LIMITATIONS OF ELECTRON BEAM CONDITIONING FOR FREE-ELECTRON LASERS

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INTRODUCTION

The most demanding requirement for future FELs in the x-ray regime [2, 3] is the generation of a sufficiently small transverse electron emittance. To mitigate this problem, ideas have been proposed to 'condition' an electron beam by increasing each particle's energy in proportion to the square of its betatron amplitude [1, 4]. This conditioning enhances FEL gain by reducing the axial velocity spread within the electron bunch. We present a new conditioning scheme using solenoid magnets, which looks promising. But a strong head-tail focusing variation arises, as in [1]. We quantify the resulting 'projected' (bunch-length integrated) emittance growth, relating it directly to the FEL parameters. We then present a general symplectic beam conditioner and show the unavoidable relation between conditioning and projected transverse emittance growth.

FEL BEAM CONDITIONING

Electron beam conditioning, as proposed in [1], increases each particle's energy in proportion to the square of its betatron amplitude. A particle with high energy travels a shorter path in an undulator (increased mean axial velocity), while a large betatron amplitude delays a particle by lengthening its path through the undulator [5]. The conditioning correlation establishes a cancellation of these two effects, resulting in a reduction of the axial velocity spread, enhancing the FEL gain. The energy conditioning requirement, for natural undulator focusing, can be written as [1]

$$\delta_u = \delta_n + \frac{1}{4\gamma_u}\frac{\epsilon_N}{\beta_u}\frac{\lambda_u}{\lambda_r}r^2 \,, \tag{1}$$

where $\delta_n(\ll \delta_u)$ is the non-conditioned component of the particle's relative energy deviation, γ_u is the electron energy in the undulator (in units of rest mass), $\epsilon_N(=\gamma_u\epsilon)$ is the normalized rms transverse emittance (equal in x and y), $\beta_u(=\beta_x=\beta_y)$ is the constant beta-function in the undulator, λ_u is the undulator period, λ_r is the FEL radiation wavelength, and r is the invariant normalized 4D betatron amplitude of the particle,

$$r^2 \equiv \frac{x^2+(\beta_u x')^2+y^2+(\beta_u y')^2}{\beta_u\epsilon} \,. \tag{2}$$

The betatron amplitude, r, is expressed in terms of a particle's transverse positions, x and y, and angles, x' and y', with natural focusing where $\alpha_x=\alpha_y=0$. A conditioner beamline is designed to imprint this $\delta_u \sim r^2$ correlation within the electron bunch, with coefficient given in Eq. (1).

In a general case, the conditioning might be performed at low energy where the bunch is still relatively long. For short wavelength FELs, the bunch is compressed and accelerated after the injector. Both effects scale the conditioning, but in the absence of mixing, do not alter its correlation. Acceleration from γ_0 to γ_u ('energy' in undulator) reduces the conditioned relative energy spread, while compression from an initial bunch-length, σ_{z_0}, to a shorter final bunch-length, σ_{z_f}, amplifies the conditioning. The relative energy deviation, δ, at the location of the conditioner, before acceleration and compression, must then be scaled by the acceleration and compression factors:

$$\delta = \frac{\sigma_{z_f}}{\sigma_{z_0}}\frac{\gamma_u}{\gamma_0}\left(\delta_n + \frac{1}{4\gamma_u}\frac{\epsilon_N}{\beta_u}\frac{\lambda_u}{\lambda_r}r^2\right) \,. \tag{3}$$

A ONE-PHASE SOLENOID CONDITIONER

As an example conditioner, and to show the limitations of conditioners, we describe here a simplified system composed of a solenoid magnet and RF accelerating sections. The conditioner is composed of a solenoid magnet sandwiched between two RF accelerating sections operated at opposing zero-crossing phases. (A similar idea was proposed at the end of reference [4].) The first RF section 'chirps' the energy along the bunch, and the final section removes the chirp. The conditioning is generated in the solenoid by the delay of particles with large amplitudes in x and y. The solenoid strength is set to produce a **+I** linear transfer matrix in 6D with the relation $|k|L = n\pi$ ($n = 1, 2, 3, ...$), where $k \equiv \frac{1}{2}B_z/(B\rho)$, L is the solenoid length, B_z is its axial magnetic field, and $(B\rho)$ is the standard magnetic rigidity (= p_0/e). The particle coordinates within the bunch at the entrance to the system are $(x_0, x_0', y_0, y_0', z_0, \delta_0)$, where $\delta_0 \equiv \Delta p/p_0$, and we assume these variables are initially uncorrelated and have zero mean. For simplicity, we use a cylindrically symmetric beam with initial Twiss parameters: $\beta_x = \beta_y = \beta$, and $\alpha_x = \alpha_y = 0$. The Twiss parameters are unchanged, to 1^{st}-order, across the solenoid and across each 'thin' RF section. The electrons are assumed to be ultra-relativistic.

