RESONANCES AND BEAM LOSS IN HIGH INTENSITY RINGS*

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Abstract

Operation of high-intensity rings requires minimal beam loss. Among numerous effects which contribute to losses in rings, the interplay of excited resonances is typically an unavoidable source of halo and beam loss. Such resonances can be driven by space charge itself, magnet errors or a combined effect of both. In this paper we review several resonant effects which can limit beam intensity in a circular machine. The space-charge limit and selection of a working point in the ring are also discussed.

INTRODUCTION

In general, beam loss can be separated into a design (or technical) and a beam dynamics part. A control of the beam loss which falls under the design can be very challenging. In addition, various effects of beam dynamics give significant contribution to the beam loss. Typically, the maximum achievable intensity is limited by beam dynamics which dominates high-intensity operation, including various space-charge effects, collective instabilities, etc. As far as radioactivation at high intensity is concerned, if the collective instability occurs, then further operation is not possible. This requires finding a cure for the instability either by implementing some design changes or by damping the instability. In this paper, we discuss the beam loss due to another fundamental intensity-stopper in the AG rings - the resonances. An overview of the specifics for the resonant driven beam loss at high-intensity is given. Here, the term ”high-intensity” is used in reference to conventional synchrotrons and storage rings. The intensity is described by a depression of the betatron tunes due to the space-charge forces. Such a tune depression is defined as $\eta = \nu/\nu_0$, where ν is the space-charge depressed tune, and ν_0 is the zero-current tune. The main focus of this paper is a conventional ring where achievable tune depression is relatively weak $\eta = 0.9 - 0.98$, so that one can treat the space-charge effect as a perturbation which leads to a small shift $\Delta\nu_{sc}$ of the betatron tunes ($\nu \approx \nu_0 - \Delta\nu_{sc}$, with $\Delta\nu_{sc} \ll \nu_0$). In linear accelerators, the words ”high intensity” usually refer to $\eta < 0.8$. In another class of circular accelerators where special measures are undertaken to compensate emittance growth due to resonance crossing (such as cooler rings), the tune depression can be very strong, with the definition of the space-charge limit corresponding to an ultimately cold beam $\eta \to 0$.

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BEAM LOSS AND HALO MECHANISMS

An uncontrolled beam loss typically occurs due to a halo surrounding the beam core. Not surprisingly, there are many mechanisms which contribute to halo generation in both linear and circular accelerators [1]. The task is to identify the most important of them and come up with possible cures. Most of the effects which are important mechanisms for halo production in linear accelerators are also relevant to rings. However, due to a significant difference in the regimes of the tune depression, the time scale (growth rates) of the space-charge driven effects becomes very different. There are also some ring-specific mechanisms due to the possibility of accumulating some effects over successive turns, as well as due to a longer storage time.

SPACE-CHARGE RESONANCES

We refer to the space-charge resonances as those which are driven by the space-charge potential itself rather than the field potential of the magnets. Their importance was first shown by Montague for the coupling resonance [2]:

$$2\nu_x - 2\nu_y = 0, \tag{1}$$

where the factor of 2 in front of $\nu_{x,y}$ reflects the driving coupling term x^2y^2 in the space-charge potential, making it a fourth-order resonance. This resonance can occur even for a lattice without any perturbation since it requires only zero-th harmonic in the Fourier component of the perturbation. Due to the fact that this resonance is a difference symmetric resonance, such coupling can lead to significant halo only for a beam with unequal emittances. A detailed analytic analysis of this type of resonance was recently presented using collective beam dynamics in connection with high-current transport systems [3]. There, one can have a ratio of the transverse and longitudinal tunes which is very different from unity thus allowing the possibility of a nonlinear asymmetric ($n_x \neq n_y$) coupling resonance with significant energy exchange between the two planes:

$$n_x\nu_x - n_y\nu_y = 0, \tag{2}$$

in the single-particle approximation, or more generally [3]:

$$n_x\nu_x + n_y\nu_y + \Delta\omega = 0, \tag{3}$$

based on the approach of collective dynamics, where the $\Delta\omega$ represent an additional shift with respect to the depressed incoherent tunes, and the integers n_x, n_y can be positive or negative.

