A FAST METHOD TO ESTIMATE THE GAIN OF THE MICROBUNCH INSTABILITY IN A BUNCH COMPRESSOR

S. Reiche and J.B. Rosenzweig

University of California Los Angeles – Department of Physics, CA 90095-1547, USA

Abstract

To reach high peak currents driving Free-Electron Lasers an initial chirped electron bunch is compressed in a bunch compressor. The interaction of the electron beam with its radiation field can yield a collective instability, which amplifies any initial modulation in the current profile. We present a model, which allows one to derive an explicit analytical expression for the gain of the microbunch instability. The results are compared to those of the more complex analytical models.

INTRODUCTION

Many FEL experiments [1] requires a bunch compressor to increase the peak current. The magnetic chicane resembles a single period of an undulator and like the bunching effect in an FEL [2, 3] the interaction between the coherent synchrotron radiation (CSR) [4] and the electron beam can enhance the amplitude of an initial current modulation[5]. The mechanism has striking similarity to the FEL process, although the electron motion is more complex than the averaged motion needed for the FEL model. The interaction with the CSR field is expressed by a potential, acting instantenously on the electrons.

MOTION IN A MAGNETIC CHICANE

A magnetic chicane consists typically of bending magnets, drifts, and, optionally, focusing quadrupoles. For a simpler comparison to the FEL we exclude the latter two components from the discussion.

Because the electron beam interacts with the spontaneous radiation, the dispersion function η has to be calculated for all positions, where the electron energy changes. We sum up all contributions to the longitudinal position, resulting in

$$\zeta(s) = \zeta(s_0) + \int_{s_0}^{s} \delta(s') \tilde{R}_{56}(s,s') ds'. \qquad (1)$$

Here s and s_0 denote the final and initial position on the design orbit, respectively, $\zeta = ct$ is the position in the frame of the moving bunch and $\delta = (\gamma - \gamma_0)/\gamma_0$ is the normalized deviation of the electron energy from the mean energy γ_0.

We consider an idealized chicane, which consists of three bending magnets with a bend radius R and no drift space separating them. The outer magnets have a length of L while the inner one, which bends in the opposing direction of the outer two, is twice as long. With the initial

conditions $\eta(s') = 0$ and $\eta'(s') = 0$ and a small bending angle $\theta_b \approx L/R \ll 1$, the dispersion function is

$$\frac{\eta}{R} = \begin{cases} \frac{1}{2}\left(\frac{s-s'}{R}\right)^2 & \text{(I,I), (II,II),} \\ & \text{(III,III)} \\ \left(\frac{s-L}{R}\right)^2 - \frac{1}{2}\left(\frac{s-s'}{R}\right)^2 & \text{(II,I)} \\ \left(\frac{s-3L}{R}\right)^2 - \frac{1}{2}\left(\frac{s-s'}{R}\right)^2 & \text{(III,II)} \\ \frac{1}{2}\left(\frac{s-3L}{R}\right)^2 + \frac{1}{2}\left(\frac{3L-s'}{R}\right)^2 - \left(\frac{2L}{R}\right)^2 \\ \quad - \left(\frac{s-3L}{R}\right)\left(\frac{L-s'}{R}\right) & \text{(III,I)} \end{cases}$$

$$\qquad (2)$$

where the pair of Roman numers indicates, in which dipole the end and start positions s and s', respectively, are located. We incorporated the bend direction of the dipoles into the sign of the bend radius with $R(s) = R$ in the first and third dipole and $R(s) = -R$ in the second dipole.

The differential equation for the longitudinal motion ($s_0 = 0$) becomes

$$\frac{d\zeta}{ds} = \int_0^s \delta(s') \frac{s-s'}{R(s)R(s')} ds'. \qquad (3)$$

To describe a microbunched distribution of electrons, we assume a coasting beam with a small modulation

$$I(\zeta, s) = I_0 \left[1 + |b(s)|\cos(k\zeta + \phi(s))\right] \quad , \qquad (4)$$

where k is the modulation wavenumber. The potential [4] seen by the electrons is

$$W(\zeta) = -\frac{2}{(3R^2)^{\frac{1}{3}}} \frac{1}{\zeta^{\frac{1}{3}}} \frac{\partial}{\partial \zeta} \qquad (5)$$

for $\zeta < 0$, and zero otherwise. The energy change of any given electron is

$$\frac{d\delta}{ds} = -\frac{I_0}{I_A \gamma_0} \frac{2\Gamma(\frac{2}{3})k^{\frac{1}{3}}}{(3R^2)^{\frac{1}{3}}} |b(s)| \sin\left(k\zeta + \phi(s) + \frac{\pi}{3}\right). \quad (6)$$

The growth of the energy modulation scales linearly with the modulation of the current.

