

# SURFACE ROUGHNESS EFFECT ON A MOVING BUNCH

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## Abstract

The effect of surface roughness on a moving electron bunch is considered by resorting to a model of a metallic structure with random perturbations on its surface. Based upon this model analytic expressions have been developed for both the average energy emitted per groove as well as for its standard-deviation. For a relativistic bunch both quantities are shown to be virtually independent of the momentum. Moreover, it has been found that the standard-deviation of the energy emitted per groove is proportional to the standard-deviation of the roughness parameter to the power of 1/4.

## 1 INTRODUCTION

The acceleration structure of a linear collider designed to operate at X-band is manufactured within an accuracy of about  $1\mu\text{m}$ . These four orders of magnitude difference between the operating wavelength and the typical surface roughness will be difficult to maintain in case of a vacuum *optical* accelerator operating at  $1\mu\text{m}$  since such a difference implies engineering at the atomic level  $1\text{\AA}$ . In fact, both the size of the bunch as well as the roughness of the structure are anticipated to be of the same order of magnitude, leading to a significantly different regime of operation when compared to that of a machine driven by a *microwave* source. It is the purpose of this paper to put forward the main results of a study aimed to the investigation of the impact of the surface roughness upon the wake-field of a moving bunch of a size comparable to the dimensions of the surface roughness.

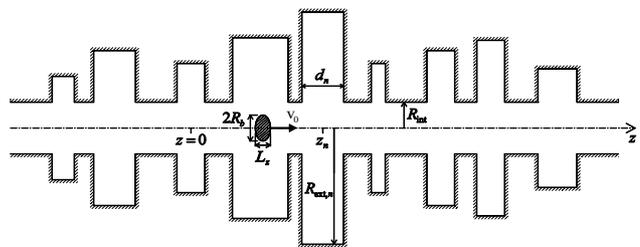
Several studies have been conducted in the past in order to investigate the surface roughness effects upon wake-fields by considering either single obstacles or periodic structures. For example, Kurennoy et al. [1] have developed a general theory of beam interaction with small discontinuities of the vacuum chamber of an accelerator. The analysis was extended [2] to obstacles protruding inside the drift pipe of an accelerator for wavelengths larger compared to the obstacle's typical size. Further, an evaluation of the coupling-impedance has been presented by Stupakov [3] employing the so-called *small-angle approximation*.

## 2 FORMULATION OF THE PROBLEM

In the framework of our investigation a quasi-analytic analysis facilitating a relatively simple evaluation of the wake-field due to surface roughness of arbitrary size has been developed. It relies on an approach published about a decade ago in the context of quasi-periodic traveling output structures for high-efficiency, high-power microwave sources [4,5]. The model relies on a cylindrical waveguide of constant internal radius to which

a series of grooves are attached; their geometric parameters i.e. width, height and location, are assumed to be *randomly* distributed, and in principle, these grooves can be large on the scale of the typical wavelength of the radiation driving the system. Details of this study will be published separately [6].

In order to analyze the wake-field generated by the surface roughness, consider a metallic structure consisting of a random number ( $N$ ) of grooves attached to a cylindrical waveguide of constant internal radius ( $R_{\text{int}}$ ), as illustrated in Fig. 1. The center of the  $n^{\text{th}}$  groove is denoted by  $z_n$ , its width by  $d_n$  and its external radius by  $R_{\text{ext},n}$ . An electron bunch of radius  $R_b$ , length  $L_z$  and a total charge  $Q$ , is moving along the symmetry axis of the structure at a constant velocity  $v_0$ , generating a current density denoted by  $J_z(r, z; t)$ .