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Collective coupling resonances

Until recently, an analytic treatment of the space-charge coupling resonances in rings was limited to a single-particle approach, although computer simulations were able to provide a more accurate description [5],[4]. An assumption of constant beam size and frozen beam potential in a typical analysis does not accurately describe the resonant beam behavior. This was realized long ago [6], and an analytic approach of collective beam dynamics was used by Sacherer to describe beam response to the one-dimensional resonances [7]. An extension to the two-dimensional isotropic beams was formulated by Gluckstern [8] and for more realistic anisotropic beams by Hofmann [3]. The latter allowed a direct application to circular accelerators where the beams are typically non-round with different transverse emittances, and to linacs, where one can have significantly different focusing constants (tunes) in different planes. This made it possible to put the space-charge coupling resonances in 2-D into a more self-consistent framework of coherent resonances for collective beam modes [3]. In conjunction with particle simulation codes, these analyses allowed exploration of the parameter space of the focusing constants in linear accelerators with the finding that nonequipartitioned beams can avoid emittance exchange as long as they are not near the stopbands of the space-charge coupling resonances [9]. In fact, such a recommendation to avoid the space-charge coupling resonances is usual in rings. The difference is that in linacs one can have very different focusing constants, which requires consideration of many asymmetric space-charge coupling resonances with the zero-th harmonic of the perturbation. In AG rings, however, the betatron tunes are typically not very different from one another, thus leaving the symmetric Montague resonance as the resonance with the zero-th harmonic. Recently, the study of such a resonance in rings, using the theory of collective beam modes, was presented [10]. Other resonances are possible when the driving force comes from the space-charge potential while the resonant harmonic results from the lattice periodicity which makes the nonlinear asymmetric space-charge coupling resonances important in rings as well [4]. When these effects are studied for a weak tune depression, it is important to remember that the dynamic rates of these effects depend on the tune depression (this effect makes it a primary concern in high-intensity linear accelerators).

Intrinsic incoherent resonances

The oscillating space-charge force can also lead to another class of resonances where individual particles inside the beam can get into resonance with an oscillating beam mode. These resonances are referred to as intrinsic or incoherent resonances of the individual particles. Such a parametric resonance mechanism was suggested as one of the dominant effects for halo generation in linacs [11]. Some literature on this subject can be found, for example, in Ref. [12]. In recent years, existence of self-consistent three-dimensional computer codes allowed systematic studies of this mechanism for realistic beam distributions [13],[14],[9].

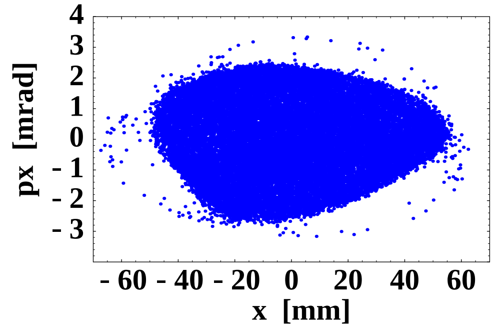

Figure 1: Incoherent resonance of individual particles with the 3rd order driven collective beam mode.

Such resonances between the motion of the individual particles and collective beam oscillations are governed by the rms beam mismatch. For example, in the case of a uniform density beam envelope, which is oscillating with the frequency Ω and the mismatch parameter μ, the equation of motion for the individual particle has the form

$$x'' + q^2 x = \mu \frac{2\kappa}{a_0^2} x \cos \Omega\theta, \tag{4}$$

where κ is the space-charge parameter, and $q^2 = k^2 - \kappa/a_0^2$ is the depressed incoherent frequency without the approximation of small space charge. This equation has a primary parametric resonance at $q = \Omega/2$. When higher order terms are included, one also gets the non-linear parametric resonances [15]. The halo extent associated with the $1:2$ parametric resonance was extensively studied for the envelope modes in high-current linacs. It is straightforward to show analytically, that for a small μ, the halo extent (finite due to the nonlinearity omitted from Eq. 4) has a linear dependence on μ and a very weak dependence on the space-charge parameter. In fact, the halo extent associated with this resonance is large not only for a strong tune depression of the order of $\eta \sim 0.5$ (typical for linacs) but also for tune depression of only a few percent $\eta \sim 0.97$ (typical for rings). However, in the limit of zero space charge, the motion near the core is very regular, and the rate at which particles are driven into the 1:2 resonance becomes very small. As a result, for space charge with $\eta \sim 0.98$, it takes significantly more time for the particles to be trapped into the 1:2 resonance [16].

