Plasma Inverse Transition Acceleration

Ming Xie, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA

Abstract

The world has not heard of thee, until now...

1 INTRODUCTION

It can be proved fundamentally from the reciprocity theorem with which the electromagnetism is endowed that corresponding to each spontaneous process of radiation by a charged particle there is an inverse process which defines a unique acceleration mechanism, from Cherenkov radiation to inverse Cherenkov acceleration (ICA) [1], from Smith-Purcell radiation to inverse Smith-Purcell acceleration (ISPA) [2], and from undulator radiation to inverse undulator acceleration (IUA) [3]. There is no exception. Yet, for nearly 30 years after each of the aforementioned inverse processes has been clarified for laser acceleration, inverse transition acceleration (ITA), despite speculation [4], has remained the least understood, and above all, no practical implementation of ITA has been found, until now.

Unlike all its counterparts in which phase synchronism is established one way or the other such that a particle can continuously gain energy from an acceleration wave, the ITA to be discussed here, termed plasma inverse transition acceleration (PITA), operates under fundamentally different principle. As a result, the discovery of PITA has been delayed for decades, waiting for a conceptual breakthrough in accelerator physics: the principle of alternating gradient acceleration [5, 6, 7, 8, 9, 10]. In fact, PITA was invented [7, 8] as one of several realizations of the new principle.

Transition radiation occurs when a charged particle transits an inhomogeneous medium, often in the form of a metal foil. To discover its inverse process for practical acceleration, one has to make two critical steps through a conceptual evolution. First, replace the metal foil by an underdense plasma layer such that a focused TM mode with longitudinal electric field and a co-propagating particle can pass through the layer with negligible reflection and scattering. Second, apply the principle of alternating gradient acceleration [10] by placing the plasma layer in the region where the particle experiences deceleration due to phase slippage. The purpose here is to make the particle slipping through deceleration phase quickly, utilizing the fact that phase velocity is larger in plasma than in vacuum, such that less energy would be lost in deceleration phase.

By willing to take losses momentarily in deceleration, PITA is awarded tremendously in overall acceleration performance. Unlike ICA which is limited in gradient by gas breakdown in the passage of an intense laser, PITA utilizes ionized gas. Unlike ISPA which requires particle beam to stay in close vicinity of a diffractive structure such as grating thus its performance is severely limited in practice, PITA operates even in free space. Unlike IUA which is limited in maximum energy by strong radiative loss, PITA can be scaled to much higher energy. Furthermore, the single stage PITA has already been implemented in many stages [7, 8] for acceleration over extended distances in over-sized open waveguides [5, 6, 7, 8, 9, 10]. As such, PITA is also a critical innovation in advanced accelerator technology.

Finally, I must point out that the concept, principle, and analytical techniques presented here have also inevitably shed new light on the understanding, interpretation, and calculation of transition radiation. According to the prevailing notion, transition radiation is attributed to the reorganization of charge field as a particle transits medium boundary [11]. Such an explanation, however, is too general to be incorrect, yet too vague to be of little use for either clarifying concept or carrying out practical calculation of the process. An alternative and transparent treatment of transition radiation as an inverse process of the inverse transition acceleration is under preparation.

2 THEORY

In this section, we first derive TM mode of a monochromatic laser beam in an inhomogeneous medium and then consider direct acceleration of a charged particle by the longitudinal electric field of such a mode. The medium of interest is specified by $\mu = \mu_0$ and $\epsilon = \epsilon_0\epsilon_r(z)$, where $\epsilon_r(z) = \nu^2(z)$ and $\nu(z) = 1 + \delta\nu(z)$. Assuming $|\delta\nu| \ll 1$ and $\left|\frac{d\epsilon_r}{dz}\right| \ll k_0\epsilon_r$, vector potential is shown to be governed under Lorentz gauge by a wave equation of the same usual form $\nabla^2\mathbf{A} + k^2\mathbf{A} = 0$, except where $k = k_0\nu(z)$, $k_0 = \omega/c$, with time dependence $e^{-i\omega t}$ understood.