**Figure 1:** A finite-size bunch moving in vacuum along the axis of a structure with random size grooves.

As the only component of the current density is parallel to the  $z$ -axis, it is sufficient to consider only the longitudinal magnetic vector-potential  $A_z$  satisfying the non-homogeneous wave equation. Its solution has two components: the so-called *primary* field determined by the current density in the absence of the metallic structure and the so-called *secondary* field accounting for the impact of the structure. Taken together, these fields satisfy the boundary conditions, establishing the unknown amplitudes. Once these amplitudes have been established all the field components may be determined; in particular the longitudinal component of the secondary electric field  $E_z^{(s)}(r, z; t)$ . As the bunch traverses the structure, the emitted power may be expressed in terms of this field component as

$$P(t) = 2\pi \int_0^{R_b} r dr \int_{-\infty}^{\infty} dz J_z(r, z; t) E_z^{(s)}(r, z; t). \quad (1)$$

Moreover, the emitted energy is given by

$$W = \frac{Q^2}{4\pi\epsilon_0 R_{\text{int}}} \text{Re} \left[ \int_0^{\infty} d\Omega S(\Omega) \right] \equiv \frac{Q^2}{4\pi\epsilon_0 R_{\text{int}}} \bar{W} \quad (2)$$

$S(\Omega)$  representing the normalized spectrum and  $\bar{W}$  the normalized energy. As already indicated, the geometrical

parameters of each groove are random and of the same order of magnitude. Explicitly, they are given by

$$\bar{R}_{\text{ext},n} \equiv \frac{R_{\text{ext},n}}{R_{\text{int}}} = 1 + \bar{g}_n, \quad \bar{d}_n \equiv \frac{d_n}{R_{\text{int}}} = \bar{g}_n, \quad (3)$$

where  $\bar{g}_n$  is a random variable that is uniformly distributed between 0 and  $\bar{\delta}$ ;  $\bar{\delta}$  will be referred to as the *normalized roughness parameter*. The center of the first groove ( $\bar{z}_1 \equiv z_1/R_{\text{int}}$ ) is being chosen as a point of reference ( $\bar{z}_1 = 0$ ) and accordingly, the center's location of the  $n^{\text{th}}$  ( $n=2,3,\dots,N$ ) groove is given by  $\bar{z}_{n+1} = \bar{z}_n + 0.5\bar{d}_n + 0.5\bar{d}_{n+1} + \bar{g}_n$ .

### 3 DISCUSSION

Although, the analysis is valid for a large variety of values of  $\bar{\delta}$ , the discussion that follows is limited to relatively small values of  $\bar{\delta}$  since if  $R_{\text{int}}$  is of the order of  $0.5\mu\text{m}$  the typical roughness is not expected to be larger than  $0.1\mu\text{m}$  therefore, we consider  $0 \leq \bar{\delta} \leq 0.2$ . Moreover, the accelerated bunch is expected to be of the order of  $30^\circ \div 45^\circ$  (namely about  $0.1\mu\text{m}$ ), and as a result,  $0.15 \leq \bar{L}_z \equiv L_z/R_{\text{int}} \leq 0.30$ ; the normalized radius of the bunch is chosen to be  $\bar{R}_b \equiv R_b/R_{\text{int}} = 0.5$ . In order to determine the characteristics of the emitted energy (average and standard-deviation), a series of simulations has been performed where each data point is a result of averaging over 80 *different distributions* for a given  $\bar{\delta}$ .

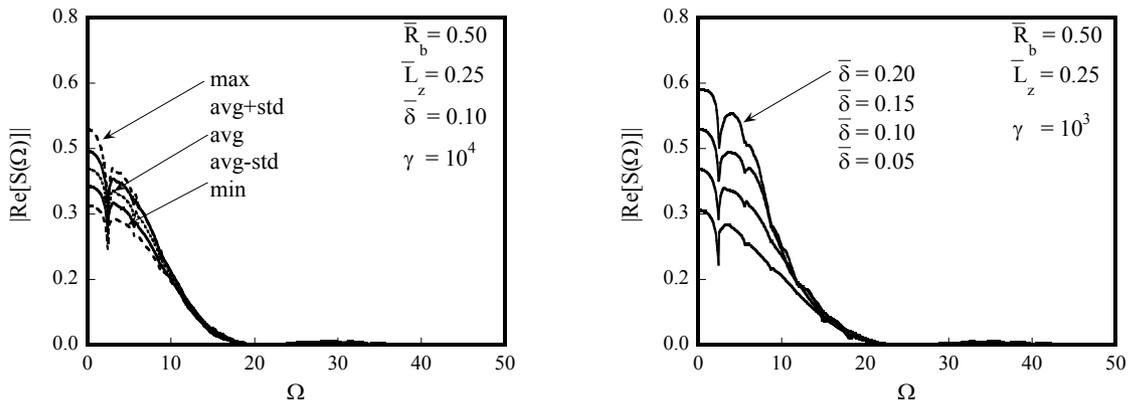
In both frames of Fig. 2 the spectrum of the emitted energy is illustrated. The left frame shows the maximum and minimum values of the normalized spectrum, its average value, and its average value plus the standard-deviation and its average value minus the standard-deviation; in all five cases  $\bar{R}_b = 0.5$ ,  $\bar{L}_z = 0.25$ ,  $\bar{\delta} = 0.1$   $\gamma = 10^4$ . Evidently, all the curves overlap for high frequencies and the deviation from the average values for

