INSTABILITIES STUDY AND IMPLICATIONS FOR THE RIA PROJECT

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Abstract

The Rare Isotope Accelerator (RIA) [1] requires a high power linac capable of accelerating all ions up to uranium to energies of 400 MeV/u with a beam power of 100 to 400 kW in a CW regime. One of the most challenging features of the proposed RIA driver linac is the simultaneous acceleration of different charge states in order to increase the final beam power. The acceleration in the last part of the accelerator is provided by elliptical cavities. Three geometrical β are used: 0.47, 0.61 and 0.81. To minimize the cost, one option is to reduce the number of cryostats. This implies a maximal number of cavities per cryostat. Assuming a lattice composed by one doublet and one cryostat, this option leads to an increase of the longitudinal phase advance if each cavity is used at the maximum field. The transverse phase advance, has to be set correctly in order to ensure stable motion. This report aims to evaluate the sensitivity to instabilities induced by the transverse to longitudinal coupling in the elliptical cavities of the RIA linac for an 88^+ uranium beam.

THEORY

This section is a brief summary of the theory developed by I.M. Kapchinsky in Reference [2]. The transverse electric and magnetic field components produce a defocusing effect. The defocusing is a function of the phase of each particle and induces a coupling term in the equations of motion. The equations of motion for the transverse plane may be reduced to

$$\frac{d^2x}{dt^2} = -\frac{ev}{m_0\gamma}G(t) - \frac{e}{2m_0\gamma}\left\{\left(\frac{\partial E_z}{\partial z}\right)(t) + \frac{v}{c^2}\left(\frac{\partial E_z}{\partial t}\right)(t)\right\}x \qquad (1),$$

where $G(t)$ is a periodic function which represents the focusing, γ is the Lorentz factor, v the speed, m_0 the mass, and the terms in the braces represent periodic functions for the defocusing induced by the accelerating gaps. After simplification, this equation may be reduced to the canonical form of the Mathieu equation:

$$\frac{d^2x}{d\tau^2} + \pi^2\left[a^2 + 2q\sin(2\pi\tau)\right]x = 0 \qquad (2),$$

with $a = 4\sigma_{0T}^2 / \sigma_{0L}^2$, $q = \Phi\left|\cot(\phi_s)\right|$, τ is a new dimensionless variable, Φ is the phase amplitude of the particle, and φ_s is the synchronous phase. The parameters σ_{0T} and σ_{0L} are the transverse and longitudinal phase advance per period at zero current, respectively. It can be shown that the unstable part of the general solution of equation 2 has the form

$$x(\tau) \propto \left[Ce^{k\tau}\cos(\phi(\tau)) + De^{-k\tau}\sin(\phi(\tau))\right] \qquad (3),$$

where $\phi(\tau)$ is the betatron phase. The parameter k can be calculated using the diagram of Figure 1 where $k = \mu\pi$.

