STUDIES ON SEXTUPOLE COMPONENTS GENERATED BY EDDY CURRENTS IN THE RAPID CYCLING MEDICAL SYNCHROTRON *

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Abstract

The Rapid Cycling Medical Synchrotron is a second generation medical accelerator that it has been designed with a repetition frequency of 30 Hz. This repetition frequency is far above the typical repetition frequency used in medical accelerators. An elliptical beam pipe has been chosen for the RCMS design in order to win as much physical aperture as possible while keeping the magnet dimensions as small as possible. Rapid Cycling induces Eddy current in the magnets. Eddy currents and elliptical beam pipes generate sextupole components that might be neccesary to consider. In this paper, the effects of these sextupoles components are evaluated, first by looking at the phase space of a bunch of particles that has been tracked for 62530 turns, and also by evaluating the dynamical aperture of the accelerator. The effect of the sextupoles component in the tuneshift is also evaluated.

First results obtained with Marylie show that the width of a phase space ellipse of a bunch of particles is slightly affected by the sextupoles due to the Eddy currents.

INTRODUCTION

The RCMS is a second generation proton therapy synchrotron offering more flexible performance in a simpler lighter and more robust implementation (see for example [1] or [2]).

The RCMS will reduce the typical treatment time and at the same time will reduce the risk of dumping a large amount of radiation into the patient. All of the above is possible thanks to the 4 design choices of the RCMS: strong focusing, rapid cycling, fast extraction, and 7 MeV energy injection.

One of the challenging aspects of the RCMS is its fast repetition frequency of 30 Hz. Due to the elliptical shape of the beam pipe chosen for the RCMS, the induced Eddy currents at that repetition frequency might induce significant sextupole components in the beam.

The effect of the induced sextupoles in the tunes and the dynamical aperture are studied in this paper. The ultimate goal of these studies is to determine whether the RCMS would require sextupole correctors or not.

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MAGNETIC MULTIPOLES COMPONENTS GENERATED BY EDDY CURRENTS

Eddy currents in the vacuum chamber induce magnetic multipole components in the beam. When the chamber is between iron poles such multipole components are [3]:

$$b_n + ia_n = \frac{\mu_0 \sigma h \dot{B}_0}{2\pi n! B_0} \left(\frac{\pi}{2g}\right)^{n+1} \int_{v.c.} x\left(\alpha_n + \beta_n\right) ds, \quad (1)$$

where σ, h, and g are the conductivity, thickness and diameter of the beam pipe respectively, and $\dot{B}_0$ and B_0 are the rate of variation of the magnetic field and the magnetic field itself.

The coefficients α_n and β_n can be found analytically and for the case of the lower non linear component (sextupoles) those coefficients are given by:

$$\begin{aligned} \alpha_2 &= -2\frac{\tanh(-\frac{\pi z_c^*}{2g})}{\left(\cosh(-\frac{\pi z_c}{2g})\right)^2}, \\ \beta_2 &= 2\frac{\coth(-\frac{\pi z_c^*}{2g})}{\left(\sinh(-\frac{\pi z_c}{2g})\right)^2}, \end{aligned} \quad (2)$$

where $z_c = x_c + iy_c$ is an arbitrary position in the vacuum chamber walls. The sextupoles components can now be found from Eq. 1 by numerical integration. Values of the b_2 have been estimated in reference [3] for different beam pipe shapes and sizes. The corresponding values for the elliptical beam pipe of the RCMS can be quickly estimated from the tabulated values in reference [3] since b_2 scale either proportional or inversely proportional with most of the parameters involved in the calculation. Using the RCMS parameters:

$$\begin{aligned} a &= 3cm \\ b &= 1.5cm, \\ h &= 0.64mm, \\ \sigma^{-1} &= 1.25\mu\Omega m, \\ \dot{B}/B &= 188.49/s, \end{aligned}$$

the sextupole strength induced in the body of the main dipole of the synchrotron ring is 0.687 $\frac{T}{m^2}$ or equivalently an integrated sextupole strength of 0.522 $\frac{T}{m}$.

MARYLIE SIMULATIONS

In order to evaluate the effect of the sextupole component in the beam, particularly dynamic aperture and tune footprints, particles were tracked with the software Marylie [4]. Marylie uses a "map" that translate the initial phase space coordinates into the corresponding phase space coordinates at the end of the ring. Simulation of the particles going through many turns are done by successively applying the mentioned maps.

