

# TWO-STREAM STUDIES FOR HEAVY ION BEAM PROPAGATION IN A REACTOR CHAMBER\*

D. V. Rose,<sup>†</sup> T. C. Genoni, and D. R. Welch, Mission Research Corp., Albuquerque, NM 87110  
C. L. Olson, Sandia National Laboratories, Albuquerque, NM 87185

## Abstract

Growth rates for the two-stream instability for a heavy-ion beam propagating in a collisionless plasma are analyzed analytically and numerically using particle-in-cell simulations. Good agreement between the analytic and simulation results is found for a wide range of parameters consistent with heavy-ion driven inertial fusion energy chamber designs.

## INTRODUCTION

Heavy-ion inertial-fusion-energy requires beams of high-energy, heavy-ions to be focused and propagated across a gas and plasma filled reactor chamber. The robust point design [1] uses the neutralized ballistic transport scheme which requires that the focusing force be applied to the ion beams outside of the reactor chamber. Prior to entering the chamber, the beams pass through a pre-ionized plasma region which provides charge neutralization. Target designs require lower energy “foot” beams to preheat the target prior to the arrival of the high energy beams which provide the final target impulse. The chamber environment for the foot beams includes a low density ( $\sim 3$  mtorr) background gas. The main beams propagate through gas which is partially ionized near the target by photo-ionization due to the preheated target. In addition, aerosols may be present along all or part of the beam path lengths, possibly impacting the transport properties [2].

A previous analysis [3] of two-stream instability growth and saturation for converging heavy ion beams propagating in a reactor chamber considered the rate of change of the maximum growing wave-number with increasing beam and plasma densities as the converging beam crosses the chamber. That analysis concluded that for a specific range of parameters, a reasonable propagation window existed in part because of the changing instability mode as the beams propagated across the chamber.

One goal of this work is to revisit the assumptions made for the analysis of Ref. [3] in light of the recent robust point design [1] parameters. As a first step towards this goal, a study of both 1-D and 2-D two-stream instability modes is examined in this paper.

\* This work is supported by the U.S. Department of Energy through the Lawrence Berkeley National Laboratory, the Princeton Plasma Physics Laboratory, and the ARIES Program.

<sup>†</sup> drose@mrcabq.com

## 1-D STUDIES

The growth rate of the electrostatic two-stream instability is investigated in 1-D for the case of cold electron and heavy-ion streams propagating through a stationary background ion population. The electron and ion beams are moving in the same direction with (possibly) different speeds. Charge neutralization is enforced in all cases by setting  $n_b + n_p = n_e$ , where  $n_{b,e,p}$  are the beam ion, electron, and plasma ion densities, respectively.

The electrostatic dispersion relation for an ion beam propagating along with an electron stream through a stationary background ion population can be written as (see, for example, Ref. [4])

$$\frac{\omega_b^2}{(\omega - kv_b)^2} + \frac{\omega_e^2}{(\omega - kv_e)^2} + \frac{\omega_p^2}{\omega^2} = 1, \quad (1)$$

where  $\omega_{b,e,p}$  are the beam ion, electron, and plasma ion plasma frequencies, respectively, and  $v_{b,e}$  are the beam ion and electron speeds.

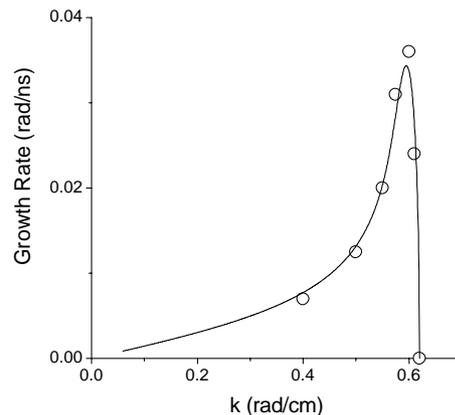


Figure 1: 1-D two-stream growth rate from dispersion relation (solid line) and 1-D LSP simulations (open circles). The parameters are  $n_e = 10^9 \text{ cm}^{-3}$ ,  $n_e/n_b = 2$ ,  $n_b = n_p$ , and  $v_e = 0.1c$ .

Electrostatic particle-in-cell simulations using LSP [5] have been carried out in 1-D with periodic boundary conditions for direct comparison to the solutions of Eq. (1). The simulation length is set to be  $2\pi/k$ , where  $k$  is the wave number of interest. The instability growth is stimulated by an initial sinusoidal perturbation of wave-number  $k$  and amplitude  $0.0001v_e$  applied to the electron velocity.

