PERIODIC ION CHANNEL LASER*

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Abstract

Radially polarized radiation is amplified by a free electron laser (FEL) in which the undulator is an ion channel with periodic density. To ensure stable beam propagation, the undulator period is much shorter than the betatron wavelength. The gain at a given distance from the axis equals that of a planar magnetostatic undulator with the same quiver velocity. When an ultrarelativistic electron beam propagates in a periodic ion density of 10^{11}–10^{17} cm^{-3}, a short-wavelength FEL may be obtained.

1 INTRODUCTION

When a round electron beam propagates in an ion channel whose density varies periodically [1], the electrons undergo forced radial oscillations in addition to damped betatron oscillations [2]. A periodic ion channel undulator may be created by ionizing a periodic gas density [1] or by using a modulated ion beam [2, 3], and may have application as an FEL [4].

A periodic ion channel FEL is similar to a non-periodic ion channel laser [5, 6] in that no magnetic field is required, while benefiting from forced transverse oscillations similar to those in a magnetostatic undulator FEL with ion channel guiding [7, 8, 9]. In contrast to an ion ripple laser where oblique propagation through a periodic ion density causes a periodic beam deflection [10], we consider propagation in the direction of the periodic density gradient. In this case, lasing results from periodic focusing rather than a periodic beam deflection.

We calculate amplification of a radially polarized wave by a periodic ion channel FEL for a cold beam in the low-gain-per-pass limit. To ensure stable beam propagation, we consider the case where the undulator period is much shorter than the betatron wavelength [2,11].

2 RADIAL MOTION

To model "force" bunching [6], radiation is included in the transverse dynamics. We consider an undulator, in which an electron's velocity deviates by less than the angle $1/\beta\gamma$ from the axis, where γ is the beam's relativistic factor and $\beta > 0$ is the beam velocity divided by the speed of light c. In an undulator, the electron motion is non-relativistic in the frame of reference moving with the beam as it enters the undulator, so we calculate the dynamics in this frame, which is related to the laboratory frame by $\gamma_{||}$ and $\beta_{||}$.

Consider a periodic ion channel undulator with entrance at $z_{lab} = 0$, whose density is given in the laboratory frame (i.e., the frame where ions are stationary) for $z_{lab} > 0$ by

$$n_{i-lab}(r, z_{lab}) = n_{0-lab}(r) + n_{1-lab}(r)\cos(k_{w-lab} z_{lab}) \,, \quad (1)$$

where z_{lab} and r are axial and radial coordinates, while $k_{w\text{-}lab} > 0$ is the undulator wave number equaling 2π divided by the undulator period $\lambda_{w\text{-}lab}$. When the undulator period greatly exceeds the beam radius, the ions' electric field in the laboratory is mostly radial [1], given by Gauss's law in SI units as $(-e/2\pi r\varepsilon_o)\int_0^r n_{i-lab}(r', z_{lab}) 2\pi r' dr'$, where $e < 0$ is the electron charge and ε_o is the permittivity of free space. In the frame moving with the beam's axial velocity as it enters the undulator, the radial electric field from the ion channel is increased by the factor $\gamma_{||}$ [12], giving [2]

$$E_w(r,z,t) = (-e/2\pi r\varepsilon_o) \times\left[\int_0^r n_0(r')2\pi r' dr' + \int_0^r n_1(r') 2\pi r' dr' \cos(k_w z + \omega_w t)\right]. \quad (2)$$

Here, z is the axial coordinate, $n_0(r) = \gamma_{||} n_{0\text{-}lab}(r)$, $n_1(r) = \gamma_{||} n_{1\text{-}lab}(r)$, $k_w = \gamma_{||} k_{w\text{-}lab}$ and $\omega_w = \gamma_{||}\beta_{||} c k_{w\text{-}lab}$. The magnetic field in the e-beam frame is in the azimuthal (ϕ) direction, with ϕ-component $B_w = -\beta_{||} E_w / c$. The axial electric field in the beam frame equals that in the laboratory; it is therefore negligible for $\gamma_{||} >> 1$.

