NEW VORTICES IN AXISYMMETRIC INHOMOGENEOUS BEAMS

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Abstract
We analyzed localized vortices in non-neutral inhomogeneous by density and velocity electron beams propagating in vacuum along the external magnetic field. These vortices distinguish from well-known vortices of Larichev-Reznik or Reznik types, which used in [1]. New types of vortex are obtained by new solution method of nonlinear equations. The new method is development of a method described in [2]. That method distinguish from standard Larichev-Reznik or Reznik method, which used in [1]. It has been found new expression for electric field potential of vortex in a wave frame. The expression is axisymmetric in a wave frame. New vortices are new solitons. New vortices are the result of external disturbances or the appearance and development of instabilities like for example a diocotron instability in hollow beams and a slipping-instability in solid beams.

1 BASIC EQUATIONS

We investigate the nonrelativistic electron beam, which propagating in vacuum along the external homogeneous magnetic field B_0 in z-direction of cylindrical coordinate system (r,θ, z). An equilibrium and homogeneous by θ and z state of the system is characterized by radial distributions of electron density $n_0(r)$ and velocity $\boldsymbol{v}_0[0,v_{0\theta}(r), v_{0z}(r)]$ and the electron field potential $\varphi_0(r)$. We assume $\omega_c^2 >> \omega_p^2$, where ω_p - the plasma electron frequency, ω_c - the electron cyclotron frequency.

We investigate the nonsteady state of the system characterized by the deviations n, $\boldsymbol{v}$, φ from equilibrium values of n_0, $\boldsymbol{v}_0$, φ_0. The solution of the motion and continuity equations for the particles and Poisson equation for the electric fields potential we choose in the form of a travelling wave in which all the parameters are functions of the variables r and $\eta=\theta+k_z z-\omega t$ with the constant wave number k_z and frequency ω. If we neglect by inertial drift of the electrons due to large value of ω_c , we obtain equation as in [3]:

$$\left\{\Delta_\perp\varphi - \Lambda\varphi + S\varphi^2, \varphi - \frac{\omega_d B_0}{2c} r^2\right\}_{r,\eta} = 0 \quad (1)$$

where

$$\{f,g\}_{r,\eta} = \frac{1}{r}\left(\frac{\partial f}{\partial r}\frac{\partial g}{\partial \eta} - \frac{\partial f}{\partial \eta}\frac{\partial g}{\partial r}\right)$$

$$\Lambda = -\frac{k_z(k_z+k_v)\omega_p^2}{\omega_d^2} - \frac{k_n\omega_p^2}{v_0\omega_d}$$

$$S = \frac{k_z}{2}\left(\frac{(k_z+k_v)e}{m\omega_d^2}\right)^2$$

$$k_v = \frac{1}{\omega_c r}\frac{dv_{0z}}{dr}$$

$$k_n = \frac{v_0}{\omega_c r}\frac{dn_0}{dr} \qquad v_0 = v_{0z}(0)$$

$$\omega_d = \omega - k_z v_{0z} - \frac{v_{0\theta}}{r}$$

m and -e - the electron mass and charge, c - is the speed of light. $\Delta_\perp$ is the transverse part of the Laplace operator.

2 LOCALIZED VORTICES

In [4-5] Larichev V.D. and Reznik G.M. solved the equation (1) only then, when neglected term $S\varphi^2$. Thus they obtain solution knows as Larichev-Reznik. But we don't neglect that nonlinear term. We obtain nonlinear equation

$$\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial\varphi}{\partial r} - \Lambda\varphi + S\varphi^2 = 0. \quad (2)$$

The nonlinear equation (2) is distinguish from KdV and Bessel. We obtain the approximate solution the equation (2) by original method. The method is the functional iteration method. The next (n+1) iteration obtain from equation:

$$\varphi^{(n+1)} = \varphi^{(n)} + \mathrm{sign}(\tau^{(n)}(0)) * \frac{1}{\Lambda}\left(\tau^{(\tau)}(r)\right) \quad (3)$$

where $\tau^{(n)}$ - the residual of $\varphi^{(n)}$ in (2):

$$\tau^{(n)} = \left(\frac{\partial^2 \varphi^{(n)}}{\partial r^2} - \Lambda\varphi^{(n)} + \frac{1}{r}\frac{\partial \varphi^{(n)}}{\partial r} + S\left(\varphi^{(n)}\right)^2 \right),$$

