CHARACTERISTICS OF GRADIENT UNDULATOR[1]

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Abstract

Undulator/wiggler having the same polarities of magnetic field in opposing though medial plane poles can be used for generation radiation which intensity is a function of the beam size and displacement. Characteristics of such device analyzed here analytically and by tracking.

INTRODUCTION

Utilization of quadrupole wiggler is an interesting option for broadening the bandwidth of pick-ups in fast feedback systems [1] and for beam alignment [2]. Such a wiggler, which is a sequence of ordinary focusing and defocusing quadrupoles, generates a radiation, which depends on the beam shape and position.

Although in such system radiation for two transverse coordinates can be easily distinguished by polarization, desire to have sensitivity to one coordinate only is always an option. In [3] a device, called *gradient undulator,* was described for the first time. Practical utilization of this device suggested in [4].

Here we will concentrate our attention on practical aspects of design and properties of magnetic fields in gradient undulator. The test was also included trajectories analyses, which was carried out numerically on the basis of real 3D model. Knowing the real trajectory allows finding radiated field instantly in principle.

ANALYTICAL CONSIDERAIONS

Let us consider first some simple aspects of symmetry of gradient undulator/wiggler in comparison with usual dipole wiggler; both are represented in Fig.1.

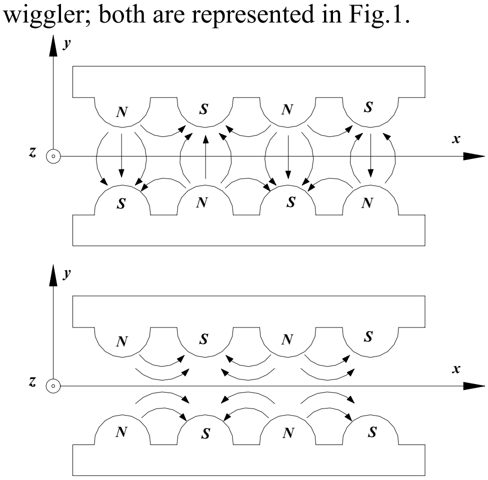

Figure 1: Dipole wiggler, upper one and gradient undulator/wiggler, lower. Beam is moving along x –axis. Normal to the drawing's plane is z –direction.

In *gradient wiggler*, the longitudinal field $B_x(y)$ has nonzero value at the plane $\{x, z\}$, but now wiggling amplitude linearly increasing with y coordinate following $B_y(y)$.

Using Maxwell's equation $div\vec{B}=0$ and $rot\vec{B}\big|_z=0$ after some algebra one can obtain analytical field expression for gradient wiggler with wide poles as the following [5, 6]

$$B_x(x,y)=\sum_{k=0}^{\infty}\frac{(-1)^k y^{2k}B^{(2k)}(x)}{(2k)!},$$

$$B_y(x,y)=-\sum_{k=0}^{\infty}\frac{(-1)^k y^{2k+1}B^{(2k+1)}(x)}{(2k+1)!} \tag{1}$$

For demonstration, fields (1) can be represented in lowest order as the following

$$B_x(x,y)=B(x)-\frac{y^2}{2}B''(x)+\frac{y^4}{2\cdot 12}B^{(IV)}(x)-... \tag{2}$$

$$B_y(x,y)=-y\cdot B'(x)+\frac{y^3}{2\cdot 3}B'''(x)-\frac{y^5}{5!}B^{(V)}(x)-...$$

One can see from here, that gradient in y direction associated with $B'(x)$. For example if dependence in x direction is periodic, $B(x)=B_0 Cos\frac{x}{\lambda_w}$, the last determines the gradient in y direction as

$$G(x)\equiv\frac{\partial B_y}{\partial y}=-B'(x)=\frac{B_0}{\lambda_w}Sin\frac{x}{\lambda_w} \tag{3}$$

So we are coming to fundamental conclusion for this type of wiggler, that the gradient is proportional to the longitudinal field and reciprocally proportional to the period.

If the field is time dependent, then the following substitution needs to be done in all formulas [6]

$$\frac{\partial^{2k}B(x)}{\partial x^{2k}}\to\left(\frac{\partial^2}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)^k B(x,t),$$

where c stands for the speed of light. Odd derivative obtained by taking integral over x in this combination.

HARDWARE ASPECTS

Let us consider now more or less realistic model of such a wiggler. Orientation of the poles is represented in Figures 1 and 2. This model has not associated with any specific project, but taken for example only.

[1]Extended version is available at http://www.lns.cornell.edu/public/CBN/2003/CBN03-1/CBN03_1.pdf . Work supported by NSF.

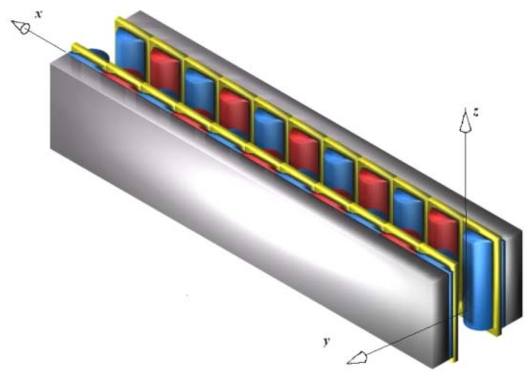

Figure 2 (Color): View to the gradient wiggler model.

