Abstract

In the last decade demand for brightness in synchrotron light sources and luminosity in circular colliders led to construction of multiple high current storage rings. Many of these new machines require feedback systems to achieve design stored beam currents. In the same time frame the rapid advances in the technology of digital signal processing allowed the implementation of these complex feedback systems. In this paper I concentrate on three applications of feedback to storage rings: orbit control in light sources, coupled-bunch instability control, and low-level RF control. Each of these applications is challenging in areas of processing bandwidth, algorithm complexity, and control of time-varying beam and system dynamics. I will review existing implementations as well as comment on promising future directions.

INTRODUCTION

In the last 10-15 years digital feedback became not only an accepted tool in the accelerator community, but a critical tool necessary for success of a modern storage ring. There many applications of the digital feedback methods in different areas of machine operation including, but not limited to, coupled-bunch instability control, low-level RF control, orbit feedback, and luminosity optimization in colliders.

FEEDBACK FUNDAMENTALS

The objective in feedback control is to make some output of a dynamic system behave in a desired way by manipulating the input of that system. A general block-diagram of such a system is shown in Fig. 1. The system consists of the physical system (plant) the output of which we want to control. The output signal \( y \) is measured by the sensors and sent to the controller. The control objective might be to keep \( y \) small (or close to some constant value) - this is defined as a regulator problem. A different objective is to make plant output \( y \) follow some reference signal \( r \) - a servomechanism problem. Controller in Fig. 1 can be a regulator - then input \( r \) is omitted - or a servo. In any case controller determines the error between plant output and desired value and, based on the knowledge of plant dynamics, computes the control output \( u \). The control signal is then applied to the plant via actuators.

Performance of a feedback system can be evaluated using many different approaches. For a servo problem time-domain response characteristics are popular. These include step response parameters such as rise time, settling time, overshoot. Tracking errors in response to constant or linearly varying inputs are often used as well [1].

For a regulator application steady-state errors in response to known disturbance spectra provide an important performance measure. Such errors can be evaluated using the root-mean-square (RMS) or peak approaches. Another important measure of feedback performance is the RMS or peak actuator effort, especially important due to the finite excursion ranges of physical actuators.

Multi-input multi-output systems

Many feedback control problems in storage rings involve multiple inputs and outputs. An example of a MIMO system is dynamic behavior of coupled-bunch instabilities. Here each bunch can be considered as a harmonic oscillator coupled to all other bunches. This results in a fully coupled MIMO system with individual bunch positions being plant outputs and bunch correction kicks as plant inputs. For a single beam position sensor and a single correction kicker the individual bunch signals are time multiplexed making the system seem to be single-input single-output (SISO).

Feedback control of MIMO systems is computationally intensive. In general, for an \( M \)-input \( N \)-output plant the feedback controller must perform \( M \times N \) transfer matrix computation. Each element of such matrix has dynamic behavior and can be implemented as an analog or digital filter. In practice one tries to avoid full transfer matrix computation by using diagonal or sparse structures. Often the controller can be reduced to a constant-coefficient matrix multiplication combined with a diagonal dynamic controller.

Digital feedback control

A digital feedback controller usually consists of one or more analog-to-digital converters (ADCs) which digitize analog sensor signals. These digitized signals are processed by a linear or non-linear control algorithm to compute actuator signals. A reference input for the servomechanism applications can be introduced in either continuous or discrete-time domains. The output of the controller is...
usually applied to actuators via the back-end digital-to-analog converters (DACs).

The feedback control algorithm is commonly implemented as a linear time-invariant system using finite or infinite impulse response (FIR or IIR) structures. Nonlinear control methods can sometimes offer better performance, but are more difficult to analyze and design than linear controllers. For certain applications, e.g. control of a double integrator plant, non-linear control structures have been well developed and analyzed [2, p. 581].

Control algorithm design methods can be separated into two main classes: emulation of continuous-time controllers and discrete-time design [2, p. 158]. The emulation approach is attractive since the design is done in the continuous domain. It is especially useful if proven continuous-time controllers already exist. However reliable discrete emulation of such controllers requires significant oversampling of the control bandwidth with suggested sampling rate to control bandwidth ratios of 20 to 30 [1, p. 601].