For a radially polarized wave traveling forward, the radial electric field in the low-gain-per-pass limit is

$$E_r(r,z,t) = E_o(r)\cos(k_r z - \omega_r t + \phi_r) \quad (3)$$

The azimuthal magnetic field B_r equals E_r/c, with wave number $k_r > 0$, phase ϕ_r, and frequency $\omega_r = ck_r > 0$.

The radial electron motion consists of forced oscillations from the undulator and radiation E-fields, in addition to damped betatron oscillations from mismatched injection [2]. For brevity, we will suppress the dependence of functions upon r in our notation. In the case where the undulator period is short compared to the betatron wavelength [$\omega_w >> \omega_\beta = (n_0 e^2/\varepsilon_o m)^{1/2}$, where m is the electron mass], we consider a small injection mismatch so that betatron oscillations are negligible. The radial velocity of an electron at radius r with constant axial velocity $<< c$ is the sum of an undulation with quiver velocity [2]

$$v_w(z,t) = -\hat{a}_w c \sin(k_w z + \omega_w t), \quad (4)$$

and a forced oscillation from the radiation

$$v_r(z,t) = a_r c \sin(k_r z - \omega_r t + \phi_r). \quad (5)$$

Here, $\hat{a}_w$ obeys

$$\hat{a}_w = \frac{e^2 \langle n_1 \rangle r}{2\varepsilon_o m \omega_w c} = \frac{e^2 \langle n_{1-lab} \rangle r}{2\varepsilon_o m \beta_{||} c^2 k_{w-lab}} \quad (6)$$

with $\langle n_1 \rangle \equiv (1/\pi r^2)\int_0^r n_1(r') 2\pi r' dr'$ (and similarly for $< n_{1\text{-}lab} >$), while $a_r = -eE_o/mc\omega_r$. Since $k_w = \omega_w/\beta_{||} c$ in eq. (4), the undulation wavelength in the laboratory is independent of the electron's axial velocity.

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Any axial velocity function may be approximated to arbitrary accuracy by constant-velocity segments, so that eqs. (4)–(6) also apply when the axial velocity is not constant. Since v_w = 0 at the undulator entrance, a matched beam has $\beta = \beta_{\|}$ and $\gamma = \gamma_{\|}$. Our assumption of nonrelativistic electron velocities in the beam frame requires $\hat{a}_w << 1$.

3 AXIAL MOTION

To describe "inertial" bunching, radiation is included in the axial dynamics [6]. An electron whose initial axial position z is 0 and radius is r obeys, to lowest order in the radiation field

$$\frac{d^2 z}{dt^2} = \frac{e}{m} v_w B_w + \frac{e}{m} v_w B_r + \frac{e}{m} v_r B_w , \qquad (7)$$

where radius = r on the right hand side (RHS). The solution with initial conditions $z(0) = dz/dt(0) = 0$ is the sum of three functions describing radiation-independent axial motion, inertial bunching, and force bunching. The radiation-independent motion obeys $d^2 z_o/dt^2 = (e/m) v_w B_w$ where $z \approx v_o t$ on the RHS of the equation, with v_o equaling the average axial velocity in the undulator. The solution with initial conditions $z_o(0) = dz_o/dt(0) = 0$ is

$$z_o(t) = \frac{e^2 \beta \hat{a}_w r}{2m\varepsilon_o \hat{\omega}_w^{\,2}} \left[\begin{array}{l} \langle n_0 \rangle (\sin \hat{\omega}_w t - \hat{\omega}_w t) \\ + \dfrac{\langle n_1 \rangle}{8} (\sin 2\hat{\omega}_w t - 2\hat{\omega}_w t) \end{array} \right] , \qquad (8)$$

where $\hat{\omega}_w \equiv \omega_w (1 + v_o / \beta c)$ is the undulator frequency experienced by an electron with axial velocity v_o. Equation (8) gives the average axial velocity as

$$v_o \approx -\frac{\hat{a}_w^{\,2} \beta c}{4} \left(1 + \frac{4 \langle n_0 \rangle}{\langle n_1 \rangle} \right) \qquad (9)$$

