APPLICATIONS OF THE CANCELLATION EFFECT IN CSR STUDIES

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INTRODUCTION

The cancellation effect in the dynamics of relativistic beams on a curved trajectory is studied in Ref. [1] based on the canonical formulation of the dynamics equations and retarded potentials. In this paper, we first discuss in a coherent manner various applications of the cancellation effect, such as a coasting beam, a short bunch in steady-state interacting with off-orbit particles, transient self-interaction of a short bunch entering a circular orbit from a straight path, and a converging beam in a bunch compression chicane. Next, spectrometer measurement and Landau damping in microbunching process are discussed based on new dynamics equations which explicitly use the cancellation effect.

REVIEW OF THEORY

According to Ref. [1], for a relativistic charged bunch moving on a curved trajectory with curvature $\kappa(s)$, the first order equation for the particle's horizontal motion is ($r = R + x$ and $R(s) = 1/\kappa(s)$):

$$\frac{d^2x}{c^2dt^2} + \frac{x}{R^2} = \frac{1}{E_0}\left(\frac{\Delta E(t)}{r} + F_x^{\text{col}}\right) \tag{1}$$

where $\Delta E(t) = E(t) - E_0$ is the deviation of the kinetic energy E from the design energy E_0, and F_x^{col} is the horizontal Lorentz force (or Talman's force [2]) due to bunch collective interaction on the curved path:

$$F_x^{\text{col}} = F^{\text{CSCF}} + F_x^{\text{eff}}, \qquad F^{\text{CSCF}} = \frac{e\beta_s A_s^{\text{col}}}{r} \simeq e\frac{\Phi^{\text{col}} + \Delta\Phi}{r} \tag{2}$$

where F^{CSCF} is the "centrifugal space charge force" (using retarded potentials), and $\Delta\Phi = A_s^{\text{col}} - \Phi^{\text{col}}$. The change of E follows

$$\frac{dE}{cdt} = \mathbf{F}\cdot\mathbf{v}/c = -e\frac{d\Phi}{cdt} + F_v^{\text{eff}} \tag{3}$$

and thus after integration one gets $\Delta E/r$ in Eq. (1)

$$\frac{\Delta E}{r} = \frac{\Delta E^{\text{tot}}(t_0)}{r} + \frac{1}{r}\int_{t_0}^{t} F_v^{\text{eff}}(t')cdt' - e\frac{\Phi^{\text{col}}}{r} \tag{4}$$

with $\Delta E^{\text{tot}}(t_0) = (E + e\Phi^{\text{col}})|_{t_0} - E_0$. The effective forces in Eqs. (2) and (4) are

$$F_x^{\text{eff}} = \frac{\partial \mathcal{L}_{\text{int}}^{\text{col}}}{\partial x} - e\frac{dA_x^{\text{col}}}{cdt}, \qquad F_v^{\text{eff}} = -\frac{\partial \mathcal{L}_{\text{int}}^{\text{col}}}{c\partial t} \tag{5}$$

where ∂_x and ∂_t only act on Φ and A_λ in the interaction Lagrangian

$$\mathcal{L}_{\text{int}}^{\text{col}} = -e(\Phi^{\text{col}} - \sum_\lambda \beta_\lambda A_\lambda). \tag{6}$$

Here $q_\lambda = \mathbf{e}_\lambda \cdot \mathbf{q}$ for $\mathbf{q}$ being $\boldsymbol{\beta}$ or $\mathbf{A}$, with $\mathbf{e}_\lambda$ the Frenet-Serret bases. Using retarded potentials, Φ^{col} and A_s^{col} exhibit logarithmic dependence on the particles' transverse position in bunch due to local interaction singularities. However, since locally the nearby particles are nearly parallel in motion for ultra-relativistic bunches, there is relativistic cancellation bewteen the local interaction contributions to A_s^{col} in Eq. (2) and to Φ^{col} in Eq. (4). Similar cancellation also occurs bewteen Φ^{col} and A_s^{col} in $\mathcal{L}_{\text{int}}^{\text{col}}$ in Eq. (6). Thus the effective forces in Eq. (5) are dominated by contributions from the long-range interactions and are basically free from the logarithmic behavior. After inserting Eqs. (2) and (4) into Eq. (1), and letting $G_{\text{res}} = e(A_s^{\text{col}} - \Phi^{\text{col}})/r$, one gets

$$\frac{d^2x}{c^2dt^2} + \frac{x}{R^2} \simeq \frac{1}{E_0}\left(\frac{\Delta E^{\text{tot}}}{r} + \frac{1}{r}\int_{t_0}^{t} F_v^{\text{eff}} + F_x^{\text{eff}} + G_{\text{res}}\right). \tag{7}$$