THE LOW GAIN MODEL

A change in the particle energy has a delayed effect on the particle's longitudinal position, which grows with the third power in s. In addition, the change in the longitudinal position is also inhibited by the change in the polarity of

the bending magnets. Particles with higher energy fall behind due to the larger bend radius, but catch up due to the shorter path length after a polarity change. Thus, for short time scales, and low gain, $\delta(s')$ can be expected to change linearly in s'. This assumption of klystron-like behavior is invalid for high currents. We discuss this limit further in the following section.

We model the initial current by a equidistant distribution plus an added sinusoidal modulation in the positions with $\Delta\zeta_j = \Delta\zeta \sin(k\zeta_{0,j} + \phi)$ for the jth electron, where ζ_0 is the initial position of the electron. Because the effective radiation potential is harmonic in ζ, the resulting modulation in the longitudinal position is harmonic as well with $\delta\zeta = Z(s)\sin(k\zeta_{j,0} + \psi)$, with $Z(0) = 0$. In our low gain model the initial offset $\Delta\zeta$ and the modulation amplitude $Z(s)$ are much smaller than the modulation wavelength. thus, the bunching factor can be approximated with $b(s) = k[\Delta\zeta e^{i\phi} + Z(s)e^{i\psi}]$.

As long as $Z(s)$ is comparable to the initial modulation amplitude $\Delta\zeta$, the bunching factor can be taken as constant and Eq. 6 can easily be integrated, giving a linear dependence on s. Inserting $\delta(s)$ into Eq. 3, the longitudinal position evolves in the chicane as

$$
\begin{aligned}
\zeta_j(s) &= \zeta_j(0) - \frac{I_0}{I_A\gamma_0} \frac{2\Gamma\left(\frac{2}{3}\right)k^{\frac{1}{3}}}{(3R^2)^{\frac{1}{3}}}|b(0)| \\
&\quad \cdot \sin\left(k\zeta_j(0) + \phi + \frac{\pi}{3}\right)\Phi(s) \quad,
\end{aligned} \tag{7}
$$

with

$$
\begin{aligned}
\Phi(s) &= \int_0^s \left[\int_0^{s''} \frac{s'(s''-s')}{R(s'')R(s')}ds'\right]ds'' \\
&= \begin{cases}
\frac{1}{24}\frac{s^4}{R^2} & \text{(I)} \\
\frac{1}{24}\frac{s^4}{R^2} - \frac{1}{2}\frac{L^2}{R^2}s^2 + \frac{2}{3}\frac{L^3}{R^2}s - \frac{1}{6}\frac{L^4}{R^2} & \text{(II)} \\
\frac{1}{24}\frac{s^4}{R^2} - 4\frac{L^2}{R^2}s^2 + \frac{52}{3}\frac{L^3}{R^2}s - \frac{56}{3}\frac{L^4}{R^2} & \text{(III)}.
\end{cases}
\end{aligned}
$$

The Roman numerals indicate the dipole in which the position s lies. The final, normalized amplitude of the sine-term in Eq. 7 is

$$
\xi = \frac{I_0\Gamma\left(\frac{2}{3}\right)}{2I_A\gamma_0}\left(\frac{8L^3k}{3R^2}\right)^{\frac{4}{3}}. \tag{8}
$$

With $kZ(4L) = \xi|b(0)| \ll 1$ the gain, defined as the ratio between final and initial amplitude of modulation, becomes

$$
G = \frac{|b(4L)|}{|b(0)|} = \sqrt{1 + \xi + \xi^2}. \tag{9}
$$

As an example of a generic magnetic chicane, modeling the first LCLS bunch compressor ($\gamma = 500$, $I_0 = 100$ A, $R = 12$ m and $L = 1.5$ m), an initial modulation with a period of 5μm would grow by a factor of 25.

