SIMULATION OF THE MUON COOLING CHANNEL IN LINEAR RF SYSTEMS

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Abstract

Super-FOFO cooling channels with absorbers of LH and LiH are considered as a cooling channel for a neutrino factory. Design and simulation of the cooling channel are performed using the ICOOL program. We describe design concepts and engineering constraints for the cooling channels. The particle losses, cooling performance, bunch lengthening and growth of momentum spread in the cooling channels are examined. The simulation result for transverse cooling is also compared with that obtained using an analytical formula. It is shown that the numerical result agrees well with that of the analytical formula.

1 INDRODUCTION

The design of a neutrino factory requires that transverse emittance of a muon beam after the capture, phase rotation and buncher channels be sufficiently reduced so that the muon beam can be accelerated and transferred to a storage ring. The cooling method that can accomplish the desired emittance reduction considering the time limitation due to short muon lifetime is ionization cooling. The muon particle loses both transverse and longitudinal momenta in an absorber. Only longitudinal momentum is then restored by acceleration through subsequent rf cavities. These sequences can reduce the divergence of a beam, resulting in a decrease in the transverse emittance. However, multiple scattering in an absorber acts as a source of heating, namely, it increases the beam emittance. Normalized transverse emittance ϵ_n is changed by passing through a length ds in an absorber at the rate

$$\frac{d\epsilon_n}{ds} = -\frac{1}{\beta^2}\frac{dE_\mu}{ds}\frac{\epsilon_n}{E_\mu} + \frac{1}{\beta^3}\frac{\beta_\perp E_s^2}{2E_\mu m_\mu c^2 L_R}, \qquad (1)$$

where s is the path length, c is the speed of light in vacuum, dE_μ/ds is the local value of the ionization-energy loss, $\beta = v/c$, L_R is the radiation length of the absorber, $\beta_\perp$ is the betatron function, and E_s is the characteristic scattering energy which is approximately 13.6 MeV. In eq. (1), the first and second terms represent the cooling and the heating due to the multiple scattering, respectively. The ionization cooling is limited by the heating due to the multiple scattering. Accordingly, we place the absorbers in low $\beta_\perp$ (i.e., strong focusing) regions and with a material with high L_R (i.e., a low Z absorber) in order to maximize the cooling since the heating is proportional to $\beta_\perp$ and $1/L_R$.

The purpose of this paper is to present Super-FOFO cooling channels with the LH and LiH absorbers and to demonstrate their performance. One cooling cell consists of two strongly focusing solenoids at either end and two weaker solenoids that generate coupling fields between them. The results of the simulation are also compared with the analytical method proposed by Kim and Wang.

2 DESCRIPTION OF THE SUPER-FOFO COOLING CHANNEL

For design of acceptable cooling channels, many conditions have to be simultaneously satisfied in terms of engineering feasibility. Liquid hydrogen (LH) and LiH are considered as absorbers because they provide the efficient cooling owing to low Z. The LH are LiH are assumed to be contained in a vessel with thin aluminum windows on both ends. The choice of thickness of the absorber windows (Al) is important because they must be thick enough to sustain the pressure from the LH and LiH. A thicker absorber yields more particle losses, but a thick absorber also makes it possible to use a thin absorber window which can reduce multiple scattering. An appropriate gap between the absorber window and rf cavity in the design is introduced for rf assembly and maintenance.

We use high-field solenoids for focusing since they provide a large angular acceptance as well as simultaneous strong focusing of a beam in both transverse directions. Engineering feasibility in magnets is also investigated by considering such parameters as magnetic field, current density, and stress on the conductor. Our design follows the engineering constraints that are shown in the feasibility study report of the neutrino factory. A conservative rule of thumb for solenoids built from a Nb_3Sn superconductor is given by $BJR <$ 350 MPa, where B is the field at the coil, J is the current density, and R is the radius of the solenoid from the z-axis. For solenoids used for strong focusing in the super-FOFO channels, the relevant values are B = 6.7 T, J = 86 A/mm^2, R = 0.41 m, giving BJR = 236 MPa. For solenoids used for coupling fields in the super-FOFO channels, the relevant values are B = 4 T, J = 65 A/mm^2, R = 0.83 m, giving BJR = 216 MPa. These BJR values in the super-FOFO channels show lower requirements than the DFOFO channel (335 MPa), alternating channel (280 MPa) and FOFO channel (310 MPa). The parameters for a solenoid in the super-FOFO cooling channel are shown in Table 1. An appropriate interval between sheets is considered for a klystron power supply.