For short bunches on a circular path, $\sigma_z/R \ll 1$, we have [1] $\mathcal{O}(G_{\text{res}}) \ll \mathcal{O}(F_x^{\text{eff}}) \ll \mathcal{O}(F_v^{\text{eff}})$. Hence G_{res} in Eq. (7) is practically negligible.

CANCELLATION IN VARIOUS CASES

Note the horizontal dynamics is driven by *both* $\Delta E/r$ and F_x^{col} in Eq. (1), and the cancellation is between the logarithmic potentials in each of these two driving terms. Including one term without the other may lead to unrealistic results.

F_x^{col} in a Storage Ring

The importance of F_x^{col} in a storage ring, due to the collective contribution of the single particle generated radiation (or acceleration) fields on the curved path, was first pointed out by Talman [2]. If only F_x^{col} in Eq. (1) is included without the $\Delta E/r$ term, the drastic dependence of F_x^{col} (due to A_s^{col} in Eq. (2)) on the particles' transverse position—as shown in Fig. 1 of Ref. [2]—could lead to horizontal tune shift and chromaticity and contribute to the appearance of nonlinear resonances (Fig. 2 of Ref. [2]). However, this effect of eA_s^{col}/r in F_x^{col} of Eq. (2) is basically cancelled by the effect of $e\Phi^{\text{col}}/r$ in $\Delta E/r$ of Eq. (4).

Cancellation for a Line Coasting Beam

The cancellation effect was first pointed out by Lee [3] for a coasting beam when both the two driving terms in Eq. (1) are included. For the line coasting beam example ($\beta_s = 1$) in Ref. [3], one gets for $x = r - R$ and $w = x/R$ (R: radius of equilibrium orbit)

$$\mathcal{L}_{\text{int}}^{\text{col}} = -e(\Phi^{\text{col}} - A_s^{\text{col}}) = e\Delta\Phi$$

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$$= -2\lambda e\left[2 - w + \frac{w^2}{8} + \frac{1}{4}w^2\ln\frac{w}{8} + \cdots\right], \quad (8)$$

$$F_x^{\rm eff} = -\frac{2\lambda e}{R}\left[-1 + \frac{w}{2} + \frac{1}{2}w\ln\frac{w}{8} + \cdots\right], \quad (9)$$

$$F_v^{\rm eff} = 0, \quad (10)$$

$$F_x^{\rm col} = \frac{2\lambda e}{R}\left[-\ln\frac{w}{8} - 1 + \frac{3}{2}w + w\ln\frac{w}{8} + \cdots\right], (11)$$

$$\frac{-e\Phi^{\rm col}}{R} = -\frac{2\lambda e}{R}\left[-\ln\frac{w}{8} + \frac{w}{2} + \frac{w}{2}\ln\frac{w}{8} + \cdots\right] \quad (12)$$

which yields

$$\frac{\Delta E}{R} + F_x^{\rm col} = \frac{\Delta E^{\rm tot}}{R} + F_x^{\rm eff}. \quad (13)$$

Note that $F_x^{\rm col}$ is a centrifugal force with a divergent gradient $\partial F_x^{\rm col}/\partial x \simeq -2\lambda e/Rx$. In contrast, the effective radial force has negligible gradient compared to $\partial_x F_x^{\rm col}$

$$\frac{\partial F_x^{\rm eff}}{\partial x} = \frac{2\lambda e}{R}\left(\frac{1}{2R} + \frac{1}{2R}\ln\frac{w}{8} + \cdots\right). \quad (14)$$