Maps used by Marylie are built with the so called Lie transformations (see for example [5], [6]). The importance of the use of Lie transformations in the construction of maps for tracking particles resides in the concept of symplecticity. Physical systems that are described by Hamiltonian flows must comply with the symplecticity condition. Since orbits of charged particle are well described by Hamiltonian flows the maps derived from such Hamiltonian must be symplectic. This is guarantee if Lie transformations are used to construct such maps. If the conventional Taylor expansions are used to construct the maps instead, such maps could not be symplectic and unphysical effects can appear when tracking simulations are done with those maps with a large number of turns.

Lattice Preparation and Main Dipole Splitting

The original design of the lattice was done in the simulation program MAD [7]. Most of the elements and commands used in MAD have their equivalent in Marylie. However, neither MAD neither Marylie has built in elements that allow to put sextupole components in the dipoles. In order to circumvent this problem, it was neccesary to divide the main dipoles and place thin multipoles at the divisions to simulate the effect of the sextupoles. The main body of the divided magnet consist then of sector bends with multipoles located at every common point between the sector bends (see Fig. 1). Since the main dipoles of the ring are combined function magnets (dipole and quadrupole at the same time) the thin multipoles are also used to introduce the required quadrupole components. The number of divisions in the dipole is determined by how

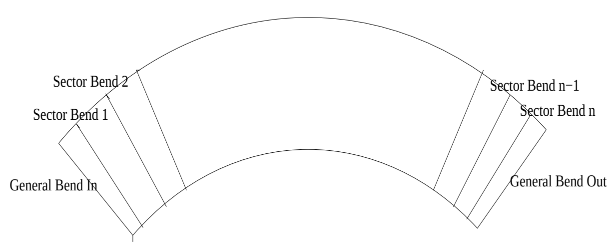

Figure 1: The main dipoles are divided in n sector bends and the multipolar elements are inserted between them. The edges of the magnet are carefully modeled with general bending magnets that allow individual pole phase rotation of the faces of the magnet. Fringe fields in the edge are also added.

much the resultant map change when the number of divisions changes. Allowing a change of 0.01% the number of division needed turn out to be 1984. Since the magnets has been designed with pole phase rotations special care has to be taken at the edge of the magnets. The edges are built with maps that represent magnets with pole phase rotation and maps that represent fringe fields in sector bends.

Once the dipoles have been appropriately divided and the whole lattice has been written in terms of Marylie elements it is neccesary to make some comparisons with MAD outputs like tunes and lattice functions.

Comparison between the tunes given by Marylie and MAD lead to small differences that start to be evident in the second significant figure. Since the tunes are one of the most determinant parameters in tracking simulations the quadrupole components of the dipole magnets were slightly re-tuned until it was possible to achieve an agreement between the MAD and Marylie tunes of at least 6 significant figures. The beta functions produced by the two simulations programs were also compared and differences below 1% were found.

Effect of the Sextupole Components on the Dynamic Aperture of the RCMS Ring

Before doing the tracking simulations neccesary to determine the dynamic aperture of the ring it is convenient to determine the effect of the sextupole in the phase space ellipses. Extensive tracking simulations were done in Marylie for this purpose. In particular, the final distribution of particles on an initial 4D 2-torus distribution for different sextupole strengths was studied (see Fig. 2). Fig. 2 shows some growth and distorsion of the the phase space ellipses when the strength of the sextupole is increased. The vertical phase space ellipses (no shown) also show some thickness growth but is smaller than for the horizontal case. Similarly, studies done with negative sextupole strengths showed that the thickness growth of the ellipses were smaller than for positive sextupole strengths. Having