It has been shown [12] that TM mode in vacuum can be derived by assuming $\mathbf{A} = A_z(r,z)\hat{\mathbf{z}}$. Following the same recipe, the vector wave equation is converted into a scalar one $\nabla^2 A_z + k^2 A_z = 0$, which is much easier to solve. Seeking slowly varying envelope solution of the form

$$A_z(r,z) = \Psi(r,z)e^{i\int^z k(s)ds}, \tag{1}$$

we then obtain the usual equation for the envelope

$$\nabla_\perp^2\Psi + 2ik\frac{\partial\Psi}{\partial z} = 0, \tag{2}$$

which admits the well-known fundamental mode

$$\Psi = \frac{\Psi_0}{1+iq}e^{-\frac{\rho^2}{1+iq}}, \tag{3}$$

where $\rho = r/w_0$, $q = z/z_r$, and $z_r = k_0w_0^2/2$ is the Rayleigh range. The difference between k and k_0 is neglected in Eqs.(2,3), since it contributes only to higher order corrections compared to the dominant one preserved in the phase factor of Eq.(1).

Given solution for $\mathbf{A}$, electromagnetic fields can be determined from $\mathbf{B} = \nabla \times \mathbf{A}$, $\mathbf{E} = i\omega\mathbf{A} + \frac{i}{\omega\mu\epsilon}\nabla(\nabla \cdot \mathbf{A})$, yielding for TM_{01} mode under paraxial approximation

$$B_z = B_r = E_\phi = 0, \quad B_\phi = \frac{\nu E_r}{c},$$

$$E_r = -\frac{i\gamma_g E_a \rho}{(1+iq)^2} e^{-\frac{\rho^2}{1+iq} + i\int^z k(s)ds},$$

$$E_z = \frac{E_a}{(1+iq)^2}\left(1 - \frac{\rho^2}{1+iq}\right) e^{-\frac{\rho^2}{1+iq} + i\int^z k(s)ds},$$

where $\gamma_g = z_r/w_0 \gg 1$ is the condition used when making the paraxial approximation.

Acceleration by the TM_{01} mode of a charged particle traveling along the axis, $\rho = 0$, is determined by

$$\frac{dW}{dq} = \frac{eE_a z_r \cos\psi(q)}{1+q^2}, \tag{4}$$

where $W = \gamma mc^2$ is the particle energy,

$$\psi = \omega t(z) - \int^z k(s)ds + 2\tan^{-1}\left(\frac{z}{z_r}\right), \tag{5}$$

and $t(z) = t_i + (1 + 1/2\gamma^2)(z/c)$ is the particle orbit. For simplicity, γ is taken as a constant when evaluating the right hand side of Eq.(4), which is valid, strictly speaking, only if the relative change of γ remains small during interaction. For laser propagation through an underdense plasma layer, we have $\delta\nu(z) = -f_p(z)/2\gamma_p^2$, where $\gamma_p = \omega/\omega_p \gg 1$, $\omega_p = c\sqrt{4\pi r_e n_0}$ is the plasma frequency corresponding to the peak electron density n_0 of a profile $n_e(z) = n_0 f_p(z)$ with $f_p(z) \le 1$ and r_e being the classical radius of electron. It is convenient to introduce the following profile function as a model for an ionized gas jet

$$f_p(z) = \frac{1}{\cosh^2\left(\frac{z-z_p}{l_p}\right)},$$

since it is analytically integrable as

$$\int_{-\infty}^{z} f_p(s)ds = l_p\left[1 + \tanh\left(\frac{z-z_p}{l_p}\right)\right],$$

where l_p is the half thickness and z_p is the location of the plasma layer. The phase from Eq.(5) is then reduced to

$$\psi = \alpha_\gamma q + \alpha_p\left[1 + \tanh\left(\frac{q-q_p}{\beta_p}\right)\right] + 2\tan^{-1} q + \psi_i,$$

where the scaled parameters are defined by

$$\alpha_\gamma = \frac{\gamma_g^2}{\gamma^2}, \quad \alpha_p = \beta_p\frac{\gamma_g^2}{\gamma_p^2}, \quad \beta_p = \frac{l_p}{z_r}, \quad q_p = \frac{z_p}{z_r}.$$

