THE MULTI-MODE ANALYSIS OF THE SLOT COUPLED ACCELERATING STRUCTURES

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Abstract

The family of accelerating structures, coupled with slots between cells, is well known in linear accelerators. It are side coupled, annular coupled, coaxial coupled, on-axis coupled. The slots are generally used to couple the structure tanks with the bridge cavities. The general electrodynamics approach is applied to describe the slot coupled system in multi-mode approximation. Such description is necessary for the mode-mixing analysis and presents the generalization of the well known Knapp's single mode coupled resonator model for standing wave accelerators tanks. General relations and effects for slot coupled cavities are described.

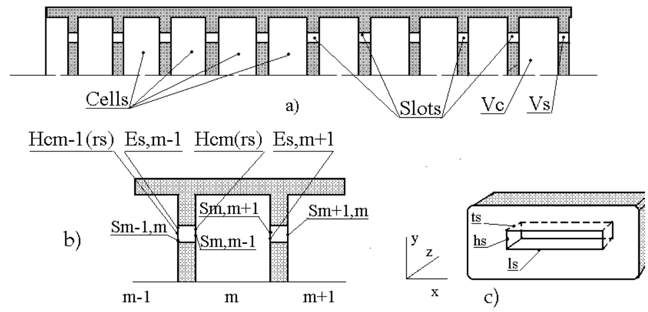

Figure 1. The scheme of the coupled chain of cavities.

1 INTRODUCTION

The cavities, coupled with slots, are widely used in particle accelerators. It are coupled cells accelerating structures, accelerating cavities with bridge couplers and so on. Some tasks in the investigation and development of accelerating structures and systems require multi-mode description. Examples are mode-mixing problem, deterioration of the dispersion curve shape. In this report the multi-mode description of the slot coupled system is presented as the solution of self-consistent electrodynamic problem - slots excite cells, cells excite slots. The conclusion obtained describe general slot effects. As a partial case of the single mode approximation well known Knapp model for standing wave accelerators tanks [1] can be obtained.

2 DISPERSION EQUATION

Considering the electromagnetic field distribution in the chain of cavities (cells), coupled into the resonant system with slots in the walls between cells (Fig. 1a), we can expand the field distribution $\vec{H}^c_m$ in the $m-th$ cell in series over own cell modes $\vec{H}^c_{mn}$ with the frequencies ω^c_{mn} in the unperturbed (with the shortened coupling slots) cells, [2]:

$$\vec{H}^c_m = \sum_n h^c_{mn}\vec{H}^c_{mn}, \mu_0 \int_{V^c_m} \vec{H}^c_{mn}\vec{H}^{c*}_{mn} dV = 2W_0. \quad (1)$$

In general case both solenoidal and potential own modes should be included in this expansion (1).

Considering the magnetic field excitation in the $m-th$ cell by tangential electric field in the slots $\vec{E}^s_{m-1}, \vec{E}^s_{m+1}$, [2], [3], one can get:

$$h^c_{mn} = \frac{j\omega(I^c_1 + I^c_2)}{2W_0(\omega^2 - \omega^{c2}_{mn})}, \quad (2)$$

$$I^c_1 = \int_{s^{m-1}_m} [\vec{E}^s_{m-1}\vec{H}^{c*}_{mn}] d\vec{S},$$

$$I^c_2 = \int_{s^{m+1}_m} [\vec{E}^s_{m+1}\vec{H}^{c*}_{mn}] d\vec{S}$$

where $d\vec{S} = \vec{\nu} dS$, $\vec{\nu}$ is the outer unit normal vector to the cell surface, S^{m-1}_m, S^{m+1}_m are the surfaces of all slots in the $m-th$ cell from the $(m-1)-th$ and $(m+1)-th$ cells (Fig. 1b). In the $m-th$ cell the magnetic field value $H^c_m(r_s)$ at the slots ($r = r_s$) is:

$$H^c_m(r_s) = \sum_n \frac{j\omega H^c_{mn}(r_s)(I^c_3 + I^c_4)}{2W_0(\omega^2 - \omega^{c2}_{mn})}, \quad (3)$$

$$I^c_3 = \int_{s^{m-1}_m} [\vec{E}^s_{m-1}\vec{H}^{c*}_{mn}] d\vec{S},$$

$$I^c_4 = \int_{s^{m+1}_m} [\vec{E}^s_{m+1}\vec{H}^{c*}_{mn}] d\vec{S}.$$

To define the slots excitation, we have to specify at first the model for the slot description. The real field pattern near the slot is enough complicated and can be described approximately. Three approximations for the slot descriptions are known, providing similar results but with different mathematical formulation. The first model is the well known representation of the slot as a part of a shortened transmission line [4], [5]. The second model represents the slot as an irradiating dipole [6]. Here we will use the third model - the slot representation as a part of a waveguide in the cut-off mode [7]. This model is more suitable for treatment in terms of cavities and allows application of the same procedure for the slot excitation.