The inertial bunching term [6] results from the axial radiation force on an electron, obeying $d^2 z_i / dt^2 = (e/m) v_w B_r$ where $z \approx z_o(t)$ on the RHS. For $\hat{a}_w << 1$, approximating $z_o(t) \approx v_o t$ on the RHS for the fundamental FEL mode gives the solution with initial conditions $z_i(0) = dz_i/dt(0) = 0$:

$$z_i(t) = \frac{e\hat{a}_w E_o}{2m} \left[\begin{array}{l} \dfrac{\sin(\omega_+ t - \phi_r) + \sin \phi_r - \omega_+ t \cos \phi_r}{\omega_+^{\,2}} \\ + \dfrac{\sin(\omega_- t + \phi_r) - \sin \phi_r - \omega_- t \cos \phi_r}{\omega_-^{\,2}} \end{array} \right] \qquad (10)$$

where $\omega_+ \equiv \hat{\omega}_w + \hat{\omega}_r$ and $\omega_- \equiv \hat{\omega}_w - \hat{\omega}_r$, in which $\hat{\omega}_r \equiv \omega_r (1 - v_o / c)$ is the radiation frequency experienced by an electron with axial velocity v_o. Since the undulation wavelength in the laboratory is independent of the electron's axial velocity, the inertial bunching is also called "axial" bunching [6].

The force bunching term [6] results from the transverse radiation force on an electron, obeying $d^2 z_f / dt^2 = (e/m) v_r B_w$ where $z \approx z_o(t)$ on the RHS. For $\hat{a}_w << 1$, approximating $z_o(t) \approx v_o t$ on the RHS for the fundamental FEL mode gives the solution with initial conditions $z_f(0) = dz_f/dt(0) = 0$:

$$z_f(t) = \frac{\beta e^2 a_r r}{4m\varepsilon_0} \left\{ \begin{array}{l} \dfrac{2\langle n_0 \rangle [\sin(\hat{\omega}_r t - \phi_r) + \sin \phi_r - \hat{\omega}_r t \cos \phi_r]}{\hat{\omega}_r^{\,2}} \\ + \dfrac{\langle n_1 \rangle [\sin(\omega_+ t - \phi_r) + \sin \phi_r - \omega_+ t \cos \phi_r]}{\omega_+^{\,2}} \\ - \dfrac{\langle n_1 \rangle [\sin(\omega_- t + \phi_r) - \sin \phi_r - \omega_- t \cos \phi_r]}{\omega_-^{\,2}} \end{array} \right\} \qquad (11)$$

For effective amplification of radiation, $\omega_- << \omega_+$, so that

$$z_i(t) = \frac{z_f(t) \omega_r}{\beta \omega_w} = \frac{e\hat{a}_w E_o}{2m} \left[\frac{\sin(\omega_- t + \phi_r) - \sin \phi_r - \omega_- t \cos \phi_r}{\omega_-^{\,2}} \right] . \qquad (12)$$

Because the undulator field is periodic, the inertial bunching and force bunching are nearly equal for an ultrarelativistic e-beam when $\omega_- << \omega_+$.

4 GAIN

The change in an electron's energy from interaction with the radiation obeys

$$\frac{d\varepsilon}{dt} = e v_r E_r + e v_w E_r \qquad (13)$$

where v_r, v_w and E_r are evaluated at radius r and the axial position $z(t)$ calculated in the previous section. The change in an average electron's energy is given by averaging over the phase of the radiation ϕ_r. To order $E_o^{\,2}$, the first term on the RHS does not contribute to this average, so that for $\hat{a}_w^{\,2} (1 + 8 < n_0 > / < n_1 >) << 8$

$$\begin{aligned} \langle d\varepsilon / dt \rangle_{\phi_r} &= \langle e v_w E_r \rangle_{\phi_r} \approx \\ &\langle z \cos \phi_r \rangle_{\phi_r} \left[\frac{-e\hat{a}_w c E_o}{2} (k_+ \cos \omega_- t + k_- \cos \omega_+ t) \right] \\ &+ \langle z \sin \phi_r \rangle_{\phi_r} \left[\frac{e\hat{a}_w c E_o}{2} (k_+ \sin \omega_- t - k_- \sin \omega_+ t) \right] \end{aligned} \qquad (14)$$