The rate of halo development (which is a function of both the mismatch parameter and tune depression) becomes the most important question when one tries to estimate an effect of such a parametric resonance on halo formation in rings. In addition, when applied to accumulator rings, one should take into account many other effects such as the fact that the beam intensity is not a constant during accumulation, the phase-mixing effect due to multi-turn injection, etc. [16]. Taking into account all these effects, simulations for the SNS ring showed that this intrinsic resonance may not be a problem [17].

The incoherent resonances, which were discussed above, and, which are also known as the mechanism for the "parametric halo", did not require any resonances with the lattice since the collective oscillation was induced by a mismatch. In rings, however, such collective beam modes may have a fast excitation as a result of both the space charge and machine resonances. The resonances of the individual particles with such driven collective beam modes are called "driven incoherent resonances", since the collective modes are resonantly driven (Fig. 1). In such a situation, the incoherent resonances may play an important role in halo formation even in the limit of weak tune depression, as will be discussed in the following sections.

MACHINE RESONANCES AND SPACE CHARGE

The beam loss associated with lattice driven resonances is typically the dominant one in conventional high-intensity rings. When choosing the operating point in the tune space, one carefully avoids resonances driven by the lattice periodicity, which are referred to as structure resonances. However, the unavoidable presence of errors in the magnetic field imposes a restriction associated with the imperfection resonances. Although technically such resonances are not the space-charge induced resonances since they are driven by the lattice harmonics, space charge plays an important role here as well. First, it creates the problem since the tunes are depressed by the space charge towards the resonances. On the other hand, the space charge introduces an effective nonlinearity which changes the beam response to the resonances.

Collective beam response

A single-particle approach with a frozen beam potential was introduced to study effects of the space-charge tune depression in the tune space [18]. This approach was later generalized to include the effect of the wall images [19]. It was quickly noted that the space-charge potential can itself change in a response to the external time dependent perturbation [6]. To include this effect, the resonance response required a treatment of collective beam dynamics [7], which takes into account the fact that the space-charge force depresses not just the single-particle frequencies but also the collective modes of the beam oscillation. As a result, the response to an external perturbation occurs when the tune of the corresponding collective mode is depressed towards the resonance condition. An extensive study of these effects recently reemerged due to the increasing interest in high-power circular accelerators [20]-[21]. Although availability of powerful computer codes allowed one to study these effects self-consistently with the prediction of beam loss due to the space-charge and imperfection resonances for a specific accelerator [4], [22], the theory of collective resonant response provided a good tool in understanding the beam behavior observed in simulations.

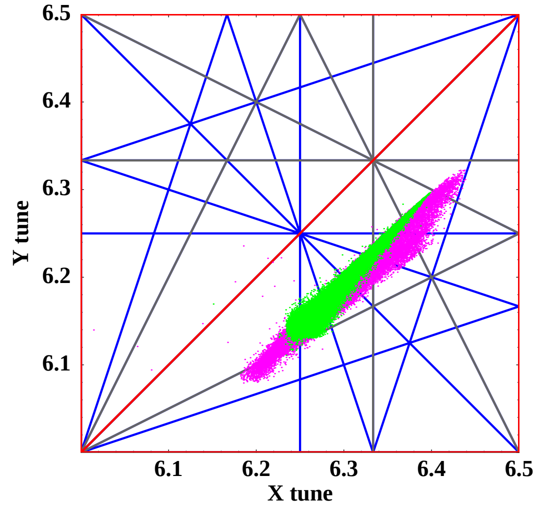

Figure 2: Tune spread of a 2MW SNS beam: only space-charge spread (green), space charge plus $dp/p = 1\%$ (pink). The tune-box is chosen free from the structure resonances. The imperfection resonances for the 2nd, 3rd and 4th orders are shown in red, black and blue, respectively.