Here the constant part of $F_x^{\rm eff}$ serves to readjust the equilibrium orbit for each particle

$$\frac{(E + e\Phi^{\rm col})|_R - E_0}{R} + F_x^{\rm eff}(R) = 0. \quad (15)$$

Non-inertial Space Charge Force

We now turn to short bunches. For a steady-state line bunch on a circular orbit with uniform charge distribution acting on off-orbit particles, the longitudinal electric field is given by Eq. (5) of Ref. [4], in which the second term gives the non-inertial space charge force $F^{\rm NSCF}$. If $F_x^{\rm col}$ is not included in Eq. (1), the kinetic energy change ΔE induced by $F^{\rm NSCF}$ could cause extra emittance growth in addition to that caused by the usual CSR forces, as shown in Figs. 7-13 of Ref. [4]. However, studies show [5] that $F^{\rm NSCF}$ is $-e\,d\Phi^{\rm col}/cdt$ term in Eq. (3). Thus once $F_x^{\rm col}$ is included on the equal footing as $\Delta E/r$ in Eq. (1), the integrated effect of $F^{\rm NSCF}$, or $-e\Phi^{\rm col}/r$ in Eq. (4), is actually cancelled by the $e\Phi^{\rm col}/r$ term in $F_x^{\rm col}$ of Eq. (2).

Transient Longitudinal Force

For a rigid-line-bunch entering a circular orbit from a straight path, the longitudinal collective force on the particles is given by Eq. (87) of Ref.[6]:

$$\frac{dE(z,\theta)}{cdt} = T_1(z,R,\theta) + T_2(z,R,\theta) \quad (16)$$

where for $K = -2e^2/(3R^2)^{1/3}$ and $z_\theta = R\theta^3/24$,

$$T_1 = K\left[\lambda(z - z_\theta) - \lambda(z - 4z_\theta)\right]/z_\theta^{1/3}, \quad (17)$$

$$T_2 = K\int_{z-z_\theta}^{z}\frac{dz'}{(z - z')^{1/3}}\frac{d\lambda(z')}{dz'} \quad (18)$$

with R the radius of the circular orbit, $z = s - \beta ct$ the longitudinal position of the particle in the bunch, λ the longitudinal charge density, and θ the angle of the bunch into the bend. Sometimes in CSR simulations, Eq. (16) is used to calculate ΔE in Eq. (1) and the consequent effect on the bunch horizontal dynamics without $F_x^{\rm col}$ as another driving term for Eq. (1). Comparing with Eq. (3), one finds a part of $dE(z,\phi)/cdt$ in Eq. (16) is related to the potential energy change. Thus the integrated effect of $-ed\Phi^{\rm col}/cdt$ in Eqs. (3) or (16)— which contributes to $-e\Phi^{\rm col}/r$ in Eq. (4)— is actually cancelled by $e\Phi^{\rm col}/r$ in Eq. (2) once $F_x^{\rm col}$ is included in these simulations. After the cancellation, the horizontal dynamics is governed by the effective forces as in Eq. (7). For example, compared to Eq. (16), the *effective* longitudinal force at entrance of a bend is

$$F_v^{\rm eff}(z,\theta) = T_1'(z,\theta) + T_2(z,\theta) \quad (19)$$

with T_2 in Eq. (18), and $T_1'(z,\theta)$ derived using Ref. [7]

$$T_1'(z,\theta) = e^2\int_{z_\theta}^{\infty}dz'\left[\frac{d\lambda(z - z')}{dz'} \times \frac{2\sin^2(\theta/2)}{\sqrt{(z' - 4z_\theta)^2 + [R(1 - \cos\theta)/\gamma]^2}}\right]. \quad (20)$$

Converging Beams in Chicanes

For a 4-bend bunch compression chicane, the maximum compression often occurs during the drift before the last bend, where the hard compression could cause significant potential energy change $\Phi^{\rm col} - \Phi^{\rm col}(t_0)$ which further increases the kinetic energy spread σ_δ, as given in Eq. (4). If only $\Delta E/r$ is used in Eq. (1) without $F_x^{\rm col}$, one would conclude [8] that significant emittance growth could result from this $\Delta\sigma_\delta$ as the bunch passes the 4th bend. However, $\Phi^{\rm col}$ in Eq. (4) is actually cancelled by that in Eq. (2). Thus according to Eq. (7), the horizontal dynamics in the 4-th bend depends on the canonical energy offset $\Delta E^{\rm tot}$ at the entrance of the bend, in addition to the effective forces. This $\Delta E^{\rm tot}$ is only changed by $F_v^{\rm eff}$ generated from the previous bend, but not by the beam convergence in the drift.