$\varphi^{(0)}$ is the solution for KdV equation:

$$\varphi^{(0)} = \frac{3}{2}\frac{\Lambda}{S}\frac{1}{\left(ch\left(\frac{\sqrt{\Lambda}}{2}r\right)\right)^2}$$

The equation for first iteration:

$$\varphi^{(1)} = \varphi^{(0)} - \frac{1}{\Lambda}\left(\frac{\partial^2 \varphi^{(0)}}{\partial r^2} - \Lambda\varphi^{(0)} + \frac{1}{r}\frac{\partial \varphi^{(0)}}{\partial r} + S\left(\varphi^{(0)}\right)^2 \right)$$

First iteration $\varphi^{(1)}$

$$\varphi^{(1)} = \frac{3\sqrt{\Lambda}\left(\sec h\left(\frac{\sqrt{\Lambda}r}{2}\right)\right)^2\left(\sqrt{\Lambda}r + \tanh\left(\frac{\sqrt{\Lambda}r}{2}\right)\right)}{2Sr}$$

That iteration $\varphi^{(1)}$ is the approximate solution the equation (2). We can obtain $\varphi^{(2)}$, then $\varphi^{(3)}$, et al. The iterations $\varphi^{(2)}$ and $\varphi^{(3)}$ is the approximate solution the equation (2). The second iteration equation $\varphi^{(2)}$

$$\varphi^{(2)} = \frac{1}{\Lambda}\left(\frac{\partial^2 \varphi^{(1)}}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi^{(1)}}{\partial r} + S\left(\varphi^{(1)}\right)^2 \right)$$

The dependence of $\varphi^{(0)}$ - dot line, $\varphi^{(1)}$ - solid line, $\varphi^{(2)}$ - dash dot line, $\varphi^{(3)}$ - dash line - on the radius r is shown in Fig. 1 for Λ=1 cm^{-2} and S=1 cm$^{5/2}$g$^{-1/2}$sec. We see that the maximum amplitude $\varphi^{(n)}$ approach to constant with increase n.

We see that the $\varphi^{(1)}$ and $\varphi^{(2)}$ are closely to $\varphi^{(3)}$. Thus the functional iteration method for the approximate solution have convergence.

Thus we obtain the approximate solution, which exponentially decreases with radius r. That approximate solution is continuous function in first differential in contrast to Larichev-Reznik solution. That approximate solution is near KdV solution at large r. It has been found new expression for electric field potential of vortex in a wave frame. The expression is axisymmetric in a wave frame. New vortices are the result of external disturbances or the appearance and development of instabilities like for example a diocotron instability in hollow beams and a slipping-instability in solid beams.

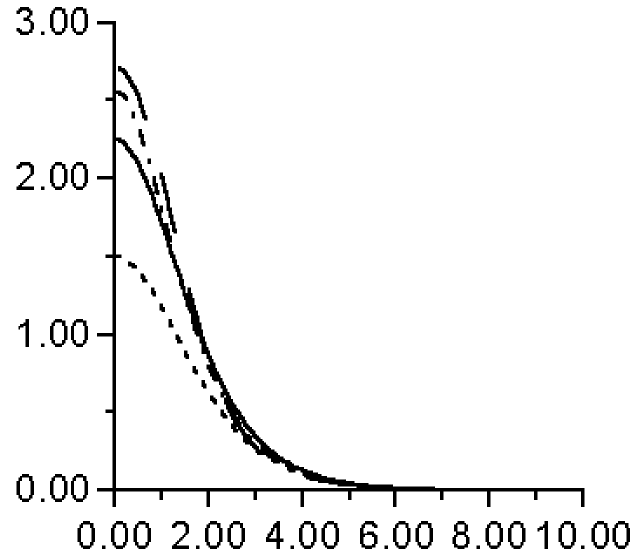

Fig.1: The dependence of $\varphi^{(0)}$ - dot line, $\varphi^{(1)}$ - solid line, $\varphi^{(2)}$ - dash dot line , $\varphi^{(3)}$ - dash line - on the radius r.

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