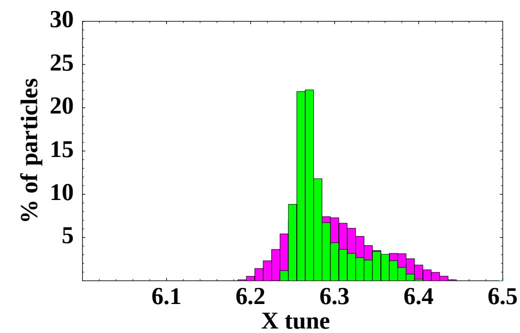

Figure 3: Horizontal distribution of the depressed tunes, corresponding to the tune-spread shown in Fig.1.

Integer and half-integer collective response

In general, in the case of non-linear imperfection errors one needs to consider the tune values near the corresponding non-linear resonances. The excited high-order modes respond to the imperfection harmonics according to

$$n = \Omega_m, \tag{5}$$

which is the coherent resonance condition for any order beam mode $\Omega_m = m\nu_0 - \Delta\Omega_{m,sc}$, derived by Sacherer [7]. To derive such condition for $m > 2$ modes, one needs to use either the high-order moment equations or the Vlasov equation. In addition, the effect of periodic focusing adds the possibility of the collective modes resonating at half-integer values, which corresponds to the parametric (half-integer) resonance [23]:

$$n/2 = \Omega_m. \quad (6)$$

For completeness, we note that in the absence of non-linear imperfections, the periodic oscillation of the high-order modes are well defined so that the condition $n = \Omega_m$ no longer applies [23], and stability is determined solely by the parametric condition $n/2 = \Omega_m$ (see Ref. [26] for discussion). This effect becomes dominant in high-current transport channels or cooler rings, if the imperfections are not important. With the harmonic n now standing for the structure harmonic, the beam encounters a whole set of the instabilities of the corresponding collective modes in the process of the space-charge tune depression. Such instabilities were first numerically explored in connection with the transport channels [24], and recently were analytically described using the terminology of the resonances with an application to cooler rings [23].

Space-charge limit

A restriction imposed by the integer and half-integer single-particle tune values ("space-charge limit") can be written, using the self-consistent condition in Eq. 5 (for the case of linear perturbations), as

$$n = \Omega_2 = 2\nu_0 - \Delta\Omega_{2,sc}, \quad (7)$$

where Ω_2 is the frequency of the 2nd order coherent mode of beam oscillations (beam envelopes). The effect of the periodic focusing adds the possibility of a parametric resonance of the beam envelopes

$$n/2 = \Omega_2, \quad (8)$$

also known as the "envelope instability" [25]. According to Eq. 8, the envelope instability can limit the allowable tune space to only 0.25. This may have an impact on the performance of a high-intensity machine. The situations when such an envelope instability should be considered, and, whether it can alter the space-charge limit governed by the Eq. 7, are discussed in [26]. If such a parametric resonance is driven by the imperfection errors, the resonance is expected to be very narrow, and the envelope growth is detuned at a very low level due to the nonlinearity [21]. In fact, this is why the effect of the envelope instability was found to be negligible in rings, provided that the tune-box is chosen free of the structure resonances, and only the imperfection harmonics are of a concern [26].

Tune spread, resonance crossing and halo growth

The dependence of frequency on amplitude introduces an important asymmetric feature in resonance crossing. Depending on the sign of the nonlinearity, the frequency can grow or decay as the amplitude increases. In the case of the nonlinearity due to the space-charge, one gets a frequency increase with amplitude. As a result, when the resonance is crossed in the direction in which the peak intensity in the bunch is decreasing, the beam will experience a sudden large (but finite) jump in oscillation amplitude. If an error term, driving the resonance, is not very big, and there is sufficient aperture to accommodate such oscillations, one can cross the resonance in this direction with relatively low losses. On the other hand, when the resonance is crossed in the direction in which the peak intensity is increasing (for example, due to beam bunching or due to a multi-turn injection), this results in an adiabatic increase of the oscillation amplitude at resonance crossing. As a result, the high-intensity crossing of resonances in this direction without significant beam loss may be possible only with proper correction [27].