Design of digital feedback controllers in the discrete-time often uses proportional-integral-derivative (PID) structures. Tuning and optimization of PID control is straightforward, however PID designs are best suited to relatively simple plant dynamics.

A more sophisticated design method is the state-space control when the actuator signal is computed from the information on the internal states of the plant. Such an approach provides the designer with independent control of all closed-loop plant poles. Since internal states of the plant are rarely available in full, a parallel model of the plant dynamics (an estimator) is commonly used to estimate the internal state of the plant. The estimator is normally used in a closed loop configuration which adjusts the estimated states using the error between estimator and plant outputs. The next step in the control design is to use optimal control methods to design both the estimator and the state-to-actuator matrix. Commonly used approaches include linear quadratic regulator steady-state optimal control which minimizes the weighted quadratic sum of state and actuator excursions. Optimal estimator design is often based on a Kalman filter which optimizes state estimation using the knowledge of process and sensor noise [2, p. 444].

Robust control design extends the notion of optimality to include the sensitivity of the closed-loop system to variations in loop parameters and other uncertain terms.

**Waterbed effect**

The plant is subject to external disturbances which affect the output \( y \). As one of the performance criteria of the control system one can consider the reduction of the transfer gain from external disturbance input to plant output.

Let \( L(j\omega) \) to be the open-loop transfer function of a SISO system. Then the sensitivity function \( S(j\omega) = (1 + L(j\omega))^{-1} \) determines transfer characteristics from an input to a summing junction to its output. The Bode integral theorem states that if the open-loop transfer function has no poles in the right-hand plane (a stable system) and there two or more poles than zeros, the following equation holds

\[
\int_{0}^{\infty} \log |S(j\omega)| d\omega = 0
\]  

(1)

According to this equation, if the sensitivity function is reduced in some band of frequencies it must necessarily increase elsewhere. For a system with a bandlimited loop transfer function it can be shown [3, p. 89] that Eq. 1 leads to a peaking phenomenon in the sensitivity function. Thus a comparison with a waterbed: when one pushes down \( |S(j\omega)| \) in one place it pops up in another.

The waterbed effect is illustrated in Fig. 2 for proportional feedback around the plant \( P(s) = \frac{100}{s(s+2)} \). Application of the feedback attenuates the unity open-loop sensitivity function at low frequencies with moderate peaking above 0.8 rad/s. When the feedback gain is raised, improvement of the low-frequency disturbance rejection is accompanied by increased peaking. Note, however, that the bottom plot shows much smaller disturbance amplification if the effect is measured at the plant’s output.

In order to achieve improvement with feedback three techniques are traditionally used for waterbed effect mitigation. The first method is to consider not just the sensitivity function, but its product with the plant transfer function \( P(j\omega) \). If sensitivity amplification occurs in a range of frequencies where plant response is small, the overall effect is attenuated. However one must remember that noise induced elsewhere in the feedback loop, e.g. additive sensor noise, will be amplified by \( S(j\omega) \). Another method is to use the knowledge of external disturbance spectra to place sensitivity function peaks away from significant excitations. Finally, for rejection of periodic disturbances one can use the feedforward approach.
FEEDBACK CONTROL OF COUPLED-BUNCH INSTABILITIES

Control of transverse and longitudinal coupled-bunch instabilities is critical for successful operation of the high-current storage rings. Designers of the bunch-by-bunch feedback systems used digital technology quite early on due to two factors. They needed a way to implement one-turn bunch delay which for large rings is more feasible digitally. Also, bunch motion is sampled at the revolution frequency by a beam position monitor (BPM) making this problem a natural fit for discrete-time processing. Early feedback systems [4] only used digital delay while the next generations of bunch-by-bunch feedback [5] combined both digital delay and filtering.

