TEMPLATE POTENTIALS TECHNIQUE WITH FULLY-PARAMETERIZED FIELD SOLVER FOR HIGH-CURRENT BEAMS SIMULATION*

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Abstract

A three-dimensional (3D) particle-in-cell (PIC) code based on the template potential concept [1-6] has been upgraded by inclusion of a fully parameterized model for 3D space charge force calculations. The dynamics of a bunched 3D beam were studied for FODO channels having a conducting chamber with an arbitrary elliptical symmetry. The simulation results were in a good agreement with conventional 3D PIC models, but were obtained with ten to one hundred shorter computational times.

1 INTRODUCTION

Envelope (KV) equations are adequate for modeling a coasting two-dimensional (2D) beam with elliptical symmetry through linear FODO channels in free space [7]. Adaptation of the KV model for rms sizes [8-10] provides inclusion of image forces and momenta dispersion [11-12]. However, extension of the envelope formalism to 3D beam configurations [13] can be inappropriate for non-linear image forces.

General 3D PIC codes provide the most complete model, but require significant computational times. An alternative approach using templates has been developed providing a nearly complete model with dramatically reduced computational times [1-6]. Similar to analytical models, this approach operates with macro objects and expresses the solution for field components via special functions, the template potentials, maintaining a good generality of dynamics simulation on a micro level.

2 HIERARCHY OF TEMPLATE METHODS

The grid density is the most time-consuming subroutine in general 3D PIC codes. The template approach does not require calculation of the 3D grid density. Instead, the space charge fields are derived from the convolution of template potentials, corresponding to beam shape functions $S_{x,y}(z)$, scaled by the charge density $\Lambda(z)$ [1,5].

The theory of templates was published in [1-6]. These template-based field solvers were included as subroutines into PIC codes to model dynamics of 3D intense beams.

1. One approach [6] used the transverse fields $E_{x,y}$ found from the 2D Poisson equation with the longitudinal field E_z determined by superposition of on-axis template potentials or uniformly charged round discs (slices).
2. A more general approach [3-5] obtained $E_{x,y}$ from a series of 2D Poison equations with a "corrected density": $\Delta u = -4\pi\rho_{corr}$ and $\rho_{corr}(x,y,z)=\rho+\partial^2 u/\partial z^2/4\pi$. The latter term was evaluated via an off-axis longitudinal field E_z derivative: $\partial^2 u/\partial z^2 = -\partial E_z/\partial z$, found from the superposition of round template potentials with arbitrary transverse density.
3. An improvement [1-2] was made with the inclusion of the driving terms $\partial^2 u/\partial z^2 = -\partial E_z/\partial z$ and corrected density for all off-axis coordinates for any templates with elliptical symmetry. The code also used a parameterization for E_z field representation. This version approaches the generality of a conventional 3D PIC algorithm and is appropriate for 3D modeling of a bunched beam without axial symmetry, propagating in an arbitrary focusing doublet, as shown schematically in Fig. 1.

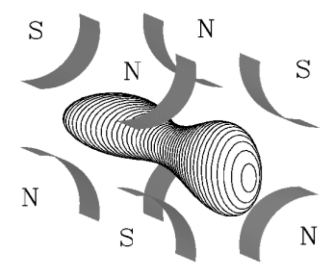

Figure 1: Non-ellipsoidal 3D bunched beam in a quadrupole doublet (different polarities are indicated).

3 PARAMETERIZATION

For the template technique [1-6], the beam is represented as a sequence of templates and the total potential $u_{beam}(x,y,z)$, including space charge and image fields, is represented as a superposition of individual template potentials $u_{tmp}(x,y,z)$.

Template potentials for geometries of different sizes and aspect ratios, as shown in Fig. 2, are stored and tabulated prior to simulations and will reconstruct the potential of a beam with an evolving distribution.

Since direct storage of all 3D template potential functions could require excessive memory demands and possibly slow the simulation process, an alternative approach was used. The approximation of individual template potentials allows an analytical representation [1]: $u_{tmp}(x,y,z)=\exp(a_0(x,y)+a_1(x,y)z+a_2(x,y)z^2)$ with the longitudinal field derived as $E_{z,tmp}=-(a_1+2a_2 z)u_{tmp}(x,y,z)$. The three coefficients a_i, i=0,1,2 define both potential and field from a template for specific x, y as a function of "z".