The first RF section changes the relative energy deviation of a particle to: $\delta_1 = \delta_0 + h_1 z_0$, where h_1 is the linear RF-induced slope ($h \equiv d\delta/dz$). For simplicity, the RF sections are treated as thin elements which do not alter the transverse coordinates. After the solenoid, the coordinates are unchanged to 1^{st}-order, but a chromatic $2^{\rm nd}$-order aberration is added to the angles with $\Delta x' =$

$2T_{216}x_0(\delta_0 + h_1 z_0)$ and $\Delta y' = 2T_{436}y_0(\delta_0 + h_1 z_0)$. All other $2^{\rm nd}$-order transverse aberrations are small in comparison for the case: $|k|\beta \gg 1$, $|k|L = n\pi$ [6].

The energy is not changed in the solenoid, but the particle is delayed by the helical trajectory according to $z_1 = z_0 + T_{511}x_0^2 + T_{533}y_0^2$ (bunch head at $z > 0$). Similarly, all other $2^{\rm nd}$-order longitudinal aberrations are small for the case $|k|\beta \gg 1$. The $2^{\rm nd}$-order coefficients of a solenoid with $|k|L = n\pi$ are related to each other by: $T_{511} = T_{533} = -T_{216} = -T_{436} = -k^2L/2$ [6], which, as shown in section , is an unavoidable connection for symplectic systems. The final RF section, h_2, changes the energy according to $\delta = \delta_1 + h_2 z_1 \approx (h_1+h_2)z_0 - \frac{1}{2}k^2Lh_2(x_0^2+y_0^2)$. The second chirp is chosen equal and opposite to the first, $h_1 = -h_2 \equiv h$, and the final coordinate map across the conditioner, to second order and for $|\delta_0| \ll |hz_0|$, becomes

$$\begin{aligned} x &= x_0\,, \qquad x' = x_0' + k^2Lhz_0x_0\,, \\ z &= z_0 - \frac{1}{2}k^2L(x_0^2 + y_0^2)\,, \\ \delta &= \delta_0 + \frac{1}{2}k^2Lh(x_0^2 + y_0^2)\,, \end{aligned} \tag{4}$$

with similar relations in y and y'. The final energy deviation, δ, is clearly conditioned (for $h > 0$) in both planes but in only one betatron phase (i.e., x_0, but not x_0'). This system provides spatial (but not angular) conditioning described by

$$\delta = \delta_0 + \frac{1}{2}k^2Lh\beta\epsilon_0 r^2\,, \quad r^2 \equiv \frac{x_0^2 + y_0^2}{\beta\epsilon_0}\,. \tag{5}$$

Two solenoids can also be used, separated by a $\pi/2$-transformer to condition both betatron phases, but here we simplify the description by considering only a one-phase conditioner.

The bunch-length coordinate, z, in Eqs. (4) also includes a non-linear distortion due to the solenoid delay of large amplitude particles. This can easily be removed, without changing the energy conditioning, by adding a four-dipole chicane with $R_{56} = 1/h > 0$, after the final RF section.

ENERGY CONDITIONING AND TRANSVERSE EMITTANCE GROWTH

The conditioning coefficient in Eq. (3) can be equated to that in Eq. (5) producing the conditioning requirement for the solenoid system

$$k^2Lh\beta\sigma_{z_0} = \frac{1}{2}\frac{\lambda_u}{\lambda_r}\frac{\sigma_{z_f}}{\beta_u} \equiv a\,, \tag{6}$$

where the solenoid-conditioner parameters are on the left side and the FEL parameters are on the right, and here we define the dimensionless conditioning coefficient, a. In the typical case of a short wavelength FEL, the conditioning parameter a is large, $a \gg 1$, (see table below).