It is convenient to model coupled-bunch instabilities as a MIMO system consisting of \( N \) coupled harmonic oscillators. Such a structure in combination with a bunch-by-bunch feedback controller is shown in Fig. 3. Longitudinal or transverse positions of bunches are the outputs of the plant while the voltage kicks are the inputs.

The goal of the feedback system is to stabilize the plant transfer \( G(s) \). A powerful control architecture in this case is diagonal, i.e. bunch-by-bunch feedback. The correction signal for a given bunch is computed based only on the motion of that bunch. It can be shown that a bunch-by-bunch feedback system that acts equally on every bunch also acts equally on every eigenmode. Since eigenmodes normally differ only parametrically, bunch-by-bunch feedback can provide simultaneous stabilization of all eigenmodes.

Three feedback designs capable of processing bunch signals at 2 ns intervals and controlling coupled-bunch instabilities in the machines with thousands of bunches and hundreds of unstable eigenmodes emerged in the 1990s. One of these is the longitudinal feedback system currently in use at ALS, BESSY-II, DA\( \Phi \)NE, PEP-II, and PLS, the second was developed for KEK-B, and the third was designed by the ELETTRA/SLS collaboration.

The SLAC/ALS/DA\( \Phi \)NE design is a longitudinal only feedback system due to its use of downsampling [6]. The system is very flexible and has been used to sample bunch motion at 238–500 MHz and to process 120–1746 bunches. The feedback correction signal is computed using either a 12-tap FIR algorithm or a 12\(^{th}\) order IIR filter.

The KEK-B feedback system processes every bunch on every turn and, therefore, can be used for either transverse or longitudinal feedback. The system parameters are matched to KEK-B RF frequency of 508 MHz and harmonic number of 5120. The control filter in this case is a much simpler two-tap FIR [7].

Finally, the ELETTRA/SLS design bridges the gap between the first two systems. It is capable of processing every bunch on every turn for transverse feedback using a 5-tap FIR filter to compute the correction signal. The system can also be reconfigured for downsampling longitudinal processing with longer, 10-tap FIR filters. Thus the ELETTRA/SLS design combines capabilities for transverse processing of the KEK-B system with the relatively complex control algorithms of the SLAC/ALS/DA\( \Phi \)NE system [8].

The value of digital feedback flexibility is seen in the longitudinal feedback system at DA\( \Phi \)NE configured to simultaneously control both dipole and quadrupole instabilities [9]. Due to large bunch length in this machine the dipole feedback system can affect the quadrupole dynamics of the beam. Frequency separation of dipole and quadrupole signals makes it possible to design feedback controllers for simultaneous stabilization of both instabilities. A filter design algorithm has been developed for this task and allows independent control of gain and phase responses at and around these two frequencies. In Fig. 4 frequency responses of two control filters are presented. The dual-peak filter has gain peaks centered at the synchrotron frequency and its first harmonic with nearly equal gains and +90 and −90 degrees phase shifts for dipole and quadrupole oscillations respectively. The second filter with a notch at the quadrupole frequency is used to allow growth of quadrupole instabilities while maintaining control of dipole motion. Such a filter was used first to verify the existence of quadrupole instabilities and rule out instability at 5120 kHz.
excitation of quadrupole motion via the dipole feedback system. In addition, these filters have been used to conduct grow/damp measurements of the quadrupole-coupled-bunch instabilities [10].

LOW-LEVEL RF CONTROL

In the PEP-II collider sophisticated low-level RF feedback loops are used to reduce the effective fundamental impedance of the RF cavities seen by the beam. This brings down the longitudinal coupled-bunch instability growth rates into the manageable range. Direct and comb feedback loops are the two main elements of low-level RF feedback providing impedance control over ±1.3 MHz band around the RF frequency. These wideband loops are complemented by multiple slower hardware and software feedback loops used to maintain a consistent operating point of the klystron, eliminate loop gain and phase changes with the klystron output power shifts, reject periodic gap transients, etc. [11].