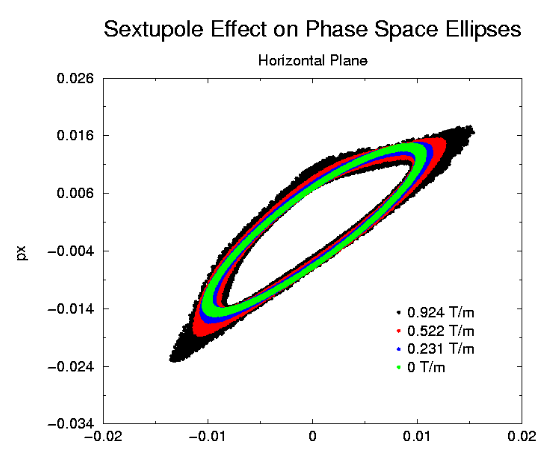

Figure 2: Horizontal phase space ellipses at one location of the ring after tracking for 62530 turns with different sextupole strengths. The nominal sextupole strength of 0.522 T/m is represented by the red ellipse

evaluated the effect of the sextupoles in the phase space

Table 1: The radius r for which particles start to get lost in the tracking simulations is found as function of the sextupole strength. The chromaticities Qx' and Qy' are also calculated in each run.

Sext. Str. [T/m]	Qx'	Qy'	r [mm]
-0.924	91.56	-9.20	10.2
-0.522	64.70	11.05	10.9
-0.231	45.32	25.66	14.1
0	29.90	37.28	10.9
0.231	14.49	48.90	10.8
0.522	-4.89	63.51	11.2
0.924	-31.76	83.75	8.7

ellipses the dynamic apertures of the ring can now be evaluated. This can be done by finding the biggest ellipse allowed in the ring before particle start to get lost. This is a process that involved tracking of particles several times in the ring, every time with a different size particle distribution.

The process was repeated for all the sextupoles strengths studied before (positive and negative) with results that are summarized on Table 1. The maximum radius of the RCMS beam is expected to be about 3 mm at one sigma. Even in the worst case when the sextupole strength is +0.924 (which is almost double the nominal strength) , the maximum transverse radius allowed for the particles before they start to get lost (r = 8.7mm) is just enough to keep most of the particles inside the beam pipe during the acceleration cycle of the RCMS.

Footprints

Non linear components in the ring can introduce some dependence of the particles tune with the amplitude of oscillation. In the RCMS it is important to know if the tune spread caused by the eddy currents and other non linear effects can excite unwanted resonances in the ring.

In order to calculate such tune spread, a uniform distribution of particles in the Jx-Jy space were tracked for 62530 turns with the Marylie software. The tunes of each individual particle were determined using the orbit information provided by the 62530 simulated turns. The particles are then sorted out according to their horizontal and vertical actions J_x and J_y given place to Fig. 3 and an equivalent figure in the vertical plane (no shown). These graphs are done with the nominal sextupole strength of 0.522 T/m and a bigger strength of 0.924 T/m. The biggest tune spread in the horizontal plane is about $8.15e^{-4}$ and $2.55e^{-4}$ in the vertical plane, both of them for the extreme situation of a sextupole strenght of 0.924 T/m. These tune spreads are no large enough to move the nominal tune into the strongest resonances of one half at one third and hence the operation of the machine is stable within the sextupole range considered in this study.

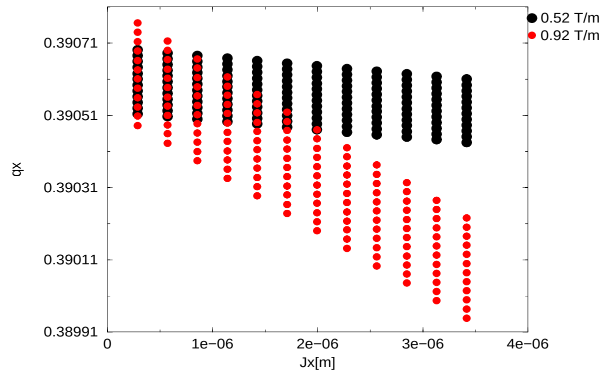

Figure 3: Horizontal tune distribution of the particles after tracking for 62530 turns.

CONCLUSIONS

The sextupole component induced by the eddy currents in the beam pipe has a visible effect in the phase space distribution of the particles but the dynamic aperture is not sensitive to this sextupole component in the range of sextupole strenghts used in this study. Even in the very extreme case in which the sextupole component is almost double the nominal strenght the dynamic aperture although reduced is big enough to hold most of the particles of the beam.

It was also shown through Marylie simulations that the tune spread of the particles due to their different amplitudes is not significant and hence it doesn't represent any risk for the stable operation of the machine. The addition of sextupoles correctors to the RCMS design doesn't seem neccesary according to the results that has been presented in this paper.

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