Particles are moving along x. Pole size in z direction is $\pm$15 cm (=30cm), radiuses of cylinders 4 cm, coil cross-section is 2×2.3 cm^2, period $\lambda_w = 24.84$ cm, gap between poles ± 3.5 cm. Coil cross section is a rectangle with dimensions $\{x \times y\} = \{2 \times 2.5\} cm^2$. Yoke plate ends at y=15 cm from medial plane y=0.
This wiggler is not sensitive to z- position of the particle, giving linear dependence in radiated field across y direction. Radiation in this wiggler is polarized along z –axis. Yoke modeled by soft Steel 1010. Total current running in central coil is 60 kA-Turns, what suggests utilization of SC windings. All field calculation done with 3D code MERMAID.

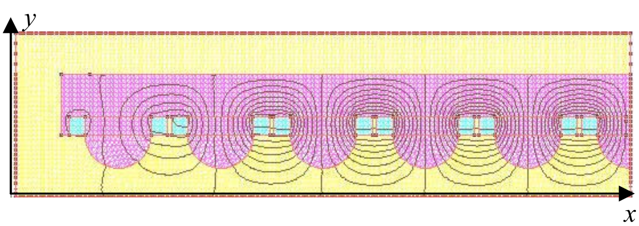

Figure 3 (Color): Lines of magnetic field in central plane of wiggler, z=0. Half of the wiggler is shown; right side of the plot is a plane of symmetry along x. Radiuses of the poles are 4 cm, period -25 cm. Gap between coils 0.5 cm. Length of the pole in z direction is $\pm$15 cm. Particles are moving from the left side. $\{xz, y=0\}$ represents medial plane.

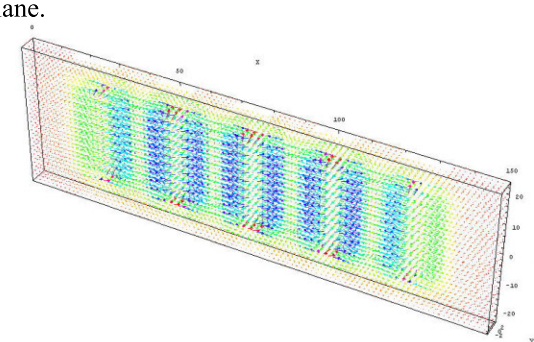

Figure 4 (Color): Vectorial representation of the fields in the gap of 11-pole gradient wiggler inside the 3D volume {x=0-150;y=$\pm$3;z=$\pm$24} cm. Medial plane of the wiggler {x, z} runs at y=0.

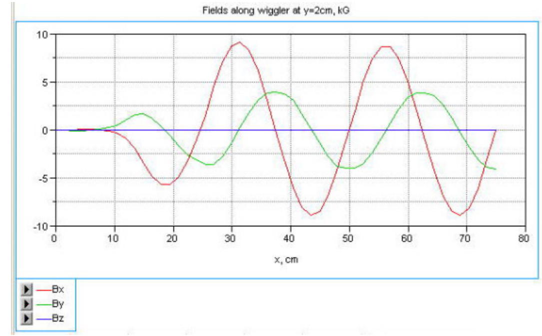

Figure 5 (Color): Field graphs for y=+2 cm, z=0. Half wiggler is shown as in Figure 3. Center is at the right side of the plot at 75 cm. B_y comes to it's maximum, B_x, indeed comes to zero. Basically B_z=0 along this axis as it must be according to symmetry properties.

Wiggler is focusing towards medial plane {x, z} in y –direction. This focusing is typical for any wiggler and is proportional to the associated angle of wiggling $\alpha \cong K/\gamma$, with K –factor depending on vertical position itself.

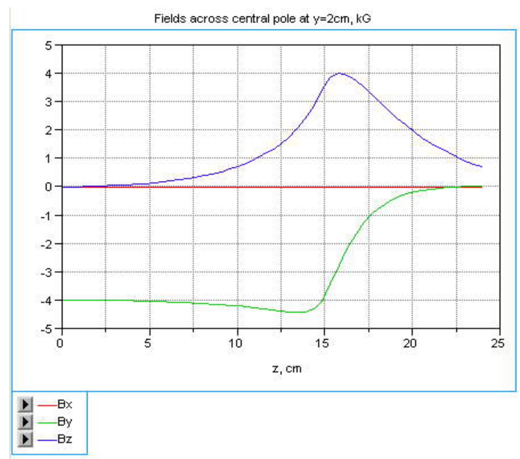

Figure 6 (Color): Fields across central pole, starting from z=0, y=2 cm. Material of yoke St1010. Points at {x=75, y=2, z=0-24} cm.