The gain growth (Eq. 9) has a singularity at a zero modulation period length. This artifact is removed if energy spread is included in the model. For a Gaussian energy distribution with rms spread σ_δ the current modulation evolves
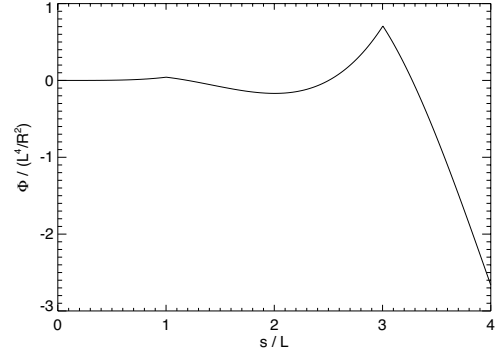

Figure 1: The function Φ (Eq. 8).

as

$$
I = I_0\left[1 + e^{-\frac{1}{2}(\sigma_\delta R_{56}k)^2}|b(0)|\cos(k\zeta + \phi)\right] \quad. \tag{10}
$$

The modulation decays if the spread in the longitudinal position $\sigma_\delta R_{56}$ is comparable with the modulation period length. The initial modulation is sheared mainly in the second dipole, where the value of matrix element R_{56} changes significantly. Because the seed for the microbunch instability – the accumulated change in the electron energy – occurs before that, the change in the longitudinal position (Eq. 8) is hardly effected. We can just apply the damping factor due to the energy spread to the previous results of Eq. 9. With $R_{56}(4L, 0) = -(4/3)L^3/R^2$, the final gain is now given by

$$
G = e^{-\alpha|\xi|^{\frac{3}{2}}}\sqrt{1 + \xi + \xi^2}, \tag{11}
$$

defining the normalized energy spread as

$$
\alpha = \sqrt{\left(\frac{I_A\gamma_0}{2I_0\Gamma\left(\frac{2}{3}\right)}\right)^3 \sigma_\delta^2}. \tag{12}
$$

For the LCLS case with an energy spread of 0.01% ($\alpha = 0.05$), the gain at 5 μm would be reduced by 7 orders of magnitudes.

THE HIGH GAIN MODEL

In our low gain model we assumed that the CSR microbunching instability does not drive any bunching within the first dipole. To qualitatively place a limit on this assumed scenario, we develop here a high-gain, exponential growth model as well. There may be situations where the exponential gain does not assert itself in the first dipole, but may, by compression and thus higher current, become notable in the last dipole.

We define the collective variables $B = -ik < e^{-i\Psi}\zeta >$ and $\Delta = < e^{-i\Psi}\delta >$, where $\Psi_j = k\zeta_{0,j}$ is the initial phase of the jth electron of a uniform distribution. The equations

of motion for a cold beam become

$$\frac{d\Delta}{ds} = -\frac{\rho_{CSR}^4}{kR^2}e^{i\frac{\pi}{3}}B \tag{13}$$

$$\frac{dB}{ds} = -i\frac{k}{R^2}\int_0^s \Delta(s-s')ds', \tag{14}$$

with the definition of the dimensionless ρ_{CSR}-parameter expressed as

$$\rho_{CSR} = \left[\frac{I_0}{I_A\gamma_0}4^{\frac{1}{3}}\Gamma\left(\frac{2}{3}\right)\right]^{\frac{1}{4}}(kR)^{\frac{1}{3}}. \tag{15}$$

The equations can be combined into a forth order differential equation. Using the ansatz $B \propto \exp[i\Lambda s]$ we obtain the dispersion relation $\Lambda^4 = (\rho_{CSR}/R)^4\exp(i5\pi/6)$. Two of the four roots have a negative imaginary part, corresponding to an exponentially growing instability. The growth rates are $(\rho_{CSR}/R)\sin(7\pi/24)$ and $(\rho_{CSR}/R)\sin(5\pi/24)$, respectively. The gain length of the high-gain CSR instability is roughly R/ρ_{CSR}. Because the calculations are based on a relatively small deflection angle ($L \ll R$) exponential gain within a single dipole becomes significant only for $\rho_{CSR} > \frac{R}{L} \gg 1$.