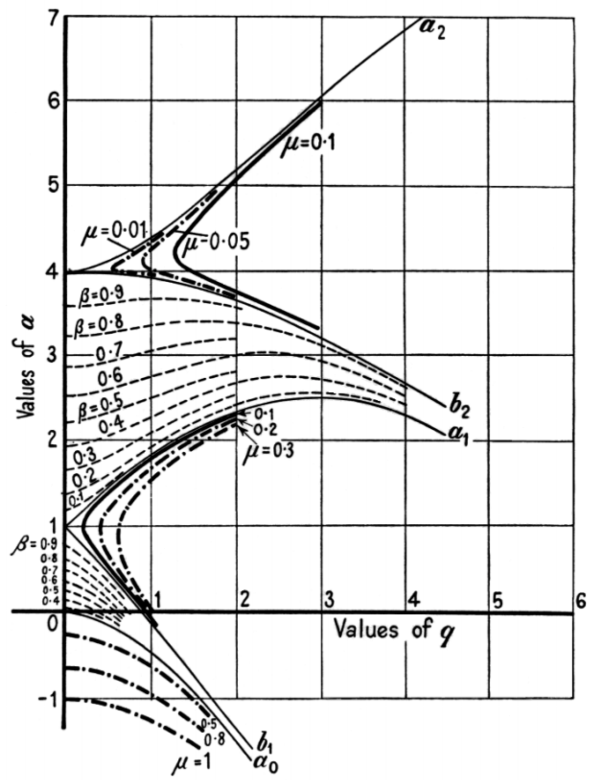

Figure 1: Stability diagram of the Mathieu equation from N.W. McLachlan, Theory and applications of Mathieu functions, Dover Publisher, 1964.

A FEW EXAMPLES

To illustrate this theory in the RIA case, let us test several configurations of the low β section of the elliptical cavities part of the linac. As was pointed out in [3], only this part is relevant, because as the velocity increases, the longitudinal phase advance decreases and pushes the beam to a stable area in the Mathieu diagram (a increases). The recommendation of this previous work [3] was that no more than three cavities per period should be included due to parametric resonance.

To explore a different part of the diagram and test our code PARTRAN [4], we use a test design with a geometrical $\beta = 0.47$, 14 periods (typical for this part of the linac),a constant synchronous phase of -30°, a constant longitudinal phase advance equal to 90° per period, a phase amplitude Φ equal to 15°, and rms normalized emittances $\varepsilon_T = \varepsilon_L/2 = 0.6\,\pi$ mm mrad. We take ε_L a factor of two larger then ε_T to maximize the effect. A higher amount of energy is then available for the instabilities. The radial dependence of the field in a gap is represented by a Bessel function in PARTRAN, following Reference [5]. With this set of parameters, q is equal to 0.45. Figure 2 shows a classical beam envelope behavior for this linac.

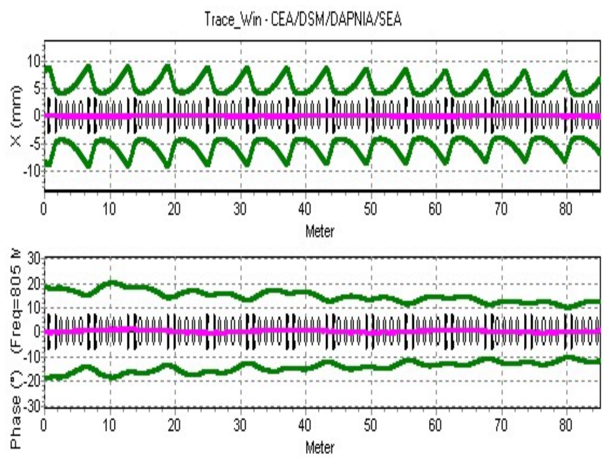

Figure 2: Beam envelope behavior in the test case.

If we set the doublets to get a constant transverse phase advance per period equal to 50°, the parameter a is equal to 1.23. The theory predicts unstable motion. Figure 3 shows the emittance behavior for this case.

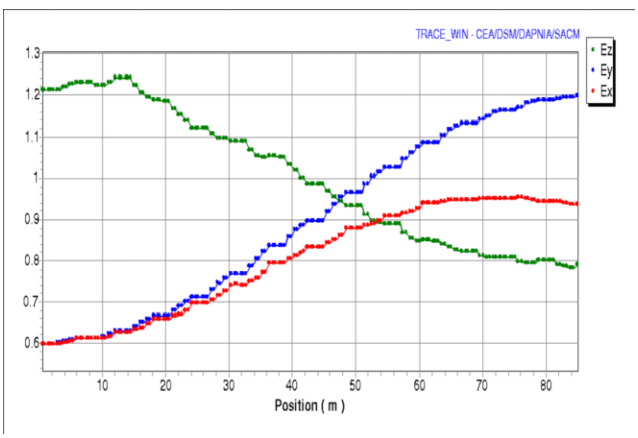

Figure 3: Rms norm. emittances for the 50° case.