low frequencies is less than 25%. Furthermore, as illustrated in the right frame of Fig. 2, the first zero of the spectrum remains virtually unchanged when varying  $\bar{\delta}$ . In fact, this zero is related to the cut-off frequency of the cylindrical waveguide i.e.  $\Omega \equiv \omega R_{\text{int}}/c = p_1 = 2.4048$ , where  $c$  is the speed of light in vacuum,  $p_1$  being the first zero of Bessel function of zero order and first kind. Other simulations indicate that the normalized spectrum is weakly dependent on  $\gamma$  and in fact, for the ultra-relativistic case the spectrum is virtually independent of  $\gamma$ .

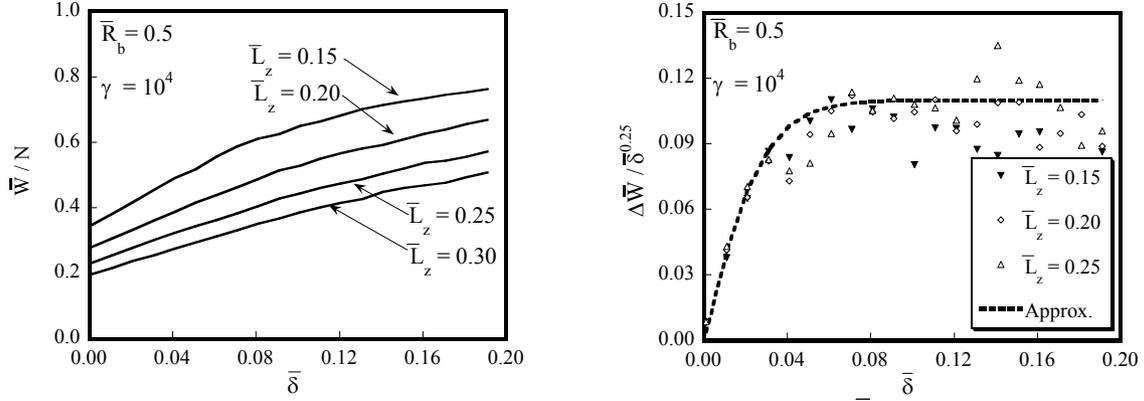
After examining the properties of the spectrum, our focus moves towards the *total energy* emitted, establishing its dependence on the roughness parameter ( $\bar{\delta}$ ) with  $\bar{L}_z$  as a parameter. The left frame of Fig. 3 illustrates the average value of the normalized energy per groove versus  $\bar{\delta}$ ; ( $\bar{L}_z = 0.15, 0.20, 0.25, 0.30$ ,  $\bar{R}_b = 0.50$ ) and  $\gamma = 10^4$ . Two facts are evident: firstly, the average as well as the normalized energy per groove ( $\bar{W} \equiv \bar{W}/N$ ) increases with the increase of  $\bar{\delta}$ . Secondly,  $\bar{W}$  increases when reducing the length of the bunch ( $\bar{L}_z$ ). Simulations reveal that the impact of  $\gamma$  for relativistic energies is virtually negligible. According to these simulations the average emitted energy may roughly be approximated by

$$\frac{\langle W \rangle}{4\pi\epsilon_0 R_{\text{int}} \times N} \approx 0.57 \tanh \left( \frac{45 \frac{\langle g \rangle}{R_{\text{int}}}}{1 + 20.72 \frac{\bar{L}_z}{R_{\text{int}}}} \right) + \frac{1.429}{1 + 20.72 \frac{\bar{L}_z}{R_{\text{int}}}} \quad (4)$$