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The approximation for the transverse fields $E_{x,tmp}$ and $E_{y,tmp}(x,y,z)$ are obtained with a similar approach.

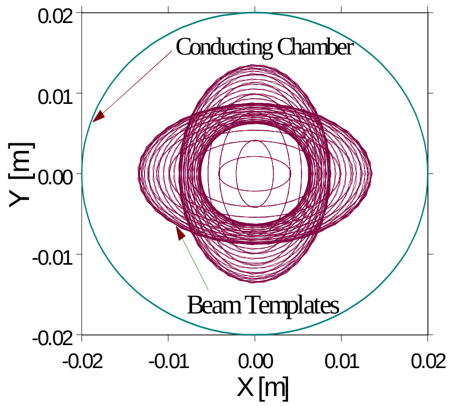

Figure 2: Elliptical templates with different sizes and aspect ratios, used to reconstruct the 3D beam of Fig. 1.

The approximating coefficients $\{a_i\}$ for all three field components $E_{x,tmp}$ $E_{y,tmp}$ $E_{z,tmp}$ can be stored in a library of modest memory size. The template fields are now represented by a smooth continuous function providing faster numerical evaluations for total field ($E_{x,y,z}$) reconstruction and the avoidance of computational errors.

4 ILLUSTRATION OF 3D FIELD SOLVER

As an example of the 3D parameterization, we evaluated the beam shown in Fig. 1, carrying a total charge of 10^{-11} C in a conducting cylindrical pipe 4 cm in diameter.

The beam was represented by $N_p=10^4$ macroparticles. Figs. 3 and 4 illustrate the procedure.

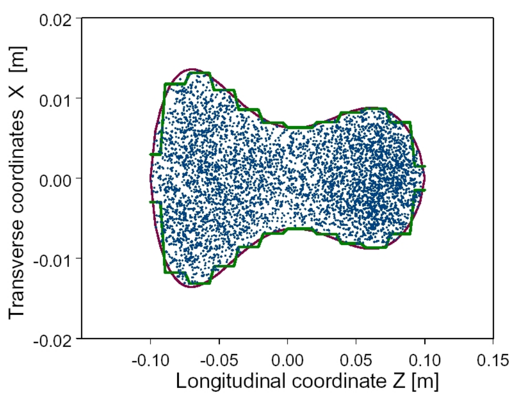

Figure 3: Macro-particle $\{x\}$-coordinates (dots), the envelopes (step-wise lines) and smoothed profiles (solid).

For the vertical direction the algorithm is similar.

From particle coordinates, we find rms envelopes $\langle x^2\rangle^{1/2}(z,p)$, $\langle y^2\rangle^{1/2}(z,p)$ and the shape functions $S_{x,y}(z,p)$, scaled by $\Lambda(z,p)=\lambda_0\cdot\lambda(z,p)$ for each i-th thick slice, having a with H_Z^T in z-direction.

$$\lambda(z_i)=\frac{N_i}{\pi\cdot r_{m,x}r_{m,y}H_Z^T} \quad \text{and} \quad \lambda_0=\frac{\int dz\int\sigma(x,y,z,p)dxdy}{N_p}$$

with $r_{m,x}=S_x(z)$, $r_{m,y}=S_y(z)$, $r_m^{\ 2}=r_{m,x}^{\ 2}+r_{m,y}^{\ 2}$ and $\sigma(x,y,z,p)=\sigma_0(z)[1-(x^2(z)+\ y^2(z))/r_m^{\ 2}]^p$, see [1,5] for details.

A superposition of template potentials yields the total beam potential u_{beam} and $E_{x,y,z}$ fields, as shown in Fig.5.

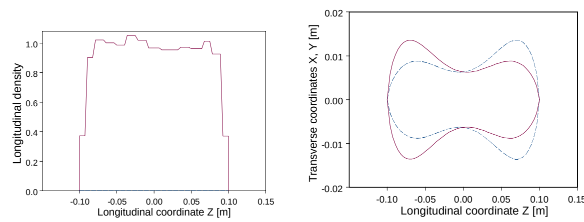

Figure 4: Charge density $\lambda(z)$ (left) and the shape functions for horizontal and vertical directions (right).