where $k_+ \equiv k_w + k_r$ and $k_- \equiv k_w - k_r$. Equation (12) gives

$$\begin{aligned} \langle z \cos \phi_r \rangle_{\phi_r} &= \frac{(1 + \beta \omega_w / \omega_r) e\hat{a}_w E_o}{4m\omega_-^{\,2}} (\sin \omega_- t - \omega_- t) \\ \langle z \sin \phi_r \rangle_{\phi_r} &= \frac{(1 + \beta \omega_w / \omega_r) e\hat{a}_w E_o}{4m\omega_-^{\,2}} (\cos \omega_- t - 1) \end{aligned} \qquad (15)$$

where for $\beta \approx 1$, $1 + \beta\omega_w/\omega_r = 2/(1 + v_o/c)$.

Let $\Delta\varepsilon \equiv \int_0^T \langle d\varepsilon/dt \rangle_{\phi_r} dt$ be the average energy change per electron from interacting with radiation. Here, T is the undulator transit time, obeying $\hat{\omega}_w T = 2\pi N$ with integer or half-integer N equaling the number of undulator periods. For $\omega_- << \omega_+$ and $\gamma >> 1$, eqs. (14) and (15) give

$$\Delta\varepsilon = \frac{-e^2 E_o^{\,2} \hat{a}_w^{\,2} c k_+ T^3}{4m\,(1 + v_o/c)} \left(\frac{2 - 2\cos \omega_- T - \omega_- T \sin \omega_- T}{\omega_-^{\,3} T^3} \right) . \qquad (16)$$

In the beam frame, the number of electrons passing through the undulator within a transverse area A_o during a time t_o is $n_e A_o \beta c t_o$, so that the energy transferred to the forward wave is $-n_e A_o \beta c t_o \Delta\varepsilon$, where n_e is the electron density. The time-averaged Poynting vector of the radiation is $< S > = \varepsilon_o c E_o^{\,2}/2$, with energy density $< S >/c$. Since the relative velocity between the forward wave and undulator is $(1+\beta)c$, the electromagnetic energy passing through the undulator is $(<S>/c)(1+\beta)ct_o A_o$. The radiation energy gain per pass at radius r therefore obeys

$$\text{gain} = \frac{-n_e A_o \beta c t_o \Delta\varepsilon}{\langle S\rangle (1+\beta)\, t_o A_o} = \left(\frac{-2\beta}{1+\beta}\right)\frac{n_e \Delta\varepsilon}{\varepsilon_o {E_o}^2}. \quad (17)$$

Equations (16) and (17) give for $\gamma >> 1$

$$\text{gain} = \frac{n_e e^2 k_+ c \hat{a}_w{}^2 T^3}{4m\varepsilon_o(1+v_o/c)}\left(\frac{2-2\cos\omega\_T - \omega\_T\sin\omega\_T}{\omega\_{}^3 T^3}\right). \quad (18)$$

In the laboratory frame, the maximum transverse velocity divided by c is obtained from the radial and axial velocities in the beam frame when $|v_w|$ is largest:

$$\beta_{\perp-lab} = \frac{a_w}{\gamma} \approx \frac{\hat{a}_w}{\gamma[1-(\hat{a}_w{}^2/2)(1+2\langle n_0\rangle/\langle n_1\rangle)]} \quad (19)$$

where a_w is the wiggler parameter. The gain at radius r is given to lowest order in the wiggler parameter a_w as

$$\frac{n_{e-lab}\, e^2 \omega_{w-lab}\, a_w{}^2 L_{lab}{}^3}{2m\varepsilon_o c^3 \gamma^3}\left[\frac{2-2\cos\omega\_T - \omega\_T\sin\omega\_T}{(\omega\_T)^3}\right] \quad (20)$$

where $n_{e\text{-}lab}$ is the e-beam density, $\omega_{w\text{-}lab} = \beta c k_{w\text{-}lab}$ is the angular frequency of electron undulations, and L_{lab} is the undulator length measured in the laboratory frame. Here,