The start-up regime determines after how many gain lengths the exponential growth becomes dominant. The two growing modes have similar growth rate but different phase slippages (real part of Λ), so that the interference between these two is still noticeable after several gain lengths (Fig. 2). It takes at least 5 gain lengths before the bunching factor has grown by one order of magnitude, compensating an initial amplitude drop.
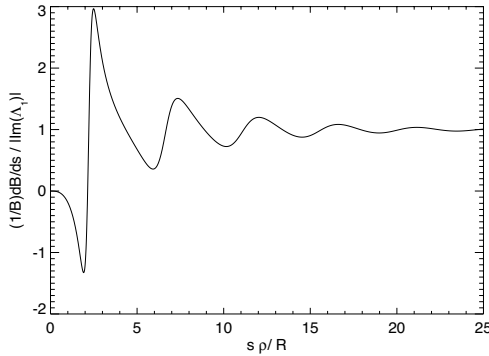

Figure 2: Growth rate of the bunching factor, normalized to the growth rate of the dominant growing mode.

For a characteristic spread σ_δ in energy the momentum dispersion couples it to a phase spread of approximately $(k/6R^2)\sigma_\delta s^3$. In units of the gain length ($\hat{s} = s\rho_{CSR}/R$) the normalized phase spread $\hat{\sigma}_\delta = (kR/6\rho_{CSR}^3)\sigma_\delta$ is independent on the bend radius and wavelength. An estimate on the mitigation of the instability by the energy spread is $\hat{\sigma}_\delta < 0.02$ by this criterion – any value larger than 0.02 would smear out the modulation and completely suppress the exponential growth of this instability.

CONCLUSIONS

We have derived a simple low-gain model to calculate the growth of an initial current modulation within a magnetic chicane. As in a klystron, the physics is split into the initial modulation in beam energy, and a change in the longitudinal position (followed by an enhanced emission level of radiation). The major assumption in our low-gain model is that the beam current modulation, which generates the coherent synchrotron radiation, is held constant over the entire chicane. The klystron-like assumption implies that the system does not have an exponential gain, but rather acts as a linear amplifier where the gain coefficient is dependent on the physical details of the beam and chicane. Fig. 3 shows the difference, however, between our klystron-like model and a self-consistent, more complex model [5].
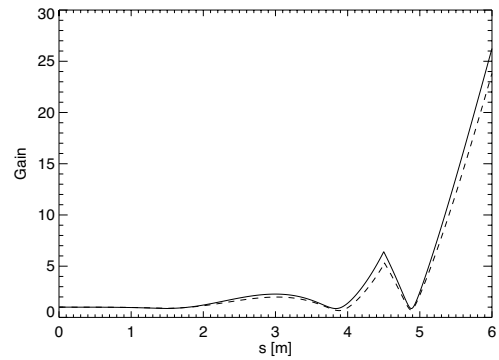

Figure 3: Evolution of the gain along the chicane, using the low gain model (Eq. 9) and a self-consistent numerical simulation (solid and dashed line, respectively). The input parameters are $I_0 = 100$ A, $\gamma_0 = 500$, $R = 12$ m and $L = 1.5$ m.

In order to extend the low-gain model to account for self-consistent behavior, we have developed a high gain model with a gain-length proportional to R/ρ_{CSR}. The parameter ρ_{CSR} must have a value in excess of unity, so that exponential gain occurs within a single dipole.

REFERENCES

[1] E.g. *Linac Coherent Light Source (LCLS) Design Study Report*, SLAC-R-521, UC-414 (1998) and *TESLA - Technical Design Report* TESLA-FEL 2001-05, DESY, Hamburg, Germany (2001)

[2] R. Bonifacio, C. Pellegrini, L.M. Narducci, Opt. Comm. **50** (1984) 373

[3] A.M. Kondratenko and E.L. Saldin, Par. Acc. **10** (1980) 207

[4] Y.S. Derbenev *et al.* TESLA-FEL 95-05, DESY, Hamburg, Germany (1995)

[5] S. Heifets *et al.* SLAC-PUB-9165, SLAC, Stanford, USA (2002)