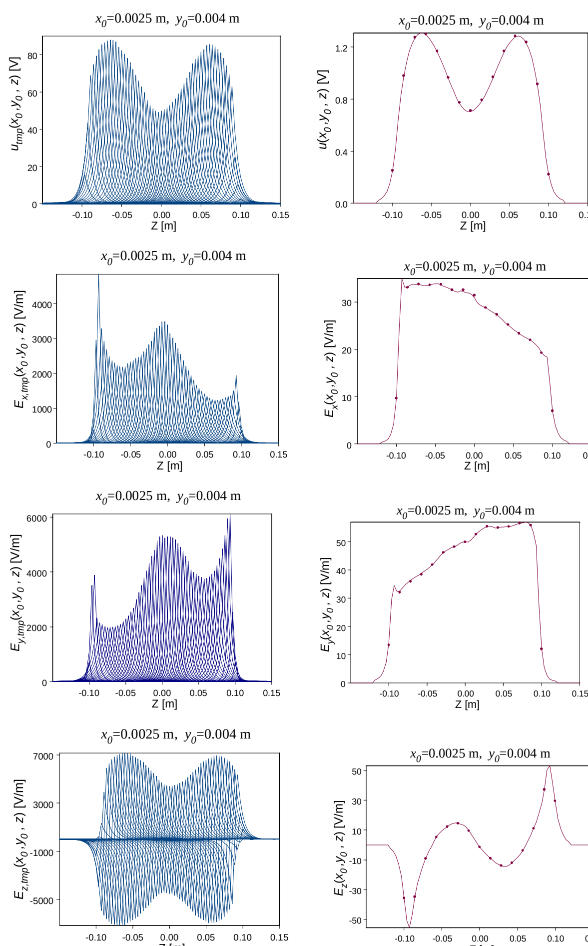

Figure 5. Template potentials and fields (left). The total potential and total fields (right, solid lines) found for off-axis coordinates, as functions of "z" and the cross-checking results obtained by SOR 3D algorithm (dots).

The number of coefficients $\{a_i\}$ for the potential and each field component is less than 9×10^3 (see [1] for details). The total array providing a full $E_{x,y,z}$ parameterization is of 2.7×10^4 in size. With the inclusion of different charge densities $\sigma(x,y,z,p)$, say for p=0,1,2,4 the total number of coefficients would be about 10^5.

5 BEAM SIMULATION BY 3D PIC CODE

The parameterized field solver was implemented in the 3D PIC code and the beam dynamics modeled for one full turn in the E-Ring [14] with a focusing lattice consisting of 36 FODO periods of length 32 cm and 36 dipole magnets each deflecting beam by 10°. The initial electron beam was assumed to have an energy of 10 KeV and a current of 10 mA (the equivalent generalized perveance for a 2D beam would be Q=1.5x10^{-4}). As an example, we considered a beam bunch with rms semi-axes 0.35cm×0.67cm and a semi-length of 5cm, with initial rms emittances $\varepsilon_{x,y}=5\cdot10^{-5}\pi$ m·rad, $\Delta p/p=0$ (i.e. $\varepsilon_z=0$), propagating along a metal pipe 4 cm in diameter through a FODO lattice. No longitudinal focusing was applied.

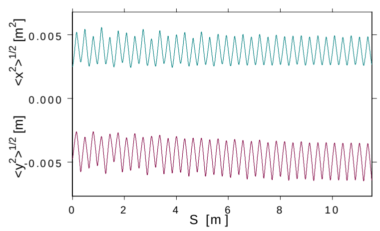

Figure 6. Rms-envelopes in the E-Ring.