NEW DYNAMICS EQUATIONS

Using (x, x', z, δ) as dynamical variables, and s as an independent variable, with $\delta = (E - E_0)/E_0$, the complete first order equations of motion for dynamics in the bending plane are ($k_\beta(s)$ is the focusing strength)

$$\left\{\begin{aligned} \frac{dx}{ds} &= x' \\ \frac{dx'}{ds} &= -k_\beta^2 x + \frac{\delta}{R(s)} + \frac{F_x^{\rm col}}{E_0} \\ \frac{dz}{ds} &= -\frac{x}{R(s)} \\ \frac{d\delta}{ds} &= \frac{1}{E_0}(F_s^{\rm col} + x'F_x^{\rm col}) \end{aligned}\right. . \quad (21)$$

Here the local (nearby particle) interaction contributions to the longitudinal and transverse collective Lorentz forces, $F_s^{\rm col}$ and $F_x^{\rm col}$, need to be computed carefully; and their effects on the bunch dynamics eventually get cancelled implicitly as Eq. (21) is integrated over time. Using equations in Sec. 2, one can have the following new description of the dynamics with the cancellation effect explicitly expressed

$$\begin{cases} \dfrac{dx}{ds} = x' \\ \dfrac{dx'}{ds} = -k_\beta^2 x + \dfrac{\delta_H}{R(s)} + \dfrac{F_x^{\text{eff}} + G_{\text{res}}}{E_0} \\ \dfrac{dz}{ds} = -\dfrac{x}{R(s)} \\ \dfrac{d\delta_H}{ds} = \dfrac{F_v^{\text{eff}}}{E_0} \end{cases} \tag{22}$$

where $G_{\text{res}} = e(A_s^{\text{col}} - \Phi^{\text{col}})/R(s)$, and (x, x', z, δ_H) are the new dynamical variables with $\delta_H = [(E + e\Phi^{\text{col}}) - E_0]/E_0$ ($H = E + e\Phi^{\text{col}}$ is the Hamiltonian conjugate to t). For short bunches ($\sigma_z \ll R$), G_{res} is negligible; and F_v^{eff} and F_x^{eff} in Eq. (22) are mainly dominated by the long range interactions. For a coasting line beam [3], G_{res} is comparable to F_x^{eff}. However, the logarithmic singularities in A_s^{col} and in Φ^{col} are still cancelled.

To study the microbunching instability in a bunch compression chicane, we change the dynamical variables (x, x', z, δ_H) to $(J, \psi, \tilde{z}, \tilde{\delta}_H)$, with J, ψ the action-angle variables, $\tilde{z}$ the initial longitudinal position of a particle in the bunch and $\tilde{\delta}_H$ the initial uncorrelated canonical energy offset for zero effective forces

$$\begin{cases} x = \sqrt{2J\beta(s)}\cos[\psi + \psi_0(s)] + D_x\delta_H \\ x' = -\sqrt{\dfrac{2J}{\beta(s)}}\,\{\sin[\psi + \psi_0(s)] + \alpha(s)\cos[\psi + \psi_0(s)]\} \\ \qquad + D_x'\delta_H \\ z = -D_x' x + D_x x' + \tilde{z} + R_{56}\delta_H \\ \delta_H = \tilde{\delta}_H - u\tilde{z} \end{cases} \tag{23}$$

with $\beta(s)$ and $\alpha(s)$ the designed twiss parameters in the horizontal phase space, u the initial linear δ-z correlation imposed on the bunch by an RF cavity, and [9]

$$\psi_0(s) = \int_0^s \frac{ds'}{\beta(s')}, \quad R_{56}(s) = -\int_0^s \frac{D(s')}{R(s')}ds',$$
$$D_x(s) = \sqrt{\beta(s)}\int_0^s \frac{ds'}{R(s')}\sqrt{\beta(s')}\sin[\psi_0(s) - \psi_0(s')]. \tag{24}$$