The chirp parameter, h, is related it to the rms relative energy spread in the solenoids by: $\sigma_{\delta_1} \approx |h|\sigma_{z_0}$, showing the transverse aberrations in Eqs. (4) as chromatic ($\delta_1 \approx hz_0$), which we now quantify as an projected transverse emittance growth. The rms emittance after the solenoid is

Table 1: FEL parameters for LCLS [2] and VISA [7].

parameter	**sym**	**LCLS**	**VISA**	**unit**
und. energy/mc^2	γ_u	28000	140	
undulator period	λ_u	3	1.8	cm
rad. wavelength	λ_r	1.5	8500	Å
und. $\beta_{x,y}$	β_u	72	0.6	m
und. bunch length	σ_{z_f}	24	100	μm
conditioning coef	a	33	1.8	

$$\epsilon_x^2 = \langle(x-\overline{x})^2\rangle\langle(x'-\overline{x'})^2\rangle - \langle(x-\overline{x})(x'-\overline{x'})\rangle^2\,. \tag{7}$$

The mean values, $\overline{x} = \langle x\rangle$, and $\overline{x'} = \langle x'\rangle$ are zero since the initial coordinates are uncorrelated and have zero mean. The correlation $\langle xx'\rangle$ is zero for the same reasons, so the x-emittance after the solenoid is:

$$\epsilon_x^2 = \langle x^2\rangle\langle x'^2\rangle \approx \epsilon_{x0}^2[1 + (k^2Lh\beta\sigma_{z_0})^2]\,, \tag{8}$$

where $\epsilon_{x0} = \langle x_0^2\rangle/\beta = \langle x_0'^2\rangle\beta$, and $\sigma_{z_0}^2 = \langle z_0^2\rangle$, with a similar form in y. The relative emittance growth after the solenoid is

$$\frac{\epsilon_x}{\epsilon_{x0}} \approx k^2Lh\beta\sigma_{z_0} = a \gg 1\,, \tag{9}$$

which is identical to the conditioning relation in Eq. (6), providing a direct connection between transverse emittance growth and FEL conditioning requirements.

For parameters of the LCLS [2] shown in Table 1 (using a beta function for natural focusing, to be consistent with Eq. 1), the relative emittance growth is extremely large at $\epsilon_x/\epsilon_{x0} \approx 33$. The parameters for the VISA FEL [7] are also included showing that conditioning may still be possible at longer wavelengths. This growth is actually an increase of the 'projected' transverse emittance integrated over the bunch length. The second line of Eq. (4) shows that the bunch head ($z_0 > 0$) is de-focused (equating: $k^2Lhz_0 = 1/f$), while the bunch tail ($z_0 < 0$) is focused.

With a chirped energy spread, the chromatic effects of the solenoid are equivalent to the effects of an RF-quadrupole (RFQ). It is interesting to compare this result with that of reference [1], where a completely different conditioner beamline, employing transverse RF cavities, produced an undesirable RFQ effect. In fact, as shown in the next section, FEL beam conditioning in a symplectic beamline always produces an undesirable RFQ-effect, which is extremely large for short wavelength FELs, as given in Eq. (9).

A GENERAL CONDITIONER

We will now show that the transverse emittance growth associated with conditioning is not related to the specific design outlined in the previous section, but is a general feature of any conditioner, and is due to the symplecticity of the map between the entrance to and exit from the conditioner. To simplify consideration, we assume that the conditioner does not introduce coupling between the vertical and horizontal planes, and consider only the horizontal plane with the initial values of coordinates (x_0, x_0') at the entrance, and the final values (x, x') at the exit. Consideration of the vertical coordinates y, y' can be carried out analogously to x, x'. We will also assume that the initial and final values of the longitudinal coordinate are the same: $z = z_0$. Instead of using the variables x_0, x_0' and x, x', it is convenient and more general to introduce new variables u_0, v_0, and u, v, such that

$$\begin{pmatrix} u_0 \\ v_0 \end{pmatrix} = Q_0 \begin{pmatrix} x_0 \\ x_0' \end{pmatrix}, \quad \begin{pmatrix} u \\ v \end{pmatrix} = Q \begin{pmatrix} x \\ x' \end{pmatrix}, \tag{10}$$

where the matrices Q_0 and Q are

$$Q_0 = \frac{1}{\sqrt{\beta_0}} \begin{pmatrix} 1 & 0 \\ \alpha_0 & \beta_0 \end{pmatrix}, \; Q = \frac{1}{\sqrt{\beta}} \begin{pmatrix} 1 & 0 \\ \alpha & \beta \end{pmatrix}, \tag{11}$$

with β_0, α_0 and β, α the Twiss parameters at the entrance and exit of the conditioner, respectively. Being symplectic linear transformations, Q and Q_0 conserve the symplecticity of the map from (u_0, v_0) to (u, v). Note, that in linear approximation this map has a form

