SIMULATIONS OF OCTUPOLE COMPENSATION OF HEAD-TAIL INSTABILITY AT THE TEVATRON

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Abstract

The proton lifetime in the Tevatron depends sensitively on chromaticities. Too low chromaticities can make the beam unstable due to the weak head-tail instability. One way to compensate this effect is to introduce octupoles to create a larger amplitude dependent betatron tune spread. However, the use of octupoles will also introduce additional side effects such as second order chromaticity, differential tune shifts and chromaticities on both proton and anti-proton helices. The non-linear effects may also reduce the dynamic aperture. We report on a simulation study to find the best combinations of polarities and strengths of the octupoles.

1 INTRODUCTION

In the Tevatron at injection energy, the transverse weak head-tail phenomenon drives proton beam unstable if it is not damped. Before the transverse dampers were commissioned, the damping was done by setting the chromaticities to relatively high values of 8 units in both planes. On the other hand, the beam lifetime at injection energy depends sensitively on chromaticity. In principle, a lower chromaticity would help to decrease the growth of higher order modes of instability, and improve the DA(Dynamic Aperture), and consequently the beam lifetime. However, too low chromaticities can make the beam unstable [2]. Currently, the Tevatron is running with tranverse and logitudinal dampers. Better beam lifetime is observed when the chromaticity decreased from 8 units to 2 units. Another way to compensate the instability of the protons is to introduce octupoles to create larger amplitude dependent betatron tune spread. If the shifted coherent tunes due to wake fields is within the width of the incoherent betatron tune spread or spread in synchrotron tunes, all unstable higher order modes can be damped by Landau damping.

Table 1 lists the number and the average beta function and the dispersion of two octupole families we used for the simulations. The use of octupoles will also introduce additional side effects on both proton and anti-protons. These effects are second order chromaticity, addtional coupling and non-linearty. Octupoles will also produce differential tune shift and linear chromaticities on both proton and pbar helices.The purpose of this simulation is to find the best combinations of polarities and strengths of the octupoles.

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Octupoles	Number	$<\beta_x>$, $<\beta_y>$[m]	$<D_x>$, $<D_y>$[m]
TOZD	24	93.6,30.2	3.67,-0.02
TOZF	12	30.5,92.5	2.07,-0.01

Table 1: Octupoles in the Tevatron used in this simulation

2 THE AMPLITUDE-DEPENDENT TUNE SHIFT DUE TO OCTUPOLES

The field due to an octupole in terms of the multipole components (b_3, a_3) is

$$B_y + iB_x = \frac{B_o(b_3 + ia_3)}{R^3_{ref}}[x^3 - 3xy^2 + i(3x^2y - y^3)] \quad (1)$$

The octupole strength parameter k_3 is defined as

$$k_3 = K_n L = \frac{n!}{(R^n_{ref}\rho)} b_n L = \frac{1}{(B\rho)} \frac{\partial^n B_y}{\partial x^n}.L \quad (2)$$

Introducing the action angle coordinates $x = \sqrt{2\beta_x J_x} cos\phi_x$, $y = \sqrt{2\beta_y J_y} cos\phi_y$ and assuming the phase and the beta functions are nearly constant over the length of a single octupole, we can find the action-dependent tune shift as follows [1]

$$\begin{aligned} \Delta\nu_x &= a_1 J_x + a_2 J_y \\ \Delta\nu_y &= a_2 J_x + a_3 J_y \end{aligned} \quad (3)$$

where

$$\begin{aligned} a_1 &= \frac{1}{16\pi}\sum_n k_3(n)\beta^2_{x,n} \\ a_2 &= -\frac{1}{8\pi}\sum_n k_3(n)\beta_{x,n}\beta_{y,n} \\ a_3 &= \frac{1}{16\pi}\sum_n k_3(n)\beta^2_{y,n} \end{aligned}$$

$J_{x,y}$ is the action,and related to the amplitude by $J_{x,y} = \frac{(a_{x,y})^2}{2} \cdot \varepsilon_{nx,y}$, and $a_{x,y}$ is the amplitidue im units of beam size σ, $\varepsilon_{nx,y}$ is the normalized emittance (1σ)in x or y plane.