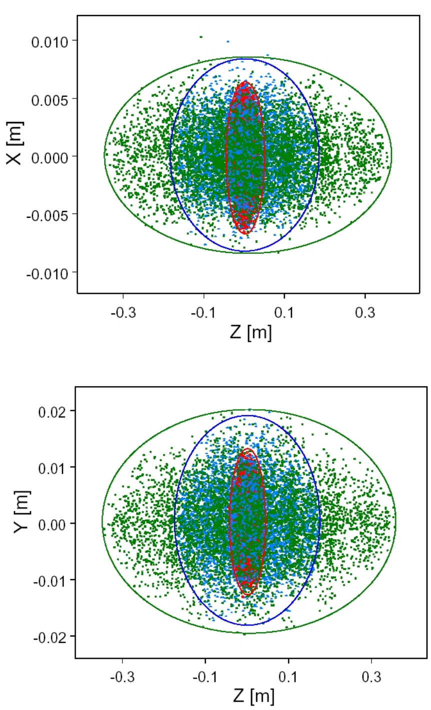

Figure 7. (Color) Particle distributions (x,z) and (z,y) for s=0, s=6.4 m (20 periods) and s=11.52 m (36 periods=1 full turn). Contour lines include 98% of the beam particles for each cross-section. Due to the absence of longitudinal focusing the beam dilutes in "z" direction and space charge forces subside.

Beam particles were tracked through one full turn of 11.52 m with the initial, intermediate and final (z,x), (z,y) distributions given in Figs. 6-7. The number of particles used for simulation was $5\cdot10^4$ and the number of templates used to reconstruct the beam potential was 50. The dipole magnet chambers were approximated by a straight chamber. In addition, the 2D charge density $\sigma(x,y,z,p)$ was assumed to have a fixed parameter "p". The typical calculation time was 30 minutes on a 600 MHz Alpha machine. To the best of our knowledge this is one to two orders of magnitude faster than using conventional 3D PIC methods.

6 DISCUSSION AND CONCLUSION

General PIC codes may be used to model nearly any beam distribution. The template-based formalism is less general and would not be appropriate for e.g., filamenting beam distributions. The template approach is, however, appropriate for those beams which have *a priory* regular shapes and when their 3D geometry may be approximately reconstructed by elliptical slices such as shown in Figs. 1,2.

Some of the template limitations can be overcome. E.G., the inclusion of changing chamber geometry (bends, etc.), off-centered beam bunches and varying parameter "p" in $\sigma(x,y,z,p)$, are feasible simply by an extension of the tabulated data and the implementation of this upgrade is planned. However, clustered, filamenting, or disintegrating beam simulation requires a standard 3D PIC algorithm with adaptive mesh refinement.

Preliminary comparisons with standard 3D PIC codes give a confidence in a validity of template-based algorithms for a large class of beam configurations with calculation times one to two orders of magnitude shorter.

7 REFERENCES

[1] L.G.Vorobiev and R.C.York, EPAC'02, Paris, June 2002, p. 1679.

[2] L.G.Vorobiev and R.C.York, ICCS 2002, LNCS 2331, Springer-Verlag, pp. 315-324 (2002).

[3] L.G. Vorobiev and R.C. York, Phys. Rev. ST Accel. Beams **3**, 114201 (2000).

[4] L.G. Vorobiev and R.C. York, PAC'01, Chicago, June 2001, pp. 3072-3075.

[5] L.G. Vorobiev and R.C. York, ibid., pp. 3075-3077.

[6] L.G. Vorobiev and R.C. York, PAC'99, New York, March 1999, p. 2781.

[7] I. M. Kapchinskij and V. V. Vladimirskij, Proc. Int. Conf. on High-Energy Accelerators (CERN, Geneva, 1959), p. 274.

[8] P. Lapostolle, CERN Report AR/Int SG/65-15, 1965.

[9] F. J. Sacherer, IEEE Trans. NS **18**, p. 1105 (1971).

[10] P. Lapostolle, IEEE Trans. NS **18**, p. 1101 (1971).

[11] B. L. Qian, J. Zhou, and C. Chen, Phys. Rev. ST Accel. Beams **6**, 014201 (2003).

[12] M. Venturini and M. Reiser, Phys. Rev. E **54**, p. 4725 (1998).

[13] T. Wangler "RF Linear Accelerators", John Wiley & Sons, New York (1998).

[14] M.Reiser et al. PAC'99, New York, March 1999, p. 234. Web site: http://www.ireap.umd.edu/ebte/ring/.