This simulation shows an unstable motion that is damped at the end of the section. A transfer of energy occurs between the transverse planes and the longitudinal plane. This transfer is damped when the equipartitioning condition is met [5].

To continue this illustration of the theory with the PARTRAN code, two supplementary cases are tested. For the first one, we set the transverse channel to a constant 80° phase advance. This is equivalent to $a = 3.16$. With q still equal to 0.45, the theory predicts stable motion. Again, a simulation with PARTRAN shows a very good agreement for this case (see Figure 4).

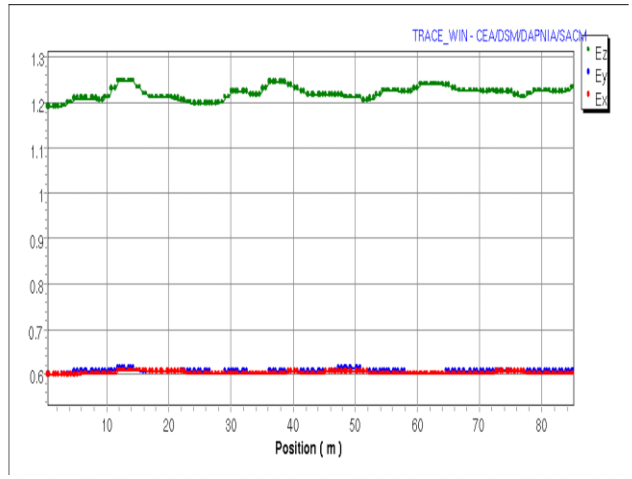

Figure 4: Rms norm. emittances for the 80° case.

For the next case, we set the transverse phase advance to 90° (a=4). The results are shown in Figure 5. The motion for this case should be unstable due to a second-order resonance.

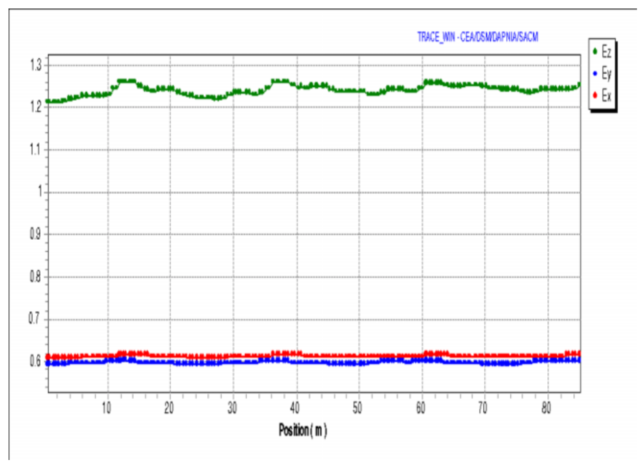

Figure 5: Rms norm. emittances for the 90° case.

Figure 5 shows that there is no significant effect on the halo and the rms emittance. This is still in agreement with the theory. Indeed, the strength of the instability may be reduced to the value of the coefficient k in the exponential (see Equation 3). In Figure 1, it can be seen that, for the same q, k is almost 20 times weaker for a second-order resonance than for a first-order one. This diagram shows that it is possible to increase the strength of the instability if q is increased. Taking into account that Φ must be lower than φ_s, the maximum value for q that we can get, decreasing φ_s, is 1, according to the q definition above. To display this resonance, a new design with a constant synchronous phase equal to -10° is studied. The number of periods is 20 in order to have enough time for the instability to occur. The phase amplitude Φ is maximum and equal to 10°. The rms normalized emittances are

$\varepsilon_T = \varepsilon_L/2 = 0.3\ \pi$.mm mrad. Parameter a is still equal to 4 and $q = 0.99$. Figure 6 shows the behavior of the emittances for this set of parameters.