We consider the pillbox type of the TM_{010} mode, with $\pi/2$ phase advance per cell, equipped with thin Be windows of 125 μm thickness and four cells per each rf section in the simulation. Rf frequency of 201.25 MHz and rf gradient of 14 MV/m are used for the cooling channel. The radius of the rf window may affect particle losses, depending on the magnitude of the betatron function at the position of the rf window.

Table 1: Parameters of super-FOFO cooling channels

	LH case	LiH case
Rf frequency	201.25 MHz	201.25 MHz
Rf gradient	14 MV/m	14 mV/m
Cell length	2.6 m	2.6 m
Magnetic field on z-axis	3.7 T	3.7 T
Total channel length	158 m	122 m
Minimum beta function	30 cm	25 cm
Maximum beta function	67 cm	80 cm
absorber length/cell	40.6 cm	8 cm
absorber window thickness	200 μm	100 μm
absorber window radius	18 cm	18 cm
Be window thickness	125 μm	125 μm
Be window radius	17 cm	18 cm

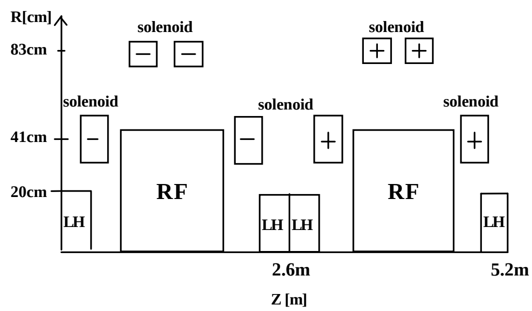

Figure 1: Configuration of the designed cooling channel. The cooling channel consists of two kinds of solenoids. Solenoids that are located at R=0.83m and R=0.41m are for strong focusing and coupling fields, respectively.

3 SIMULATION RESULTS

One cell configuration of the cooling channel with 2.6 m length is shown in Fig. 1. Fig. 2 shows the magnetic field on the z-axis for one cell of the cooling channel. The minimum values of magnetic field B_z occur at the midpoint of absorbers. The magnetic field increases from 3.7 T on the z-axis to 6.7 T at the coils. The lattice shows magnetic fields that have a dip at the positions of the rf cavity and thus acceptance could be extended. The beta function in the super-FOFO channel is minimal at midpoints of absorbers. In the cooling channel with the LH absorber, the matched beam has a minimum beta function of 30 cm and a maximum of 67 cm. Fig. 3 shows the beta function in one cooling cell with LH absorber. Major parameters of the super-FOFO cooling channel are shown in Table 2.

3.1 The input beam

A beam with a longitudinal momentum of P_z = 212 MeV/c with a rms momentum spread of 8.5% is used. A correlation between the longitudinal momentum and the transverse amplitude of the initial beam distribution is necessary to properly match the beam into the rf system. This

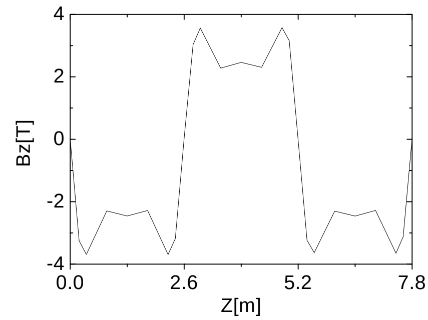

Figure 2: Magnetic field B_z[T] in the cooling channel.

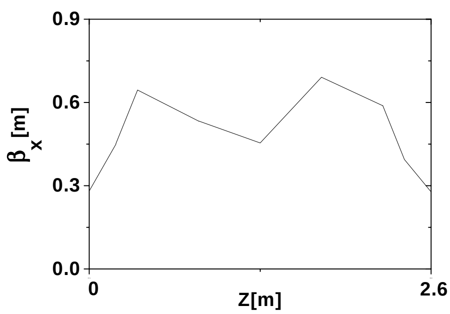

Figure 3: Horizontal beta function β_x (m) in one cell of the super-FOFO cooling channel with the LH absorber.

is the only non-Gaussian modification in our initial beam distribution. The correlation between longitudinal momentum and transverse amplitude is given by the relationship

$$P_z = P_{zi} + a\left[\frac{x_i^2 + y_i^2}{b^2} + \frac{P_{xi}^2 + P_{yi}^2}{P_{zi}^2}\right]. \quad (2)$$