The achievable gain of the direct loop is determined by the total group delay in the system. In order to minimize the controller delay the direct loop processing is analog and has a total of 86 ns of delay. Compare this with a single sample delay of 100 ns if using digital processing at 10 MHz. To improve impedance reduction at the synchrotron sidebands of revolution harmonics a double-peaked comb filter loop is used. This comb filter applies significant additional loop gain in a narrowband manner thus avoiding the group-delay limitation of the direct loop. Such filter is adjusted for a full turn of delay to obtain proper (periodic) phasing at all revolution harmonics. One of the two channels of this filter is illustrated in Fig. 5. The filter samples cavity I and Q signals at 10 MHz resulting in 72 samples per turn. The second order IIR filter is used to generate peaks at the synchrotron sidebands as well as notches at the revolution harmonics. The IIR filter is followed by a 32-tap FIR filter which implements a group-delay equalizer as well as a low-pass filter. The system allows for 25 ns steps in DAC clock edge placement for improved one-turn delay matching.

GLOBAL ORBIT FEEDBACK

Application of feedback formalism to orbit feedback started with the pioneering work of R. Hettel on local orbit control in 1983 [12]. By 1989 a global orbit feedback system was implemented and tested at NSLS VUV ring [13]. This system used analog signal processing and was limited to 4 position sensors and 4 corrector magnets. Later systems used fast digital feedback capable of sampling at 1 kHz or faster and supporting tens and hundreds of BPMs and correctors. Such systems were implemented and commissioned at the APS [14], ESRF [15], and many other storage rings.

Global orbit feedback control algorithms utilize the information in response matrix R which relates small-signal corrector changes Δ⃗c and the resulting orbit shifts Δ⃗x:

\[ Δ⃗x = R Δ⃗c \]

The BPM-to-corrector transformation matrix R_{inv} is computed to minimize the error term \(|R R_{inv} Δ⃗x - Δ⃗c|\) using direct matrix inversion or singular value decomposition [16].

A general block diagram of a global orbit feedback system is shown in Fig. 6. Transverse position of the beam is measured at N BPMs distributed around the ring. The measured orbit is digitized and subtracted from a reference orbit. The error signal is processed by the compensation filter and transformed from the BPM space to the corrector space using R_{inv}. Resulting correction terms are added to reference magnet settings and applied to the corrector magnets via DACs and power supplies.

Main technical challenges in fast global orbit feedback are due to the distributed nature of the sensors and the actuators. Correction computation generally requires the information from all of the BPMs leading to adoption of reflective memory [14] and fast networking [17] communication schemes. The choice of the control structure is by no means obvious. While static correction is addressed by the inverse response matrix, the compensation filter is very important for achieving good dynamic performance, e.g. external disturbance rejection. Most designs to date have used variants of PID control as a compensation filter, optimal and robust controller designs should be explored. Placement of
the dynamic controller in the BPM, eigenvector, or corrector basis strongly affects the closed-loop behavior of the system. Control algorithm in the eigenvector basis would allow one to better filter small eigenvalues which are more sensitive to individual BPM errors. Finally, control filters in the corrector basis provide a way to equalize system response between fast and slow corrector magnets. Corrector saturation issues are important in a practical system and are partially addressed by the SVD algorithm.

**SUMMARY**

Table 1 summarizes the digital feedback applications in high-current storage rings that were considered in this paper. These applications cover a wide range of sampling rates and input-output dimensions as well as a wide range of control algorithm complexities. Diagonal control dominates the MIMO feedback architectures, mostly due to computational complexity limitations; even fully coupled implementations separate dynamic control into a diagonal structure. Analog feedback is still important, especially for ultra-low group delay medium-to-wideband applications. At the same time even analog feedback channels benefit from integrated digital diagnostics.

Promising future directions for digital feedback in storage rings involve higher sampling rates and ADC resolutions. Faster sampling, in turn, leads to wider use of digital receiver structures to detect beam signals. Explosive growth in commercial digital signal processing architectures in the last 10 years resulted in powerful off-the-shelf signal processing products which can be used to accelerate feedback development cycles. Application of optimal and robust control methods can help to improve both performance and reliability of feedback systems.

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**REFERENCES**


