ELECTROMAGNETIC GREEN'S FUNCTION BASED SIMULATIONS OF PHOTOCATHODE SOURCES

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Abstract

We show the results of beam simulations for photocathode sources using a newly developed Green's function based code called IRPSS (Indiana Rf Photocathode Source Simulator). In general, a fully electromagnetic treatment of space-charge fields within simulations of photocathode sources is typically difficult since the beam is most often tightly bunched. The problem is further complicated by the inclusion of nearby conducting structures, i.e. cathode and cavity walls, from which the fields are reflected. The entire problem can be solved self-consistently using an electromagnetic Green's function method. Since Green's functions are generated by a Delta function source while simultaneously satisfying the boundary conditions of the system, they are an effective tool when solving for fields within photocathode source simulations. Using IRPSS we show examples of electromagnetic field calculations and benchmark studies.

INTRODUCTION

IRPSS is a fully electromagnetic simulation code which is capable of modeling the physics of electron sources such as photoinjectors. The novel feature of this code is its' usage of Green's function methods to compute the electromagnetic fields generated by the beam, i.e. space-charge forces. Green's function methods allow for the resolution of fields due to arbitrarily tight bunches, while correctly incorporating the effects of the source's conducting surface. This is in contrast to other codes such as, PARMELA [1] which computes the space-charge fields electrostatically, and TREDI [2] which utilizes Lienard-Wiechert methods to compute the electromagnetic space-charge fields but can only handle the effects of a conducting cathode – not the side walls. And while, electromagnetic PIC algorithms can compute the time-dependent fields for arbitrary shaped conductors, the field resolution is limited by the choice of length and time scales in the simulation grid, i.e. the bunch dynamics must occur on length and time scales which are sufficiently large compared to the grid size.

In its' current operating mode, IRPSS simulates the space-charge fields of a disk-like (zero-thickness) bunch for the cathode geometry shown in Fig. 1. In this paper, we will show how IRPSS computes the electromagnetics associated with the Fig. 1 geometry. We will also show recent results which illustrate how IRPSS models a bunch of finite size and discuss benchmarking studies of IRPSS. We are actively working to improve IRPSS to include the effects of one or more irises, which are commonly found in photoinjectors. However, we will not describe those improvements in the present paper.

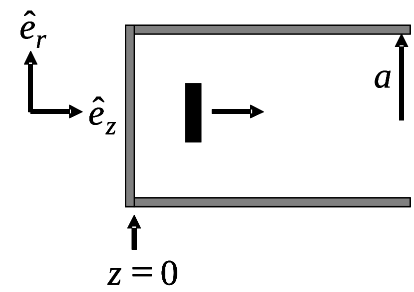

Figure 1: Schematic of system including conductor geometry and accelerating bunch.

Our paper is organized as follows. In Sec. 2, we describe the Green's function method which IRPSS utilizes to compute the electromagnetic potentials for Fig.1. In Sec. 3, we show the potentials computed by IRPSS corresponding to the design parameters of the BNL 2.856 GHz 1.6 cell photocathode gun. [3] In Sec. 4, we describe a series of benchmarking studies of IRPSS with analytical methods. In Sec. 5, we give a summary of the paper.

ELECTROMAGNETIC POTENTIALS

Fig. 1 shows a cross-section of the cathode geometry that IRPSS simulates. This geometry is assumed to be circularly symmetric. The cathode is located at the point $z=0$, and the pipe radius is denoted by $r=a$. In its' present form, IRPSS calculates the electromagnetic potentials, and hence the fields, due to a beam which is moving in the longitudinal direction with charge and current densities, $\rho(\vec{r},t)$ and $J_z(\vec{r},t)$, respectively. Future versions of IRPSS will include the effects of transverse currents. However, since the current density in a photocathode source is predominately in the longitudinal direction, the simulation results should be quite good compared to experimental results.

In the Lorentz gauge, the electromagnetic potentials due to $\rho(\vec{r},t)$ and $J_z(\vec{r},t)$ are given by

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\begin{Bmatrix}\phi \\ A_z\end{Bmatrix} = \begin{Bmatrix}-\rho(\vec{r},t)/\varepsilon_0 \\ -\mu_0 J_z\end{Bmatrix} \tag{1}$$

These potentials satisfy the boundary conditions,

$$\phi\big|_{r=a} = A_z\big|_{r=a} = 0 \quad (2a)$$

$$\left.\frac{\partial A_z}{\partial z}\right|_{z=0} = 0\,. \quad (2b)$$

In a previous work [3], we showed that the potentials can be written in terms of Green's functions, i.e.