A second important feature of the emitted energy is its normalized standard-deviation given by the expression  $\Delta \bar{W} \bar{\delta}^{-0.25} \equiv \sqrt{\langle \bar{W}^2 \rangle - \langle \bar{W} \rangle^2} / (\langle \bar{W} \rangle \bar{\delta}^{0.25})$ . The latter is shown in the right frame of Fig. 3 versus  $\bar{\delta}$ , with  $\bar{L}_z$  as a



**Figure 2:** Normalized spectrum versus the normalized frequency. Left: Maximum, minimum, average value for the normalized spectrum and average value plus/minus one standard deviation. Right: Average value of the normalized spectrum for different values of  $\bar{\delta}$ .



**Figure 3:** Left: Average value of the normalized energy per groove versus  $\bar{\delta}$ . Right: Normalized standard-deviation of the energy out of 80 different distributions versus  $\bar{\delta}$ .

parameter. A best fit of the simulation results reveals that the standard-deviation may be approximated by

$$\frac{\sqrt{\langle W^2 \rangle - \langle W \rangle^2}}{\frac{Q^2}{4\pi\epsilon_0 R_{\text{int}}} \times N} \approx 0.15 \left( \frac{\Delta g}{R_{\text{int}}} \right)^{0.25} \tanh \left( 121.2 \frac{\Delta g}{R_{\text{int}}} \right) \quad (5)$$

$$\times \left[ 0.57 \tanh \left( \frac{45 \langle g \rangle}{1 + 20.72 \frac{L_z}{R_{\text{int}}}} \right) + \frac{1.429}{1 + 20.72 \frac{L_z}{R_{\text{int}}}} \right].$$

Simulations indicate that the last approximation is adequate for variety of values for  $\bar{R}_b$ ,  $\bar{L}_z$  and  $\gamma$ . The expression in Eq. (5) constitutes a generalization of the conditions imposed in Eq. (3) since based on the latter the roughness parameter  $\bar{\delta}$  determines both  $\langle \bar{g} \rangle$  as well as  $\Delta \bar{g}$ . In order to identify the role of each one of the last two parameters, the simulation was repeated for fluctuations in the parameters of a periodic structure e.g.  $\bar{R}_{\text{ext},n} = \bar{R}_{\text{ext},n}^{(0)} + \bar{g}_n$ ,  $\bar{d}_n = \bar{d}_n^{(0)} + \bar{g}_n$  and  $\bar{z}_{n+1} = \bar{z}_n + 0.5\bar{d}_n + 0.5\bar{d}_{n+1} + \bar{g}_n$ .

## 4 CONCLUSIONS

The characteristics of the electromagnetic energy emitted by a bunch traversing a number of grooves of random geometry were established. Of special interest are the average and the standard-deviation of the emitted energy per groove in terms of the average roughness  $\langle \bar{g} \rangle$  and its standard-deviation  $\Delta \bar{g}$ . The main result of the present study may be expressed in terms of these two quantities as given in Eqs. (4) and (5). These relations are valid for relativistic energies ( $\gamma > 50$ ) and independent of the radius of the bunch provided  $R_b \leq R_{\text{int}}/2$ . For a point-charge ( $L_z = 0$ ) and for practical structures where  $\bar{\delta} \approx 0.1R_{\text{int}}$  the expression for the average energy per groove reads

$$\frac{\langle W \rangle}{N} \approx \frac{Q^2}{4\pi\epsilon_0 R_{\text{int}}} \times 2. \quad (6)$$

It should be pointed out that the expression for the average energy per groove is identical to that developed for the case of a point-charge moving in a cylinder of radius  $R_{\text{int}}$  bored in a dielectric or metallic medium [7], and is almost equal to that obtained for the case of a point-charge moving in a cylindrical wave-guide with a periodic wall of arbitrary, yet azimuthally symmetric geometry [8].

## 5 ACKNOWLEDGEMENT

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## 6 REFERENCES

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