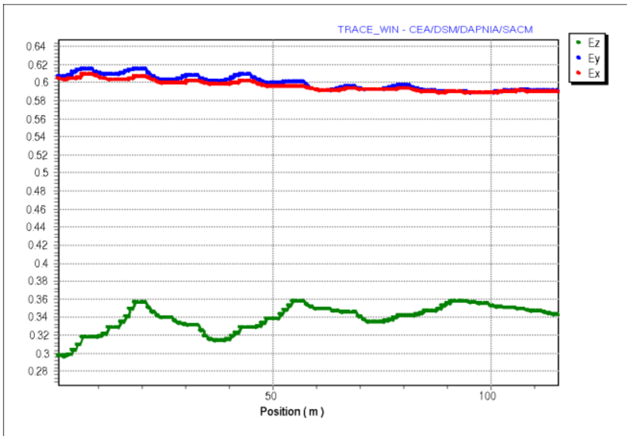

Figure 6: Rms norm. emittances for a=4 and q=0.99.

A clear exchange occurs. The transverse rms emittances lose 2% compared to their initial values. The effect is weak, but can be simulated with PARTRAN. All these results show that the code is capable of correctly illustrating the theory.

FOCUS ON RIA DESIGN

The problem of parametric resonance is more important at geometrical β equal to 0.47 than at $\beta_g = 0.61$ or 0.81 if we take into account that the longitudinal phase advance decreases with $1/\beta^2$. The $\beta_g = 0.47$ section of the RIA linac uses four cavities per cryostats, a constant synchronous phase of about -30° and 14 periods [6]. Figure 7 shows the behavior of the transverse and longitudinal phase advance for this section.

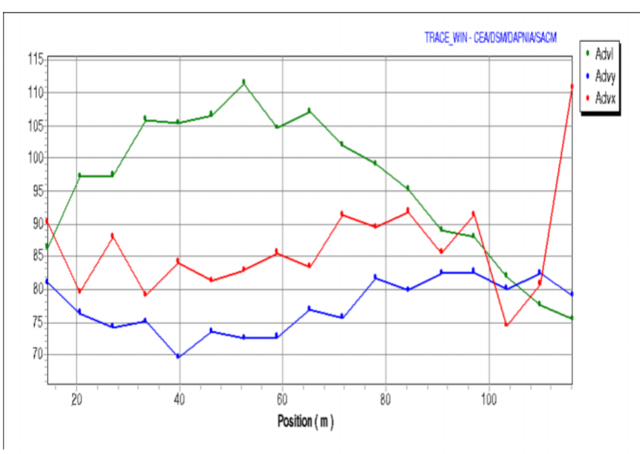

Figure 7: Behavior of phase advances for transverse and longitudinal planes.

Computing the parameters a and q according to the equations described above, it appears that 1-2 periods at the beginning and 3-4 periods at the end of this section are in the second-order resonance part of Figure 1. But the main part is located in the stable area of this diagram. A simulation of this section of the linac shows no emittance growth [6].

This result is in agreement with the above study. The second order resonance is so weak in this case, that it cannot be observed in the simulation. Although our code is able to simulate a 2% emittance growth, that is in any case negligible for this linac.

CONCLUSION

This study shows that there is no reason to decrease the number of cavities per cryostat from 4 to 3 in order to avoid instabilities due to the defocusing effect in accelerating gaps. Indeed, the number of cavities is not the issue. The phase advance per period is the relevant parameter. With the present accelerating field, the choice of 4 cavities is safe. If it is possible to increase the field due to new technological progress, it is necessary to increase the transverse phase advance via higher gradient in the quadrupoles to keep the motion stable. A new limit will then appear. If the transverse phase advance per lattice is higher than 180°, an unstable motion occurs [5]. One could think that this limitation would be lower if we take into account the coupling induced by the space charge and would become 90°. In Reference [7], it is clearly shown that it is relevant only for high tune depression, which is clearly not the case for the RIA driver linac. These results and the theory of parametric resonances are not in agreement with Reference [3], which shows emittances growth of 20% with 4 cavities. In this study, the author assumes that a second order resonance is responsible for this effect. We find here that is not possible to get such emittance growth with a second order resonance for this part of the linac.

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