The net energy change of the particle traveling from $-\infty$ to ∞ along the trajectory is given by

$$\Delta W = eE_a z_r \Lambda, \tag{6}$$

$$\Lambda \equiv \Lambda(\alpha_\gamma, \alpha_p, \beta_p, q_p, \psi_i) = \int_{-\infty}^{\infty} dq F(q), \tag{7}$$

$$F(q) = \frac{\cos\psi(q)}{1+q^2}. \tag{8}$$

When expressed in practical units, Eq.(6) becomes

$$\Delta W[\mathrm{MeV}] = 31\sqrt{P[\mathrm{TW}]}\,\Lambda,$$

where $P = \pi E_a^2 z_r^2/8Z_0$ is the laser power, and Z_0 is the vacuum impedance.

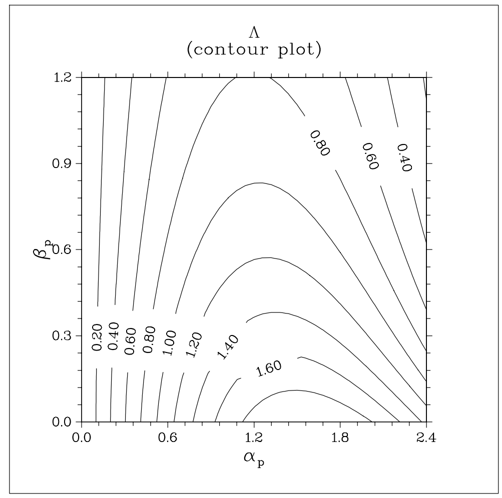

Figure 1: Λ as a function of $\{\alpha_p, \beta_p\}$ with $\{\alpha_\gamma = 0, q_p = 0\}$ and ψ_i optimized everywhere to maximize Λ.

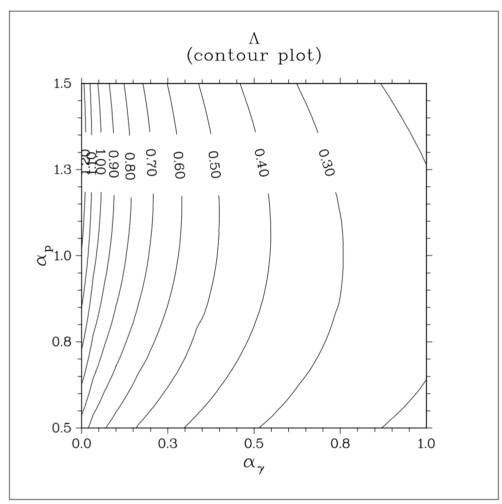

Figure 2: Λ as a function of $\{\alpha_\gamma, \alpha_p\}$ with $\{\beta_p = 0.5, q_p = 0\}$ and ψ_i optimized everywhere to maximize Λ.

3 PERFORMANCE

Due to phase slippage, a particle moving on the axis sees a varying field, Eq.(8). It is well understood that the integral effect or the net energy change of the particle is identically zero, $\Lambda = 0$, in vacuum where $\alpha_p = 0$, regardless of

ψ_i. This result is often known as Lawson-Woodward theorem [13, 14, 15]. We will see in this section how much net acceleration can be achieved with $\alpha_p > 0$. Let's first examine the dependence of Λ on the five scaled parameters through Eq.(7) and then look at a specific example given in terms of real parameters.