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} u_0 \\ v_0 \end{pmatrix}, \tag{12}$$

with ψ the betatron phase advance across the conditioner. Also note that the contribution of the x-coordinate $x_0^2/(\beta\epsilon_0)$ to the parameter r in Eq. (5) is equal to u_0^2/ϵ_0, and the conditioning requirement Eq. (5) can be written as

$$\delta = \delta_0 + \frac{1}{2} b u_0^2, \tag{13}$$

where $b = a/\sigma_{z_0}$, and the conditioning constant a is given by Eq. (6).

To derive a general symplectic map which in linear approximation reduces to the linear map Eq. (12) and also includes the conditioning given by Eq. (13), we will use a method of generating functions [8]. We choose a generating function which depends on initial coordinates u_0 and z_0 and final momenta v and δ, $F(u_0, z_0, v, \delta)$. The map is defined by the relations

$$v_0 = \frac{\partial F}{\partial u_0}, \; \delta_0 = \frac{\partial F}{\partial z_0}, \; u = \frac{\partial F}{\partial v}, \; z = \frac{\partial F}{\partial \delta}. \tag{14}$$

In paraxial approximation all coordinates and momenta are considered small and we can expand F in a Taylor series. The linear terms in this expansion vanish because zero initial coordinates and momenta map to zero final ones. The expansion begins from the second order terms $F \approx F_2 + F_3 + \dots$, where F_2 is a quadratic, and F_3 is a cubic function of the coordinates and momenta. The function F_2 should generate the linear map Eq. (12) for u and v with a unit transformation for z and δ—a direct calculation shows that

$$F_2 = \frac{1}{2}(u_0^2 + v^2)\tan\psi + u_0 v \sec\psi + \delta z_0. \tag{15}$$

The function F_3 generates 2^{nd}-order abberations in the system, out of which we choose only a term responsible for the conditioning:

$$F_3 = -\frac{1}{2} b z_0 u_0^2. \tag{16}$$

Indeed, using the second of Eqs. (14) with Eqs. (15) and (16) we find $\delta_0 = \delta - \frac{1}{2} b u_0^2$, in agreement with Eq. (13). At the same time the first and the third of Eqs. (14) yield $v_0 = u_0 \tan\psi + v \sec\psi - b z_0 u_0$, $u = v\tan\psi + u_0 \sec\psi$. These equations can be easily solved for u and v:

$$\begin{aligned} u &= u_0\cos\psi + v_0\sin\psi + b z_0 u_0 \sin\psi, \\ v &= -u_0\sin\psi + v_0\cos\psi + b z_0 u_0\cos\psi. \end{aligned} \tag{17}$$

We emphasize here that the same term in the symplectic map Eq. (16) that is responsible for the conditioning of the beam also introduces in Eq. (17) the transverse deflection that varies along the bunch. This also means that adding a system that 'fixes' this deflection downstream of the conditioner would inevitably remove the conditioning itself.

To calculate the emittance increase of the beam due to the conditioning we use Eq. (7) for the emittance, with $\bar{u} = \bar{v} = 0$, we find $\epsilon_x^2 = \langle u^2\rangle\langle v^2\rangle - \langle uv\rangle^2$. Substituting the map, Eqs. (17), into this yields

$$\epsilon_x^2 = \epsilon_{x0}^2(1 + b^2\sigma_{z_0}^2) = \epsilon_{x0}^2(1 + a^2), \tag{18}$$

in agreement with Eq. (8), but now in a general case with arbitrary phase advance, ψ, and non-zero initial alpha function, α_0. For the specific conditioner described above, we have $\psi = 2n\pi$, $\beta_0 = \beta$, $\alpha_0 = \alpha = 0$, and Eqs. (17) reproduce the first two of Eqs. (4).

CONCLUSIONS

We have demonstrated for a general one-phase conditioner that a strong head-tail focusing variation always accompanies the energy conditioning correlation, and that this focusing variation is set solely by the FEL parameters, and not the conditioner. A two-phase conditioner is more complicated, but does not qualitatively change the arguments presented here.

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