Converting Eq. (22) to equations for the new variables, we have the Vlasov equation for the distribution function $\rho(J, \psi, \tilde{z}, \tilde{\delta}_H, s)$

$$\frac{\partial \rho}{\partial s} + \frac{F_v^{\text{eff}}}{E_0}\cdot A + \frac{F_x^{\text{eff}}}{E_0}\cdot B = 0 \tag{25}$$

with

$$A = \eta_1\frac{\partial \rho}{\partial J} + \eta_2\frac{\partial \rho}{\partial \psi} - R_{56}\frac{\partial \rho}{\partial \tilde{z}} + (1 - uR_{56})\frac{\partial \rho}{\partial \tilde{\delta}_H},$$
$$B = -\left[f_1\frac{\partial \rho}{\partial J} + f_2\frac{\partial \rho}{\partial \psi} + D_x\frac{\partial \rho}{\partial \tilde{z}} + uD_x\frac{\partial \rho}{\partial \tilde{\delta}_H}\right],$$

for

$$f_1(J, \psi, s) = \sqrt{2J\beta(s)}\sin[\psi + \psi_0(s)],$$
$$f_2(J, \psi, s) = \sqrt{\beta(s)/2J}\cos[\psi + \psi_0(s)], \tag{26}$$

$$\eta_1 = D_x' f_1 - D_x f_1', \qquad \eta_2 = D_x' f_2 - D_x f_2' \tag{27}$$

where $f_{1,2}' = \partial f_{1,2}/\partial s$ and $D_x' = dD_x/ds$. At steady-state, the effective forces yield impedances [9]

$$\frac{F_{v,x}^{\text{eff}}(z)}{E_0} = \frac{Nr_e}{\gamma_0}\int_{-\infty}^{\infty}\lambda(k)Z_{v,x}^{\text{eff}}(k)e^{ikz}dk \tag{28}$$

$$Z_v^{\text{eff}} = \frac{ik^{1/3}}{R(s)^{2/3}}(-0.94 + 1.63i), \qquad Z_x^{\text{eff}} = -\frac{2}{R(s)}.$$

For $(kR)^{1/3} \gg 1$, one has $\mathcal{O}(Z_x^{\text{eff}}) \ll \mathcal{O}(Z_v^{\text{eff}})$.

DISCUSSIONS

Eq. (22) shows that the dispersion effect is related to the canonical energy offset δ_H instead of the kinetic energy offset δ. This is because in Eq. (21), F_x^{col} contains F^{CSCF} (as in Eq. (2)) which represents the dispersion effect for the potential energy $e\Phi^{\text{col}}/E_0$, just as $\delta/R(s)$ in Eq. (21) represents the dispersion effect for the kinetic energy δ. Consequently, for a single bend spectrometer, due to the existence of Talman's force F_x^{col}, the horizontal particle distribution from the spectrometer measurement is actually related to the canonical energy spread instead of the kinetic energy spread. These measurements also include effects of F_v^{eff} and F_x^{eff}. Therefore the spectrometer data needs to be carefully interpreted when the collective interactions are strong, especially when the potential energy spread is no less than that of the kinetic energy spread.

For an achromatic bending system, the initial $\delta_H(s_0)$ in Eq. (22) does not directly cause emittance growth. However, $\delta_H(s_0)$ could play a role of Landau damping in the microbunching process. Previous studies [9] of the microbunching in a chicane, based on Eq. (21) assuming $F_x^{\text{col}} = 0$ and $\Phi^{\text{col}} = 0$, show Landau damping due to $\rho(J, \psi, \tilde{z}, \tilde{\delta}, 0)$. However, with $F_x^{\text{col}} \neq 0$ and $\Phi^{\text{col}} \neq 0$, Eqs. (25)-(28) show that it is really $\rho(J, \psi, \tilde{z}, \tilde{\delta}_H, 0)$ which causes the decoherence for the microbunching process. Here the distribution over $\tilde{\delta}_H$ at s_0 needs to be carefully determined. The effect of transient F_v^{eff} and F_x^{eff} in Eq. (25) will be further studied.

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