Shown in Fig.1 is a contour plot of Λ as a function of $\{\alpha_p, \beta_p\}$ with $\{\alpha_\gamma = 0, q_p = 0\}$ and ψ_i optimized everywhere to maximize Λ. It is noted that (1): $\Lambda = 0$ for $\alpha_p = 0$, as expected; (2): $\partial\Lambda/\partial\beta_p < 0$; (3): optimal value of α_p decreases from $\pi/2$ as β_p increases from 0; (4): the absolute maximum, $\Lambda_{max} = 2$, verified over the entire parameter space, is reached at $\{\alpha_\gamma = 0$, $\alpha_p = \pi/2$, $\beta_p = 0$, $q_p = 0$, $\psi_i = \pi/2\}$. Shown in Fig.2 is a contour plot of Λ as a function of $\{\alpha_\gamma, \alpha_p\}$ with $\{\beta_p = 0.5, q_p = 0\}$ and ψ_i optimized. It is noted that (1): $\partial\Lambda/\partial\alpha_\gamma < 0$; (2): the reduction of Λ due to finite particle energy can be neglected if $\alpha_\gamma \ll 1$ or $\gamma^2 \gg \gamma_g^2$, a condition equivalent to requiring that the phase slippage caused by finite particle energy is negligibly small compared with Guoy phase shift. In addition, we note without showing the plot due to page limit, that $\partial\Lambda/\partial q_p < 0$, and the most effective location to place a plasma layer is at the waist $q_p = 0$.

A specific example is given in Table 1 for electron acceleration, and the integrand $F(q)$ of Λ from Eq.(7) is plotted in Fig.3. For comparison, $F(q)$ is also plotted in Fig.4 for the same case, but without a plasma layer. It is noted that $\Lambda = 1$ with the plasma layer and $\Lambda = 0$ without. The effect of the plasma layer is to decrease the region over which the particle experiences a deceleration field by enhancing phase slippage over that region. In the extreme case when the thickness of the plasma layer is reduced to 0 while still maintaining an overall phase shift of π by increasing plasma density, the area of the negative field region in Fig.3 would be reduced to 0, leading to $\Lambda_{max} = 2$.

In conclusion, a new acceleration mechanism is made transparent and established decisively from now on. Stimulating discussions with Max Zolotorev about the intimate relationship between radiation and acceleration are acknowledged. This work was supported by the U.S. Department of Energy under contract No.DE-AC03-76SF00098.

Table 1: A PITA Example

ΔW	MeV	31	Λ	1
W_0	MeV	100	α_γ	0.058
E_a	GV/m	44	α_p	1.23
P	TW	1	β_p	0.5
λ	μm	1	q_p	0
w_0	μm	15	ψ_i	1.91
z_r	μm	707	γ	196
l_p	μm	353	γ_g	47
n_0	$10^{18}/\mathrm{cm}^3$	1.2	γ_p	30

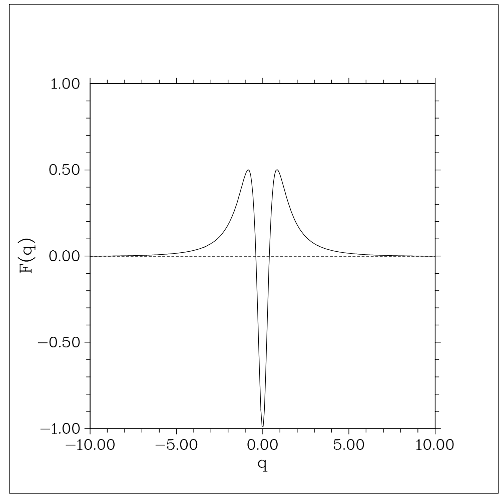

Figure 3: F(q) for the example in Table 1, $\Lambda = 1$.

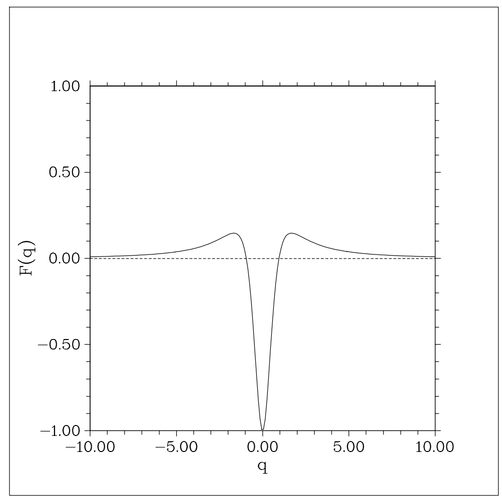

Figure 4: F(q) corresponding to the same example, but without the plasma layer, $\psi_i = \pi$ for this case, $\Lambda = 0$.

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