$$\begin{Bmatrix} \phi(\vec{r},t) \\ A_z(\vec{r},t) \end{Bmatrix} = \int_{-\infty}^{t}\!\!\int\!\!\int \begin{Bmatrix} G_\phi(\vec{r},t;\vec{r}\,',t')\rho(\vec{r}\,',t')/\varepsilon_0 \\ \mu_0 G_A(\vec{r},t;\vec{r}\,',t')J_z(\vec{r}\,',t') \end{Bmatrix} d^3\vec{r}\,'dt' \quad (3)$$

where $G_\phi(\vec{r},t;\vec{r}\,',t')$ and $G_A(\vec{r},t;\vec{r}\,',t')$ are time-dependent Green's functions whose solution can be formed from an expansion which satisfy Helmholtzs' Equation for the Fig. 1 geometry. The electromagnetic fields, $\vec{E}$ and $\vec{B}$, can be readily computed from the potentials.

BEAM SIMULATION STUDIES

Eq. (3) forms the basis of the IRPSS electromagnetic field solver. For a given $\rho(\vec{r},t)$ and $J_z(\vec{r},t)$ which satisfy the continuity equation, the potentials, and hence the fields, are automatically solved. IRPSS constructs the charge and current densities of the beam using slices, i.e. zero thickness disks. In particular, the choice of $\rho(\vec{r},t)$ and $J_z(\vec{r},t)$ which is utilized in IRPSS is of the form

$$\rho(\vec{r},t) = \sum_{i=1}^{N} \sigma_i(r,t)\delta(z - z_i''(t)) \quad (4a)$$

and

$$J_z(\vec{r},t) = \sum_{i=1}^{N} \sigma_i(r,t)\frac{dz_i''}{dt}\delta(z - z_i''(t)), \quad (4b)$$

where $\sigma_i(r,t)$ is the charge per area of the ith slice, $z_i''(t)$ is the longitudinal location of the ith slice, and N is the number of slices on the simulation.

To illustrate this numerically, we show the results of IRPSS corresponding to the parameters of the BNL 1.6 cell photocathode gun [3]. We make the simplifying assumption that the trajectories, $z_i''(t)$, are specified by the external rf-field. Physically, one could view this simulation as the space-charge fields being small compared to the external rf-fields. Future simulations with IRPSS will involve incorporating the space-charge effects when calculating $z_i''(t)$.

The trajectory of each of the slices can be computed from the relativistic equations of motion for the slice in the external rf-field.

$$dz''/dt = P_z/m_e \quad (5a)$$

$$dP_z/dt = -eE_0\cos(kz'')\sin(\omega t + \varphi)\,. \quad (5b)$$

The head of the bunch which is injected at time t=0 has a trajectory which is shown in Fig. 2. The red curve in Fig. 2 denotes the slices trajectory $z''(\tau)/\lambda$ where $\lambda = c/f$ is the free-space wavelength of the injector, and the blue curve shows the light line.

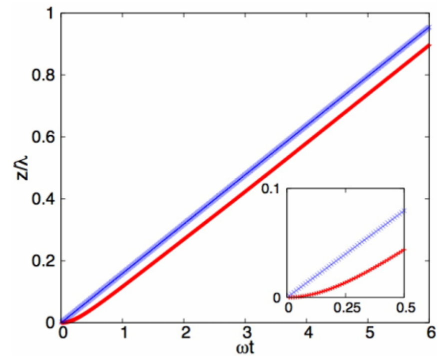

Figure 2: Plot of the bunch trajectory (red) and light line (blue) using the BNL 1.6 cell photocathode gun [3] parameters.

In Figs. (3a) and (3b), we show the potentials for two cases: 1) the bunch has zero thickness and 2) the bunch has a 10 ps bunch length. For the case of zero thickness, we only need one slice to model the bunch. However, for the case of the 10 ps bunch, multiple slices are necessary. For this case, we assumed that the bunch is uniformly distributed in the z-direction over its' bunch length, and we performed simulations using N=10 and N=30 slices. Since the beam is short compared to the rf period, we assumed that each $z_i''(t)$ was identical to $z_1''(t)$ except that it was displaced in time, i.e. $z_i''(t) = z_1''(t - t_b(i-1)/N)$ where t_b =10 ps is the bunch length. The radial charge density of the slice is assumed to be a quadratic function, i.e.

$$\sigma_i(r,t) = \left(2Q/\pi r_b^2\right)\theta(r_b - r)\left(1 - r^2/r_b^2\right)$$

(6).