$$\omega\_T = [k_{w-lab}(1+v_o/c) - k_{r-lab}(1-v_o/c)/2\gamma^2]cT_{lab} \quad (21)$$

where $T_{lab} = L_{lab}/\beta c$ is the undulator transit time and $k_{r\text{-}lab}$ is the radiation wave number in the laboratory. For optimal amplification, $\omega\_T = 2.61$ [4], so that for $N >> 1$, $\gamma >> 1$ and $a_w << 1$, maximum gain occurs for

$$k_{r-lab} \approx \frac{2\gamma^2 k_{w-lab}}{1+\frac{a_w{}^2}{2}\left(1+\frac{4\langle n_{0-lab}\rangle}{\langle n_{1-lab}\rangle}\right)}. \quad (22)$$

When $n_{1\text{-}lab}(r)$ and $n_{0\text{-}lab}(r)$ are proportional to $1/r$ and $n_{e\text{-}lab}(r)$ is independent of r, the electron quiver velocity, gain, and wavelength at which maximum gain occurs are independent of r, giving ideal undulator performance. For an ultrarelativistic beam, the gain equals that of a planar magnetostatic undulator with the same quiver velocity [13], while the wavelength experiencing maximum gain is modified because $<n_{0\text{-}lab}> \neq 0$ in eq. (22).

5 APPLICATION

To maximize FEL gain while minimizing the ion density, a strong undulator with $a_w \approx 1$ at the beam radius and a strongly modulated ion channel with $n_{1\text{-}lab}(r) \approx n_{0\text{-}lab}(r)$ may be utilized. To ensure stable propagation, we consider an undulator period much shorter than the betatron wavelength [2, 11]. The ion density required for $a_w \approx 1$ is given by eq. (6). For a relativistic e-beam, a periodic ion density $<n_{1\text{-}lab}>$ of 3.5×10^{11} cm^{-3} is required for a beam radius r_b of 1 cm and $\lambda_{w\text{-}lab} = 10$ cm, while $<n_{1\text{-}lab}> = 3.5\times10^{13}$ cm^{-3} is required for $r_b = 1$ mm and $\lambda_{w\text{-}lab} = 1$ cm. A periodic density of $<n_{1\text{-}lab}> = 3.5\times10^{15}$ cm^{-3} is required for $r_b = 100$ µm and $\lambda_{w\text{-}lab} = 1$ mm, while $<n_{1\text{-}lab}> = 3.5\times10^{17}$ cm^{-3} is required for $r_b = 10$ µm and $\lambda_{w\text{-}lab} = 100$ µm. In all cases, the undulator period is much shorter than the betatron wavelength for $\gamma >> 3$.

One method of obtaining a periodic ion channel is to create a periodic plasma channel by ionizing a periodic gas density [1]. When an e-beam propagates in the channel, the plasma electrons are expelled, provided that the electron beam density exceeds the peak ion density. For a strongly modulated ion channel, this requires a beam current exceeding (17 kA)$(k_{w\text{-}lab} r_b)\beta^2 a_w$. For the above examples, the beam current must exceed 11 kA. The parameters for $r_b = 1$ cm are comparable to those of a magnetostatic X-band FEL with ion channel guiding [7], suggesting that a periodic plasma channel FEL may be operated in the ion-focusing regime.

When a strongly modulated ion beam is used as a channel, the electron beam density may be smaller than that of the ions, since ejection of plasma electrons is not required. Transporting ions out of the FEL within an ion bounce period may limit the ion hose instability [3].

6 SUMMARY

A cold electron beam propagating in a periodic ion channel amplifies radially polarized radiation. When the undulator period is much shorter than the betatron wavelength, the gain at a given distance from the axis equals that of a planar magnetostatic undulator with the same quiver velocity. Our analysis suggests that an X-band FEL may operate in the ion-focusing regime when an electron beam expels plasma electrons from plasma with periodic density. When an ultrarelativistic electron beam propagates in a periodic ion channel with density of 10^{11}–10^{17} cm^{-3}, a short-wavelength FEL may be obtained.

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