The increase in B_y component (lower curve in Fig.6) at the side of the pole explained by the flux concentration here. The flux expelled from central region escapes from the sides. This effect also generates B_z component.

TRAJECTORIES

Formally, as formulas for magnetic field is known, (5), it is possible to make analytical calculations of particle's trajectory. We will continue here our numerical exercises, however. Trajectories calculated numerically by using code UMKA [7] on the basis of MERMAID 3D field calculations.
One can expect, as the field in vertical direction is linearly varying, trajectories with zero angle and displacement at output for one vertical input will have different resulting kicks and angles for other vertical input.

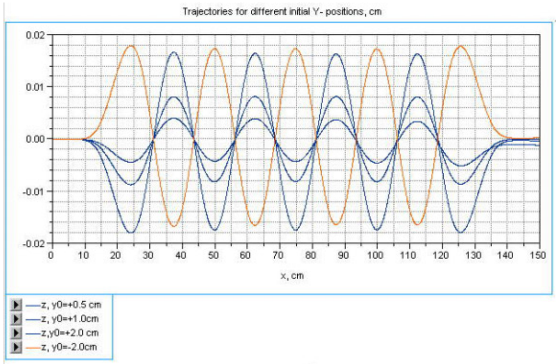

Figure 7 (Color): Trajectories for different initial positions in Y direction. There are shown ones for $y_0 =$+0.5 cm, +1.0 cm, +2.0 cm, -2.0 cm. The last one corresponds the start point below median plane. Beam energy 2 GeV.

One can see, that this type of wiggler can also be used for fine-tuning of dynamic aperture of damping ring for linear collider, typically overloaded by dipole wigglers.

RADIATION AND ACTION TO THE PARTICLE

As trajectory $z(x)$ is known, this allows calculation of second derivative $\dot{\vec{v}} \cong \ddot{z}(x = ct)$ and hence, electric field. Basically to obtain time structure at the observation point one needs to take second derivative with graph in Fig.7 and shrink this curve by Doppler factor. One can see that a single particle spectrum is the same as in a dipole wiggler with the same trajectory and formulas are the same (see for example [4]).

Evidently, the frequency of radiation

$$\omega = \frac{c/\lambda\!\!\!^{-}_w}{1-\vec{n}\vec{v}} \cong \frac{2\gamma^2 \cdot c/\lambda\!\!\!^{-}_w}{1+\gamma^2\vartheta^2 + K^2/2} \qquad (4)$$

is a function of magnetic field through $K = 93.4 \cdot B_y(y)[Tesla] \cdot 2\pi\lambda\!\!\!^{-}_w[m]$, ϑ is an angle to observer (from *x*-direction). So with variation of *y*, *K* factor also changes. If one restricts the width at the level ~50% from maximal, then $K^2/2 \le 1$, and hence, $K \le \sqrt{2}$. This limits either B_0 for given amplitude *y* or amplitude, if B_0 is given. For the wiggler with given field at the axis B_0 the broadening will be less than 50% for amplitudes

$$y \le \sqrt{2} \cdot \frac{mc^2}{eB_0} \equiv \sqrt{2} \cdot \lambda\!\!\!^{-}_w \frac{mc^2}{eB_0\lambda\!\!\!^{-}_w} = \frac{\sqrt{2} \cdot \lambda\!\!\!^{-}_w}{K_0}, \qquad (5)$$

where $K_0 = eB_0\lambda\!\!\!^{-}_w / mc^2$. For the wiggler considered above, B_0 ~$0.8T$, $2\pi\lambda\!\!\!^{-}_w \cong 0.25$ *m*, $K_0 \cong 18.7$ and amplitude must be $y \le 0.3cm$ i.e. below 3 *mm*. For narrower width, the amplitude must be restricted proportionally.

In Optical Stochastic Cooling, OSC, dependence of frequency of radiation on amplitude, might be useful for specific tuning for cooling the particles with large amplitudes only. Period of the wiggler must be adjusted properly, together with angle of observation.

For OSC, implementation of quadrupole like devices might be interesting for reduction of radiation from cold central part of the bunch. This device, similarly to the quadrupole wiggler, provides the energy shift as a function of particle displacement at location of kicker.

$$\Delta E = \frac{eE_0L}{2}\frac{K}{\gamma} \cong \frac{eE_0L}{2}\frac{K_0}{\gamma}\frac{y}{\lambda\!\!\!^{-}_w}, \qquad (6)$$

where E_0 stands for electric field strength in co-directionally propagating electromagnetic wave, the length of wiggler is *L*. One can see that action is also linearly dependent of position of particle in the gradient undulator or in quadrupole wiggler.

CONCLUSIONS

The device analyzed –gradient undulator/wiggler can be interesting not only for the purposes of beam size measurements, but also for nonlinear correction of magnetic properties in the damping ring or in a transport channel, acting in one particular direction only. Minimal number of poles for this purposes –three–give closed bump.

Device can be interesting for implementation into OSC, serving as a pickup and/or a kicker linearly (re)acting to the particle's instant position in one direction.

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