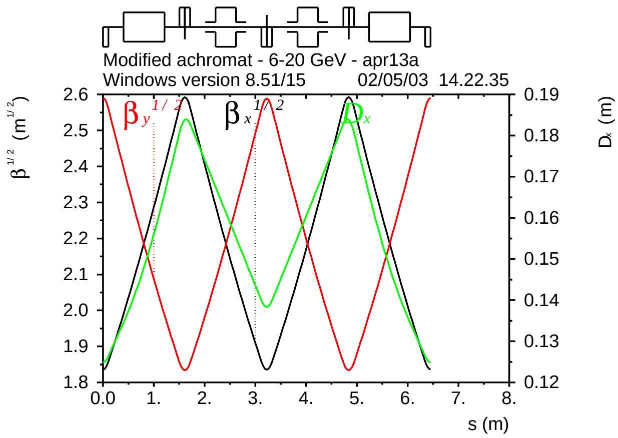

Figure 3: Layout and optical functions of period apr13a

SCENARIO 3: RACETRACK

Racetrack rings consist of closely-packed arcs that bend the muon, and transport it from one straight section, with ample space for accelerating RF cavities, to the other. Between arcs and straight sections are adiabatic transitions.

Adiabatic Transitions

Adiabatic transitions match the orbit functions between the arcs and straight sections over the whole energy range of the FFAG ring. This is achieved by smoothly varying the period length and the bending angle between the typical arc and straight section cells. We use the Fermi function $F(n, B, N) = [1 + \exp(B(2n/N - 1))]^{-1}$ with number of transition periods N, running index n, and parameter $B \approx 2 \ldots 5$ describing the smoothness. The factor 2 in F ensures that the transition happens near $n = N/2$. For $B > 1$ but not $B \gg 1$, we have $F(0, B, N) < 1$ and $F(N, B, N) > 0$. We overcome this problem by using

$$G(n, B, N) = \frac{F(n, B, N) - F(N, B, N)}{F(0, B, N) - F(N, B, N)} \tag{1}$$

with $G(0, B, N) = 1$ and $G(N, B, N) = 0$. In the n-th FODO cell, length L_n, dispersion D_n, and bending angle/period $\varphi_n = \varphi_1 D_n / L_n$ become, with indices 1 and 2 referring to arc and straight section cells:

$$L_n = L_2 + (L_1 - L_2) G(n, B_\beta, N) \tag{2}$$

$$D_n = D_1 G(n, B_D, N) \tag{3}$$

Racetrack Lattice

We study adiabatic transitions in a lattice that consists of drift spaces and thin elements. The FODO arc periods have

Table 1: Parameters of Johnstone-Koscielniak [5] lattice apr09r, modified Johnstone lattice apr10g, modified achromats apr13a and racetrack lattice apr29o. The phase advances are about 0.1 in apr09r, apr10g, and apr29o, and 0.2 in achr06.

	apr09r	apr10g	apr13a	apr29o	
Total reference energy E	16.5	16	16.5	16.5	GeV
Energy range	6…20	6…20	6…20	6…20	GeV
Range of $\delta p/p$	−.636…+.212	−.625…+.250	−.636…+.212	−.636…+.212	
Range of hor. offsets in F quads x	−76…79	−45…50	−54…46	−137/144	mm
Period length L_p	6.5	3.25	6.449	5…10	m
Number of periods N_p	314	280	140	264	
Circumference C	2041	910	902.8	1610	m
Magnetic gradients G_F/G_D	75/−32	141/−71	1.15/−1.15	–	T/m
Dipole field B	3.1	4.0	1.53	–	T
Path length spread	535	410	397	1685	mm
RF frequency	184.5	184.5	185.9	186.2	MHz
Threshold/typical circumf. RF accel. V	1036/1884	812/1680	–/1680	–/	MV
Typical number of turns	10	10	11		

super-imposed quadrupoles and dipoles, and $L_1 = 5$ m, $\varphi_1 = 31.4$ mr and phase advance $\mu/2\pi = 0.1$. The straight section cells have $L_2 = 10$ m and $\mu/2\pi = 0.1$. An arc surrounded by transitions bends the muon beam by about π. We use a 112.5 m long transition with $N = 16$ and $B = 3$. Four such transitions are in a racetrack machine. The number of cells in a straight section, 14, will be adjusted later to optimize number and voltage of the RF cavities.

Without acceleration, the residual non-adiabaticity causes oscillations of D_x, β_x and β_y, and stopbands. In the straight sections at $p_t = 0$, the amplitude of the D_x oscillations increases with decreasing N. For fixed N, it has minima near $B = 3$. At $N = 16$ and $B = 3$, $|D_x| < 0.1$ m. The β-beating is less than 7.5% for $N \geq 12$. Both $|D_x|$ and $|x| \to \infty$, and momentum compaction $\alpha_c \to \pm\infty$ at even horizontal tunes Q_x; β_x and $\beta_y \to \infty$ at integral Q_x.

The racetrack lattice has a much weaker second-order variation of ct than those in Sections and . Hence, $ct \neq 0$ for $-0.6 < p_t < 0$, the muons must be accelerated inside buckets with stable fixed point $ct = 0$ and $p_t = 0$, and not on trajectories similar to those in Fig. 1. We have studied acceleration over a large range of RF cavity voltages. We have not found a value with the bucket height needed. In addition, the synchrotron motion is chaotic, and the edges of the bucket are fuzzy. On might either attempt to increase the second-order variation of ct, or frequency modulate the RF cavities as in a synchrotron.

SCENARIO 4: SUPER-PERIODS

We studied a lattice with 10 superperiods, each consisting of 10 arc cells and 9 straight section cells from Section , i.e. without transitions. For $-0.6215 \leq p_t \leq +0.138$, the phase advances in the superperiod cover the ranges $5.911988 \geq \mu_x \geq 1.522851$ and $5.788112 \geq \mu_y \geq 1.605385$. In the absence of acceleration, we find wide stopbands at the 4 integral and the 4 half-integral phase advances within these ranges. The extreme absolute values of dispersion $|D_x|$ and orbit offset $|x|$ tend towards ∞ at the edges of the integral stopbands, while β_x and β_y reach ∞ at the edges of the integral and half-integral stopbands. We have not succeeded in accelerating muons through these stopbands. We have not studied super-periodic lattices with adiabatic transitions between arcs and straight sections.

CONCLUSIONS

Two styles of FODO lattices, suitable for super-conducting and room-temperature RF systems, respectively, accelerate muons outside buckets from about 6 to about 20 GeV in about 10 turns with less than about 2 GeV circumferential acceleration V. This value of V is at most one half, and the number of arcs is 2/7, of those in an RLA with four turns, and there are no spreaders and combiners. The circumferences C are ≈ 2 km and ≈ 1 km, respectively. A modified achromat accelerates muon inside buckets from about 8.25 to about 19.8 GeV in 11 turns with a similar circumferential RF voltage, at $C \approx 1$ km, and can probably be improved to cover the whole momentum range. We have not succeeded in accelerating muons in a racetrack machine with adiabatic transitions between arcs and straight sections and a super-conducting RF system. We have not succeeded in accelerating in a super-periodic machine. We have presented FFAG rings with very interesting properties. We leave their optimization for the future, as well as further studies of scenarios 3 and 4.

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