Let $k_3(F)$ be TOZF's strength, $k_3(D)$ be TOZD's strength. At Tevatron injection, the action-dependent tune-shifts due to octupoles are obtained as follows:

$$\begin{aligned} \Delta\nu_x &= 2091.5251([k_3(F) + 0.2124k_3(D)]J_x \\ &\quad -0.6444[k_3(F) + 1.9961k_3(D)]J_y) \end{aligned} \quad (4)$$

$$\begin{aligned} \Delta\nu_y &= 1349.6594(-[k_3(F) + 1.9961k_3(D)]J_x \\ &\quad +3.0269[0.0533k_3(F) + k_3(D)]J_y) \end{aligned} \quad (5)$$

At present machine conditions, the proton emittance is 25πmm- mrad(95%). For the particle with initial amp-plidude of 5σ, we can get the expression for the octupole's strengths which are needed to compensate the decrease of tune spread due to lowering the chromaticities as follows:

$$k_3(F) = -4.5726 \times 10^3[0.6194 \times \Delta\nu_x + \Delta\nu_y] \quad (6)$$

$$k_3(D) = -2.3044 \times 10^3[\Delta\nu_x + 0.6570 \times \Delta\nu_y] \quad (7)$$

3 QUANTITIVE CALCULATION FOR THE OCTUPOLE STRENGTH

All unstable higher order modes of the weak head-tail effect can be damped if the spread of betatron frequency which has the incoherent tune shift included,or the spread of synchrotron frequency are larger than the coherent tune shift due to the impedance [2].

The growth rate of head tail modes measured at 150 GeV is about $110/sec$, and the coherent tune shift is $\Delta\nu_{coh}$=$-1. \times 10^{-3}$ [3]. On the other hand, the synchrotron tune spread at 150 GeV is calculated to be $\Delta\nu_s$= 2.2×10^{-4}. The incoherent linear tune shifts in both x and y planes due to the space charge for the particle near the center of the proton bunch with 3-D Gaussian density distribution are also calculated, and they are $(\Delta\nu_x)_{SC} = -0.36 \times 10^{-3}, (\Delta\nu_y)_{SC} = -0.70 \times 10^{-3}$.

Therefore, in order for the landau damping to work, the betatron tune spread should be

$$\begin{aligned} (\Delta\nu_x)_\beta &= [\Delta\nu_{coh} - (\Delta\nu_x)_{SC} - \Delta\nu_s] \\ &= -0.42 \times 10^{-3} \quad (8) \\ (\Delta\nu_y)_\beta &= [\Delta\nu_{coh} - (\Delta\nu_y)_{SC} - \Delta\nu_s] \\ &= -0.08 \times 10^{-3} \quad (9) \end{aligned}$$

Then, the octupole strengths calculated from Eq. 7 for TOZF and TOZD are

$$k3(F) = 1.55, \qquad k3(D) = 1.09 \quad (10)$$

At tevatron injection,

$$k3 = K_3L = 1.2267506/m^3/Amps \times I \quad (11)$$

I is the magnet currents of octupoles.

4 DYNAMIC APERTURE TRACKING WITH AND WITHOUT OCTUPOLES

Before the octupoles were introduced into the lattice, we calculated the dynamic aperture at different chromaticities. Fig. 1 shows DAs of protons and pbars in function of momentum deviation at different chromaticities. The particles were tracked 100,000 turns by code MAD. We can see that the DAs of pbars at chromaticity of 2 units are 0.5σ larger than those of 8 units, although the DAs of protons seems not much improved.

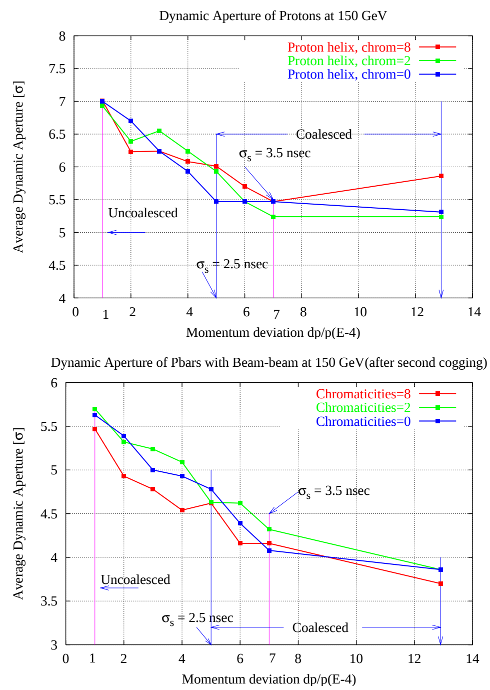

Figure 1: DA of protons and anti-protons vs. momentun deviations at different chromaticities

Using the code SIXTRACK we have also calculated the chaotic border which separates regular librator motion from fast amplitude growth. Table 2 lists DAs and chaotic borders at different chromaticities, calculated using code SIXTRACK. The momentum deviation in this calculation is dp/p=4.3e-4. Small chromaticities help to improve long term dynamic aperture since chromaticity sextupole strength are reduced, and synchro-betatron resonances are weaker.

