MEASUREMENT OF THE NONLINEAR MOMENTUM COMPACTION FACTOR IN RHIC *

C. Montag†, BNL, Upton, NY 11973, USA

Abstract

During gold beam acceleration in the Relativistic Heavy Ion Collider (RHIC), the transition energy has to be crossed at $\gamma_t \approx 23$. Since close to γ_t the longitudinal slip factor $\gamma_t^{-2} - \gamma^{-2}$ becomes very small, the longitudinal momentum compaction factor α_1 becomes significant. Measurements of this factor using longitudinal phase space tomography will be reported.

INTRODUCTION

The Relativistic Heavy Ion Collider (RHIC) consists of two superconducting storage rings, capable of accelerating hadron beams from protons to fully stripped gold ions up to energies of $100\,\mathrm{GeV/nucleon}$ in the case of gold. Ion species other than protons have to cross transition energy around $\gamma_t = 23.2$ during acceleration in RHIC, which is accomplished by a set of γ_t-quadrupoles equipped with special power supplies that can switch the sign of the magnetic field within $30\,\mathrm{msec}$, thus providing a γ_t jump of $\Delta\gamma_t = 1.0$.
To mimimize longitudinal emittance blow-up during the γ_t jump due to bucket mismatch, a detailed undestanding of the beam dynamics is required. Here we report an attempt to measure the nonlinear momentum compaction factor α_1 using tomographic phase space reconstruction.

TOMOGRAPHIC PHASE SPACE RECONSTRUCTION

To fully reconstruct the n-dimensional picture of an object, tomography requires a set of $n-1$-dimensional projections of this object, taken at different angles spanning at least 180 degrees. In the case of tomographic reconstruction of the longitudinal phase space in a storage ring, this rotation is provided by phase space dynamics. However, this dynamics is not just a simple rotation of a rigid object, but is intrinsically nonlinear with the rotation frequency (synchrotron frequency) being a function of the phase space amplitude. This difficulty can be overcome by taking into account the exact equations of motion which can be arbitrarily complex [1].
Taking the exact equations of motion, a set of test particels launched on a regular grid in phase space are tracked and sorted into N_{bins} bins that correspond to the binning of the measured profiles each time a longitudinal bunch profile i was obtained by the wall current monitor. Each particle k is therefore assigned a bin number $N_i^{\mathrm{bin}}(k)$. Next, the number $N_{i,j}^{\mathrm{population}}$ of test particles in the jth bin at the time the ith profile was taken is determined.
During the reconstruction process an intensity i is assigned to each test particle by an iterative back-projection algorithm according to the measured bunch profiles. This process increases the intensity I_k of all test particles that fall into a certain profile bin j at a specific time when the ith profile was taken by

$$\Delta I_{k:N_{\mathrm{bin}}(k)=j} = \frac{1}{N_{\mathrm{profiles}} N_{i,j}^{\mathrm{population}}} \sum_{i=0}^{N_{\mathrm{profiles}}} h_{i,j}^{\mathrm{meas}}, \quad (1)$$

where $h_{i,j}^{\mathrm{meas}}$ is the measured profile height of the jth bin in the ith profile, and N_{profiles} is the total number of profiles used for the reconstruction.
When this has been done for all profiles, the algorithm calculates the projections of the resulting distribution that correspond to the profiles measured by the wall current monitor. The difference between measured and reconstructed profiles is then iteratively back-projected.
The remaining discrepancy between measured and reconstructed profiles after a fixed number of iterations is then used as a quantitative measure of the quality of the reconstruction, which allows for parameter fitting [1].

THE NONLINEAR MOMENTUM COMPACTION FACTOR

The frequency-slip factor η which characterizes the chromatic behavior in the longitudinal phase space is defined as the relative change of revolution frequency ω per unit change of the relative momentum $\delta = \Delta p/p$,

$$\eta = -\frac{1}{\omega_s}\frac{\mathrm{d}\omega}{\mathrm{d}\delta}. \quad (2)$$

Here ω_s denotes the revolution frequency of the synchronous particle.
In general, the slip-factor η is a nonlinear function of δ,

$$\eta = \eta_0 + \eta_1\delta + \mathcal{O}(\delta^2), \quad (3)$$

with [2]

$$\eta_0 = \alpha_0 - \frac{1}{\gamma_s^2}, \quad (4)$$

$$\eta_1 = 2\alpha_0\alpha_1 - 2\eta_0\alpha_0 + 3\frac{\beta_2^2}{\gamma_s^2}, \quad (5)$$

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† montag@bnl.gov

where $\gamma_s = (1-\beta_s^2)^{-1/2}$ is the Lorentz factor of the synchronous particle, $\beta_s = v_s/c$, and $\alpha_0 = 1/\gamma_t^2$.
In the vicinity of γ_t, $1/\gamma_s^2 = 1/\gamma_t^2 = \alpha_0$, and therefore

$$\eta_1 = 2\frac{\alpha_1}{\gamma_t^2} + 3\frac{\beta_s^2}{\gamma_t^2}. \tag{6}$$

While the linear part η_0 of the slip factor η changes sign when the transition energy is crossed, the nonlinear contribution η_1 does not. This effect leads to bucket mismatch at the transition jump unless α_1 can be specifically chosen. With $\beta_s = 1$ for relativistic beams, the contribution of η_1 vanishes for $\alpha_1 = -3/2$.