Comparisons were carried out over a wide range of parameters, including electron speed, density, and wavenumbers. Very good agreement over all parameter ranges was found. A sample comparison is shown in Fig. 1 for the specific case of  $v_e = 0.1c$ , and ion masses of  $200M$  for the beam ions and  $12M$  for the initially stationary plasma ions, where  $M$  is the mass of a proton. (These mass values are used throughout this paper.) Note that the beam ion speed,  $v_b$ , is fixed at  $0.2c$  throughout this work. The solid line is the maximum growth rate as a function of  $k$ , with peak growth found for  $k \sim \omega_e/v_e$ . The data points are individual LSP simulations carried out for different periodic lengths  $2\pi/k$ . The peak growth rate in the simulations is slightly different than that found from Eq. (1), but the overall  $k$ -dependence is reproduced.

## 2-D SURFACE MODE

Surface wave two-stream growth is assessed for the limit of a “hard-edge” ion beam propagating with an electron stream through a background ion density. In the electrostatic limit, an eigenfunction analysis leads to the dispersion relation

$$\begin{aligned} 1 & - \frac{\omega_b^2}{(\omega - kv_b)^2} - \frac{\omega_{e1}^2}{(\omega - kv_e)^2} - \frac{\omega_p^2}{\omega^2} \\ & = -\alpha \left( 1 - \frac{\omega_{e2}^2}{\omega^2} - \frac{\omega_p^2}{\omega^2} \right), \end{aligned} \quad (2)$$

where  $\alpha$  is a geometric factor given by

$$\alpha = \frac{I_0(ka)}{I_1(ka)} \left[ \frac{I_0(kR)K_1(ka) + I_1(ka)K_0(kR)}{I_0(kR)K_0(ka) - I_0(ka)K_0(kR)} \right]. \quad (3)$$

(A fully electromagnetic formulation for an annular relativistic electron beam in a background plasma has been given by Jones [6]. In the electrostatic limit, a dispersion relation similar to Eq. (2) was obtained by Fukano, *et al.* [7] for the case of no outer radial boundary.) The electron density is again set such that  $n_e = n_p + n_b$ , which leads to a higher electron density inside the beam (“e1”) than outside (“e2”). The outer conducting wall is at  $R$  and the beam radius is  $a$ . These values are fixed at  $R = 2$  cm and  $a = 1$  cm throughout this paper.

For the case of a smooth density gradient at the edge of the beam a simple analytic dispersion relation such as Eq. (2) no longer applies. In this more general case, the dispersion analysis is carried out by setting the determinant of the discretized system of linearized electrostatic equations equal to zero. We choose a simple form for the ion beam density profile as a function of radius

$$n_b(r) = n_{b0}g(r) = n_{b0} \frac{1}{e^{(r-a)/f} + 1}, \quad (4)$$

where  $n_{b0}$  is approximately the on-axis value of the beam density and  $f$  determines the inclination angle of the normalized beam density profile  $g(r)$  at  $r = a$  [this angle in

radians is  $\tan^{-1}(0.25/f)$ ]. The radial profile of the axial electron velocity is obtained from the condition for current neutralization

$$v_e(r) = \frac{n_b(r)v_b}{n_b(r) + n_p}. \quad (5)$$

In the limit of very small values of  $f$ , this analysis converges to the same result as obtained from the solution of Eq. (2), the “hard-edge” limit.

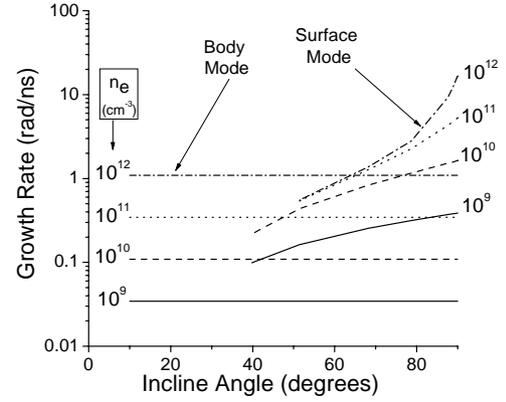


Figure 2: The maximum two-stream growth rates for the 1-D “body” and 2-D “surface” modes as a function of density and inclination angle. The parameters are  $n_e = 10^9 \text{ cm}^{-3}$ ,  $n_{e1}/n_b = 2$ ,  $n_b = n_p$ , and  $v_e = 0.1c$ .

A comparison between the body mode given by the solution of Eq. (1) and the surface mode is shown in Fig. (2) for  $v_e = 0.1c$ . The maximum body mode growth rate (continuous spectrum) is independent of inclination angle while the maximum surface mode growth rate increases with increasing inclination angle. For the range of parameters considered here, the maximum growth rate of the surface mode at large inclination angle is always greater than the maximum body mode growth rate for a given value of  $n_e$ .

## 2-D SIMULATIONS

Periodic, electrostatic, 2-D LSP simulations were carried out for direct comparison with the results presented above. The simulations are periodic in the axial direction with a conducting boundary at  $R$ . All velocities are initialized in the axial direction only. The axial electron velocity is perturbed as described above.