Table 2: Dynamic aperture and chaotic border of protons(p) and anti-protons ($\overline{p}$) at injection

	Chromaticities	(8,8)	(2,8)	(2,2)	(0,0)
p	Chaotic border(σ)	4.8	5.0	5.2	5.4
	DA (σ)	6.4	6.9	6.8	6.9
$\overline{p}$	Chaotic border(σ)	2.9	4.5	4.6	4.7
	DA (σ)	5.1	5.4	5.4	5.3

In the machine, when we reduce the both or one plane chromaticities from 8 units to 2 units, pbar lifetime was observed significately increased(from 2hrs to 12hrs), while proton lifetime of protons increased not so much. This may be because the proton beam is tickled by the damper kicker.

We used octupole strength $|k_3(F)|$=2, $|k_3(D)|$=1 with different combinations of polarities to test their impact on the Dynamic Aperture. These values are closed to the estimated values in Eq. 10. The results are listed in Table 3. From Table 3 we can see that if TOZF is positive, the dynamic aperture and chaotic border are larger than the other

chromaticities	(ξ_x=2, ξ_y=2)				(ξ_x=2,ξ_y=2)	(ξ_x=8,ξ_y=8)
octupole strength (k_3(F),k_3(D))(m^{-2})	(+2,+1)	(+2,-1)	(-2,+1)	(-2,-1)	(0,0)	(0,0)
$a_1 = \frac{\partial \nu_x}{\partial (a_x^2)}$ (10^{-5})	-15.38	-8.15	-4.29	10.60	-23.21	-4.29
$a_3 = \frac{\partial \nu_y}{\partial (a_y^2)}$($10^{-5}$)	6.34	- 5.04	- 4.05	-14.91	2.82	- 5.29
$a_2 = \frac{\partial \nu_x}{\partial (a_y^2)}$ or $\frac{\partial \nu_y}{\partial (a_x^2)}$ (10^{-5})	5.00	- 0.03	- 5.29	-12.36	6.69	- 4.05
Average DA (σ)	7.0	7.1	6.6	6.2	6.8	6.4
minimum DA (σ)	6.2	6.3	5.9	5.4	6.3	6.0
Chaotic border (σ)	5.8	5.5	4.5	4.4	5.2	4.7

Table 3: Dynamic Aperture and chotic boder with/without octupoles

combinatons. We also analyzied the coefficents a_1,a_2,a_3 in Eq. 5 for each combination. We expect that a_1 and a_3 must be much larger than a_2, so that the tune spread will be less dependent on the coupling term. On the other hand, a_1 and a_3 increase with octupole strength. It was found that both a_1 and a_3 are increasing but a_2 is decreasing if TOZD is also positive. This implies that the tune spread will be increased for larger amplitude particles, while the coupling effects will be lower.

With $k_3(F)$=2, $k_3(D)$=1, and chromaticity set at (ξ_x=ξ_y=2), we calculated the dynamic aperture and chaotic border of the anti-protons. It was found that the average DA is 5.6σ, the chaotic border is 5.1σ.

The machine study of octupoles at the Tevatron injection energy has been done by P. Ivanov in Tevatron department. A single coalesced proton bunch with relative higher intensity was injected onto the central orbit, and then two octupole families TOZF and TOZD were introduced to suppress the tranverse coherent instability while reducing the chromaticities in both x and y planes. Different combinations of polarities of two octupole families have been tested, and it was found positive polarity for both families are the best, which confirmed our calculations by simulation. So far, high intensity proton beams(N_p=280-300e9 per bunch) have been successfully injected onto the central orbit with small(zero) chromaticity setting and octupole settings $k_3(F)$=1.6, $k_3(D)$=4.1. Nevertheless, beam loss occured while going from injection bump to the proton helix - probably because of significant tune changes and strong coupling due to orbit motion. Further study in the machine is in progress.

5 CONCLUSION

The optimized combination is found by Dynamic Aperture tracking that both octupole families need to be posotive, it was confirmed by machine studies at the Tevatron.

Stronger octupole streangths have been tried and it's found that they are not better for the compensation. Besides, the tune shifts feeddown from octupoles on the helices are larger, and it is more difficult to make feeddown sextupoles work.

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7 REFERENCES

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