SIMULATIONS

To test the feasibility of measuring the nonlinear momentum compaction factor α_1 tomographically, simulations were performed. A set of 100000 particles with gaussian distributions in phase ϕ and energy deviation δ were tracked using the parameter setpoints as given in Table 1. To potentially improve the convergence of the subsequent parameter fit, a quadrupole oscillation was induced by launching the particles with a deliberate bucket mismatch, namely $\sigma_\delta = 0.2 \cdot \delta_{\max}$ and $\sigma_\phi = 0.1 \cdot \phi_{\max} = 0.1 \cdot \pi$, where $\delta_{\max}$ denotes the bucket height. Projections (wall current monitor profiles) of the evolving distribution were calculated every 125 turns.
First, a one-parameter fit for the nonlinear momentum compaction factor α_1 was performed, assuming all other parameters as exactly known. As shown in Figure 1, several local minima exist in the vicinity of the correct value of $\alpha_1 = 0.35$, which may be explained by the limited number of particles used to generate the profiles, and/or the binning of those profile data. Smoothing the curve shown in Figure 1 by regarding those small fluctuations as some sort of noise results in a nonlinear momentum compaction factor around $\alpha_1 \approx -0.8$, which significantly differs from the correct value.

In a second step, the profiles were used to reconstruct the initial phase space distribution, simultaneously fitting for three unknown parameters as it is required in the case with measured data, the nonlinear momentum compaction factor α_1, the RF voltage U_{RF}, and the bin position of the synchronous phase in the profiles using a simulated annealing technique [3, 4].

The nonlinear momentum compaction factor was fitted as $\alpha_1 = 0.32 \pm 0.14$, where the error is taken as the rms deviation form the average over several fitting runs with different inital parameters. This showed that phase space tomography together with simultaneous fitting of the three unknown parameters may indeed be a feasible method to measure the nonlinear momentum compaction factor α_1.

MEASUREMENTS

For the measurements, two RF voltage jumps were introduced $1.0\,\mathrm{sec}$ and $0.5\,\mathrm{sec}$ before the transition jump to create some longitudinal quadrupole oscillation, as shown in Figure 2. 50 profiles taken every 125 turns were used for the α_1 masurement, starting $0.1\,\mathrm{seconds}$ after the second RF jump.

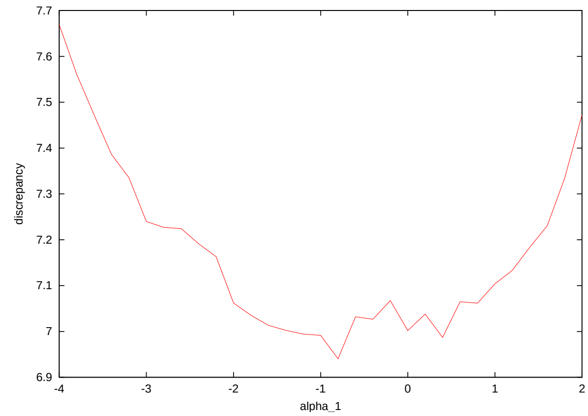

Figure 1: Remaining discrepancy as function of α_1, using simulated data. The RF voltage is set to the exact value during this one-parameter fit.

Table 1: Parameter table for the simulation test. All parameters are set according to the situation in the "blue" RHIC ring. α_1 is chosen according to model calculations with both chromaticities set to $\xi_{x,y} = -2$.

γ_t	23.7647
γ_i	22.85775
$\mathrm{d}\gamma/\mathrm{d}t$	$0.4556/\mathrm{sec}$
U_{RF}	$145\,\mathrm{kV}$
n_{harm}	360
α_1	0.35

As in the simulation test, a simulated annealing technique was applied to simultaneously fit for the three unknown parameters α_1, U_{RF}, and the position of the synchronous phase in the wall current monitor profiles. The resulting value for the nonlinear momentum compaction factor is $\alpha_1 = -1.37 \pm 0.11$, which significantly differs from the value calculated by the model, $\alpha_1 = 0.35$.
This discrepancy may have several causes. First of all, tomographic phase space reconstruction may not be sensitive enough to changes of α_1 on the order of ± 1. This explanation is backed by a similar observation during the simulation tests, where α_1 was found with a similar discrepancy during one-parameter fits.
A poor sensitivity of the resulting discrepancy to small changes in α_1 leads to additional problems during multi-parameter fits. As Figure 3 shows, different values of α_1 result in different local minima for different setpoints of the RF voltage during the reconstruction. In the vicinity of $\alpha_1 \pm 1$, the remaining discrepancy is clearly dominated by fluctuations, which may be interpreted as noise.
Finally, the nonlinear momentum compaction factor de-