Another important feature of resonance crossing with space charge (when the peak intensity is increasing) is that small amplitude particles within the beam core experience the largest tune shifts. A typical tune spread of a 2MW beam in the SNS is shown in Fig. 2, where the tune spread in a bunched beam solely due to the space-charge is shown with a green color while the total tune spread with an additional $dp/p = 1\%$ is shown with a pink color. In Fig. 3, the corresponding single-particle tune distribution is shown, for example, in the horizontal direction. The resonances are first crossed with small-amplitude particles. Already the single-particle approximation shows that the current can be further increased until the resonance condition is reached by the large-amplitude particles. A more self-consistent description can be obtained via a collective approach for a beam with a non-linear distribution. In this case, the particles with small amplitudes are depressed below the resonance. At a critical intensity one gets a coherent resonant response. The frequency of the corresponding collective oscillation is increasing with amplitude so that the large-amplitude particles can be trapped into the resonance with the corresponding oscillating beam mode (Fig. 1). Some insights into the mechanisms of particle trapping into the islands and thus corresponding emittance growth, can be obtained analytically with an appropriate treatment of the nonlinear terms which provide bounded motion [28]. However, other important effects like the redistribution process due to resonance crossing, the dynamical redistribution as a result of multi-turn injection, the longitudinal effects of a bunched beam, etc, cannot be included analytically in a simple way. As a result, a more self-consistent description is obtained with realistic simulations. One of such important dynamic effects is shown, for example, in Fig. 4. Due to emittance increase, the effective space-charge depression becomes smaller, which allows a further increase in the intensity with a further subsequent growth of the emittance which can be seen as a saturation of the maximum tune shifts in the tune diagram.

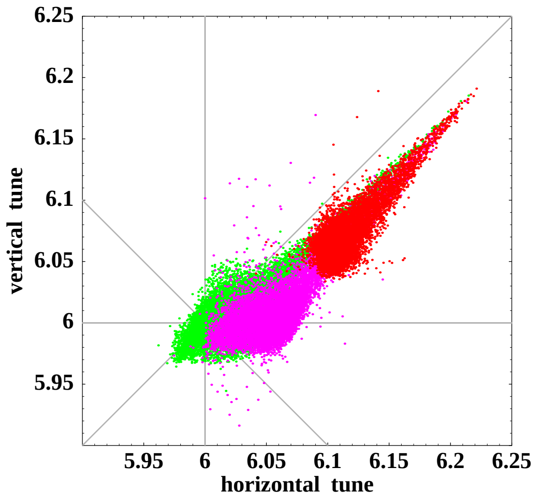

Figure 4: Space-charge tune spread of a bunched beam in the SNS for three beam intensities: $N = 2 \cdot 10^{14}$ (red), $3 \cdot 10^{14}$ (pink) and $4 \cdot 10^{14}$ (green) protons in the beam.

Selection of working point in high-intensity rings

Selection of a working point in a high-intensity ring requires careful consideration of both the space-charge and lattice driven resonances. First, tune-box which is free from structure resonances is selected. Then one considers whether the space-charge coupling resonances may give undesirable effects, which depends on the tune splitting and the ratio of the emittances in different planes. The best point is typically below the $1/4$ tunes which minimizes loss due to the high-order resonances. The harmonic $2n$ for an integer n below the fractional tune of 0.25 and the harmonic n for the half-integer $n/2$ tune below 0.75 preferably should be the imperfection harmonics, which may allow a space-charge shift in excess of the incoherent value of 0.25. If the working point is selected above the low-order nonlinear resonaces ($1/4$ and $1/3$ tune values), then all the required correctors should be available for an independent correction of these resonances. In the latter case, the exact level of loss will depend on the nature of the resonance, the beam intensity, the correction scheme, etc.

Resonance correction for high-intensity

The theory of imperfection resonances and their correction was developed based on the motion of single particles. As a result, it was successfully applied when space-charge effects are negligible. In the absence of a good physical picture of the space-charge effects in resonance crossing, any unsuccessful correction of a resonance for high-intensity operation, can be easily blamed on complicated space-charge effects. Now that we have some analytic understanding of the space-charge effects in resonance crossing and more importantly, the tools of the self-consistent computer simulations, we may attempt to resolve some of the questions related to the correction of the nonlinear resonances for high intensity [27]. Our conclusion is that one can provide good correction for the nonlinear imperfection resonances even for high-intensity operation.

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