Here, a and b are coefficients that determine the intensity of correlation. P_{xi}, P_{yi} and P_{zi} are initial horizontal, vertical and longitudinal momenta, respectively. In our simulation 0.1 and 0.16 for a and b are used, respectively. Thus, the

Table 2: Parameters of super-FOFO cooling channels

	LH case	LiH case
Rf frequency	201.25 MHz	201.25 MHz
Rf gradient	14 MV/m	14 mV/m
Cell length	2.6 m	2.6 m
Magnetic field on z-axis	3.7 T	3.7 T
Total channel length	158 m	122 m
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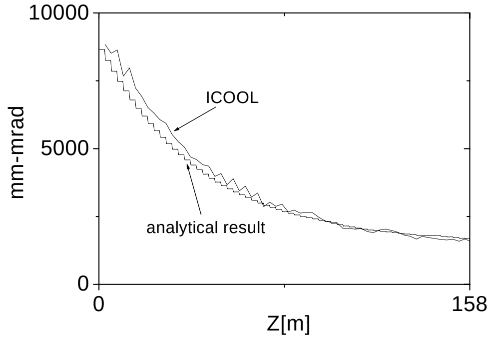

Figure 4: Transverse emittances for the cooling channel with LH absorber by ICOOL simulation and analytical formula.

correlation is sufficiently strong to push up the longitudinal momentum of the initial beam by 10%, increasing the momentum spread as well.

3.2 Cooling performance

In the designed cooling channel with the LH absorber, the transverse emittance in both the x and y planes is reduced to 17% of its initial value, while the longitudinal emittance grows by a factor of 1.6. The overall six-dimensional emittance is reduced to 17% of its initial value. The cooling channel shows the particle transmission of 54%. Without the correlation, beam transmission is reduced from 54% to 40%. Table 3 also shows the final beam parameters of rms beam size σ_x, rms bunch length σ_z and rms longitudinal momentum spread. Cooling performance of the cooling channel with the LiH absorber is also shown in Table 4.

Table 3: Cooling performance of the cooling channel with the LH absorber.

	unit	Initial	Final
Particles tracked		1000	541
Transverse emittance (ϵ_{nx})	mm-mrad	9000	1569
Longitudinal emittance (ϵ_{nz})	mm	15	24
6-D emittance ($\times 10^{-8}$)	(m-rad)^3	130	22
Rms bunch length (σ_z)	cm	10.2	14
Rms beam size (σ_x)	cm	4.1	1.5
Rms momentum spread	%	8.5	18.6

Table 4: Cooling performance of the cooling channel with the LiH absorber.

	unit	Initial	Final
Particles tracked		1000	600
Transverse emittance (ϵ_{nx})	mm-mrad	10000	2000
Longitudinal emittance (ϵ_{nz})	mm	15	24

3.3 Comparison with analytical result

The simulation result by the ICOOL program is also compared with that of an analytical method. The analytical formula for the reduction of the transverse rms beam emittance is given by

$$\begin{aligned}\epsilon(s) = & \frac{e^{-\zeta_-(s)}}{2}[\epsilon^0 + L^0 + \mathcal{D}^-(s)] \\ & + \frac{e^{-\zeta_+(s)}}{2}[\epsilon^0 - L^0 + \mathcal{D}^+(s)], \end{aligned} \quad (3)$$

where

$$\zeta_\pm(s) = \int_0^s d\bar{s}\eta(\bar{s})[1 \pm \kappa(\bar{s})\beta_\perp(\bar{s})] \quad (4)$$

$$\mathcal{D}^\pm(s) = \int_0^s d\bar{s} e^{\zeta_\pm(\bar{s})}\beta_\perp(\bar{s})\xi(\bar{s}). \quad (5)$$

η is the scaled parameter specifying the energy loss per unit length in the absorber material and $\xi(s)$ is the angular excitation due to the stochastic kick arising from multiple scattering. ϵ^0 is the initial transverse emittance. For the designed Super-FOFO cooling channel with LH absorber, eq. (3) is plotted together with the simulation result in Fig. 4. It is evident that they are in good agreement.

4 CONCLUSIONS

The purpose of our study is to obtain efficient transverse cooling channels that satisfy engineering constraints. Super-FOFO cooling channels with LH and LiH absorbers are designed and their cooling performances are calculated using the ICOOL code. Detailed simulation studies are also performed to obtain optimal parameters for each cooling channel. These studies will be valuable in selecting the best design, based on cooling performance, engineering constraints and cost. It is also shown that the simulation results agree well with those of the analytical method.

5 REFERENCES

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