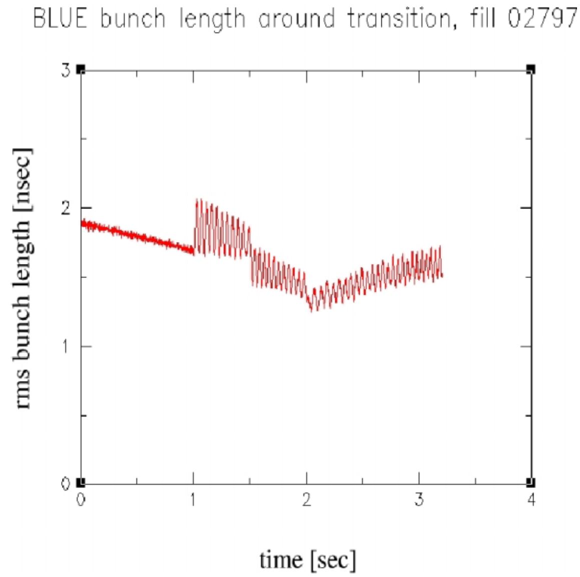

Figure 2: Measured bunch length with two RF voltage jumps (at $1.0\,\mathrm{sec}$ and $1.5\,\mathrm{sec}$, respectively) to induce a quadrupole oscillation. The transition jump occurs at $2.0\,\mathrm{sec}$, indicated by the minimum bunch length.

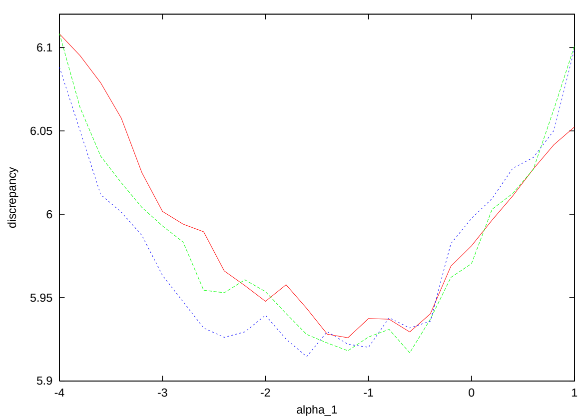

Figure 3: Remaining discrepancy as function of α_1 for different RF voltages. The red line corresponds to $U_{\mathrm{RF}} = 140\,\mathrm{kV}$, the green one to $U_{\mathrm{RF}} = 145\,\mathrm{kV}$, and the blue one to $U_{\mathrm{RF}} = 150\,\mathrm{kV}$.

pends on various machine parameters such as the chromaticity, as Figure 4 shows. Since the superconducting RHIC dipoles have a non-negligible field-dependent sextupole component, exact modeling of the machine is not trivial. This sextupole component contributes to the chromaticity, leading to a different setting of the regular sextupoles than in an ideal machine without sextupole contributions from the dipoles. Furthermore, the sextupole component of the dipoles also results in tune change due to on overall radial orbit shift of about $1.3\,\mathrm{mm}$, which in turn requires adjustment of the tune quadrupoles to keep the tunes at their target values [5]. All these corrections to the ideal model contribute to changes in the nonlinear momentum compaction factor.

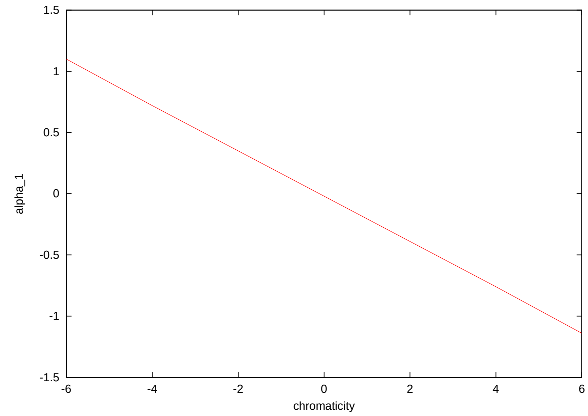

Figure 4: Nonlinear momentum compaction factor α_1 as calculated from the model, for different chromaticities, $\xi_x = \xi_y$.

CONCLUSION

An alternative approach to measuring the nonlinear momentum compaction factor α_1 of RHIC has been made, namely the use of tomographic reconstruction of the longitudinal phase space. The discrepancies with the model may be due to discrepancies between the model and the real machine itself as well as due to insufficient sensitivity of the quality factor of those parameters fits to small changes in α_1.

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