Figure 3 compares the calculations and simulations for the hard-edge beam limit at  $n_e = 10^9 \text{ cm}^{-3}$  and  $v_e = 0.1c$  as a function of  $k$ . Qualitative agreement in the overall  $k$ -spectrum is found, which is significantly broader than the similar  $k$ -spectrum found in the 1-D or body mode results (Fig. 1). The disagreement between the calculations and the simulation results around the maximum growth values is attributed to the finite radial zoning in the LSP simulations that gives an effective inclination angle to the beam once the electrons gain some radial velocity spread. Finer

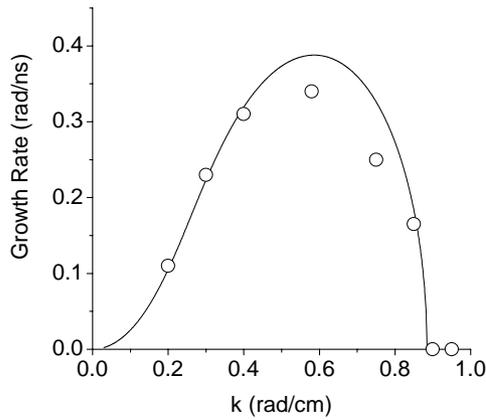


Figure 3: 2-D two-stream growth rate from dispersion relation (solid line) and 2-D LSP simulations (open circles) as a function of  $k$ . The parameters are  $n_e = 10^9 \text{ cm}^{-3}$ ,  $n_{e1}/n_b = 2$ ,  $n_b = n_p$ , and  $v_e = 0.1c$ .

resolution radial zoning and better particle statistics produces growth rates which tend towards the calculated values.

A comparison of the calculated and simulated surface mode growth rate as a function of inclination angle is shown in Fig. 4 for  $n_e(0) = 10^9 \text{ cm}^{-3}$  and  $v_e(0) = 0.1c$ . The calculated growth rate increases linearly with inclination angle for our choice of beam radial profile, as given in Eq. (4).

## DISCUSSION

Simulations presented here are idealized in order to facilitate direct comparison to analytic models of two-stream growth. Future work will examine the impact of a converging ion beam on these growth rates.

Recent analysis of current neutralization for ion beams propagating in a background plasma [8, 9] shows limits on the degree to which current neutralization can be obtained. These limits may be related to the growth and saturation of the two-stream instability for the electron return current.

Also, we note that recent simulation results [8] show a beneficial ion beam pinching effect near the target driven by a reduction in the return current near the beam focus. Again, the growth and saturation of the two-stream instability may be partially responsible for the abrupt decrease in the electron return current in this region. Extensions to the work presented here are being directed towards examining this issue.

We note that a considerable body of work on two-stream instabilities for heavy ion beams propagating through low density background plasmas, applicable to a variety of periodic focusing accelerators and transport systems, can be found in the literature; see, for example, Ref. [10], and references therein.

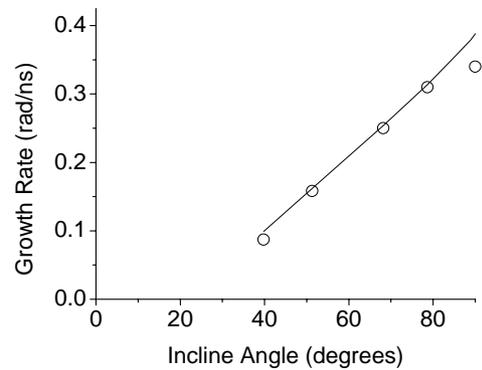


Figure 4: Two-stream surface mode growth rate calculations (solid line) and 2-D LSP simulations (open circles). The parameters are  $n_e = 10^9 \text{ cm}^{-3}$ ,  $n_e/n_b = 2$ ,  $n_b = n_p$ , and  $v_e = 0.1c$ .

## REFERENCES

- [1] S. S. Yu, *et al.*, submitted to Fusion Sci. Technol. (2003).
- [2] D. V. Rose, *et al.*, submitted to Fusion Sci. Technol. (2003).
- [3] P. Stroud, Lasers and Part. Beams **4**, 261 (1986).
- [4] R. Briggs, in Advances in Plasma Physics, edited by A. Simon and W. B. Thompson (Interscience, New York, 1971), p. 43.
- [5] LSP is a software product of Mission Research Corp., Albuquerque, NM 87110 (<http://www.mrcabq.com>).
- [6] M. E. Jones, Phys. Fluids **26**, 1928 (1983).
- [7] A. Fukano, *et al.*, J. Phys. Soc. Jap. **60**, 2627 (1991).
- [8] D. R. Welch, *et al.*, Phys. Plasmas **9**, 2344 (2002).
- [9] I. Kaganovich, *et al.*, Phys. Plasmas **8**, 4180 (2001).
- [10] R. C. Davidson, *et al.* Nucl. Instrum. Meth. Phys. Res. A **464**, 493 (2001).