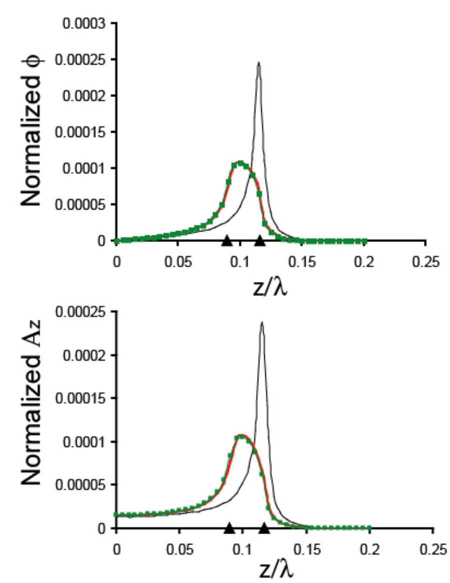

Figure 3: Plots of (a) normalized ϕ and (b) normalized A_z for $\omega t = 1.0$ and $r = r_b$ for a single slice (black), N=10 slices (red), and N=30 slices (green with dots).

Figs. 3(a) and 3(b) show the potentials ϕ and A_z normalized to $(aE_0)^{-1}$ and $(caE_0)^{-1}$ respectively, as a function of z/λ at $\omega t = 1.0$ and $r = r_b$. The black curve represents the case of a zero thickness bunch modeled with one slice. The red curve and green curve with dots represents the 10 ps bunch length case using *N*=10 and *N*=30 slices, respectively. The black arrows on the z-axis denote the location of the front and back of the bunch. It is obvious from Figs 3(a) and 3(b) that it is necessary to model a 10 ps bunch with multiple slices since the single slice method would overestimate the electric field by at a factor of ~2-3. Moreover, the figures show that a 10 slice model yields excellent results since the *N*=10 and *N*=30 cases agree extremely well. We are actively working on particle simulations using the fields computed from the multislice model.

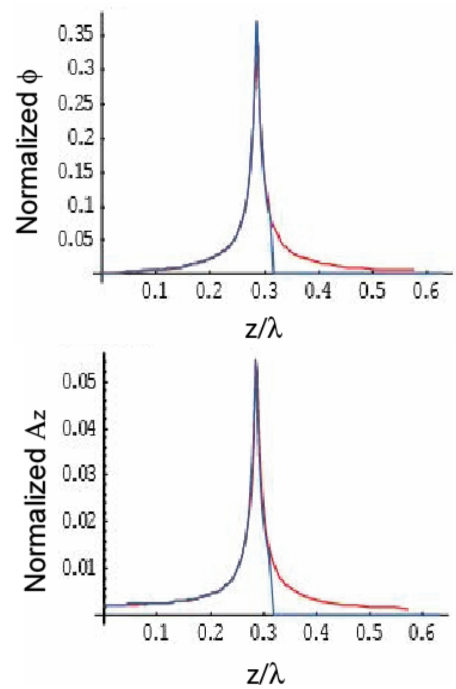

Figure 4: Plots of (a) normalized ϕ and (b) normalized A_z for the analytical case (red) and IRPSS (blue).

BENCHMARK STUDIES

We have also been running a series of benchmark studies of the code. These studies include comparisons to analytical results as well as to other codes. We show an example of one benchmark, which was a comparison of IRPSS to an analytical system. In the IRPSS simulation, we calculated the potentials for a single disk slice of charge Q being emitted at time t = 0 from the cathode with a uniform velocity v. The analytical system consisted of two disk bunches, one with charge Q moving with velocity v, and an image bunch of charge –Q moving with velocity –v. In the analytical system, the two bunches intersect at t = 0. In certain regimes of space and time, such as when reflection from the side wall has not occurred (t < 2ac for r = 0) and when the electromagnetic transient shock front has passed (z < ct), IRPSS and the analytical case will agree exactly.

Figs. 4(a) and 4(b) show a comparison of the normalized potentials for the analytical case (red) and IRPSS (blue) at r =0 and at a time before reflection from the side wall has occurred. It is obvious from the figures that for short distances, i.e. z < ct, IRPSS and the analytical case are in excellent agreement (<1%). At the point z = ct, which for the figures corresponds to $z/\lambda \cong 0.3$, the IRPSS potentials go to zero which correctly characterizes the causality condition of the potentials. The point z = ct is the electromagnetic shock at the front of the bunch. The ability of IRPSS to resolve this discontinuity is a direct consequence of the Green's function method.

SUMMARY

In summary, we have shown how the Green's function based algorithm, IRPSS, can be used for simulating the electromagnetic fields within a photocathode source. Specifically, we have computed the electromagnetic potentials for the parameters corresponding to the BNL 2.856 GHz 1.6 cell photocathode gun using a multislice method. We have also shown excellent benchmark results that demonstrate not only IRPSS high-accuracy, but also its' ability to resolve the transient effects such as the electromagnetic shock near the front of a bunch. We are working to utilize these fields for extensive multiparticle simulations of experimental sources and comparisons to other codes. In the future, IRPSS will be improved to include the effects of irises, as well as to self-consistently calculate the trajectories due to both the external fields and the space-charge fields.

ACKNOWLEDGEMENTS

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