Let consider the slot (Fig. 1c) with the length l_s, height h_s and the deepens t_s. We also can expand the field in the $(m-1)-th$ slot in a series over own slot modes $\vec{E}^s_{mn}$ as:

$$\vec{E}^s_{m-1} = \sum_n e^s_{mn}\vec{E}^s_{mn}, \epsilon_0 \int_{V_s} \vec{E}^s_{mn}\vec{E}^{s*}_{mn} dV = 2W_0. \quad (4)$$

Because the slot are excited by tangential magnetic field, the potential terms are not required here.

Considering the electric field excitation in the slot by tangential magnetic field of the $(m-1)-th$ and $m-th$ cell one get:

$$e^s_{mn} = \frac{-j\omega(I^s_1 + I^s_2)}{2W_0(\omega^2 - \omega^{s2}_{mn})},$$

$$I^s_1 = \int_{s^m_{m-1}} [\vec{E}^s_{m-1}\vec{H}^{c*}_{m-1}]d\vec{S}, I^s_2 = \int_{s^{m-1}_m} [\vec{E}^s_{m-1}\vec{H}^{c*}_{m}]d\vec{S}.$$

As a role, the slots have a match smaller dimension as cells and own slots frequencies ω^s_m are match more higher than cells (and passband) frequencies. This way we can take into account in the expansion for $\vec{E}^s_{m-1}$ only one lowest TE_{100} mode in the slot with the frequency $\omega_s = \frac{\pi c}{l_s}$. For narrow slots $h_s \gg l_s$ the next mode TE_{200} has a twice frequency and zero coupling integral, so only third mode has a sense, but with very high frequency and reduced coupling. Taking into account normalization, distributions of the cells field on the slots $H^c_m(r_s)$, the field distribution for TE_{100} mode is - $\vec{E}^s_{m-1} = E^s_{m-1,1}\sin(\frac{\pi x}{l_s})\vec{y}_0$, where x is a current coordinate along l_s. For simplicity we will suppose all slots with the same dimensions and every cell has the same number of slots M_s. Than all coupling integrals I^c_1, I^c_2 for all cells will be the same. The method allows extension for different number of slots in the cells and different slot dimensions. But it should be foreseen in the procedure and leads to more complicated equations. Supposing the distribution of the magnetic field of the cell $H^c_m(r_s)$ as an uniform on the slot (the non-uniform distribution also can be considered) :

$$\int_{s^m_{m-1}} [\vec{E}^s_{m-1,1}\vec{H}^{c*}_{m-1}]d\vec{S} = E^s_{m-1}H^c_m(r_s)\frac{2h_s l_s}{\pi}$$

and taking into account normalization for the stored energy $W_0 = \frac{\epsilon_0 E^{s2}_{m-1,0} l_s h_s t_s}{4}$, one can get for the field in the $(m-1)-th$ slot (Fig. 1b):

$$E^s_{m-1} = \frac{-4j\omega(H^c_{m-1}(r_s) + H^c_m(r_s))}{\pi\epsilon_0 t_s(\omega^2 - \omega^2_s)}\sin(\frac{\pi x}{l_s})\vec{y}_0. \quad (5)$$

Substituting E^s_{m-1} in expansion for h^c_{mn} (2) and taking into account description for $H^c_m(r_s)$ (3), one get the uniform system of equations for h^c_{mn}:

$$h^c_{mn}\frac{(\omega^2 - \omega^{c2}_{mn}}{A_s\omega^2} - H^{c2}_{mn}(r_s)/_{S^{m-1}_m} - H^{c2}_{mn}(r_s)/_{S^{m+1}_m}) -$$
$$-(H^c_{mn}(r_s)\sum_i h^c_{m-1,i}H^c_{m-1,i}(r_s) +$$
$$+H^c_{mn}(r_s)\sum_{i\neq n} h^c_{mi}H^c_{mi}(r_s))/_{S^{m-1}_m} -$$
$$-(H^c_{mn}(r_s)\sum_{i\neq n} h^c_{mi}H^c_{mi}(r_s) +$$
$$+H^c_{mn}(r_s)\sum_i h^c_{m+1,i}H^c_{m+1,i}(r_s))/_{S^{m+1}_m} = 0,$$

where $A_s = -\frac{4l_s h_s}{\pi^2\epsilon_0 t_s W_0(\omega^2-\omega^2_s)}$ and can find the frequencies and related field distributions in the chain of slot coupled cells.

In this description we neglect the mutual slot influence and if several slots are in the wall, only independent summation over all slots has to be performed during integration in the expansion for h^c_{mn} (2). The way to take into account mutual slot influence is pointed out in [5] and also lead to more complicated equations.

To simplify the system for analysis, let define the coupling constant between the $i-th$ and the $j-th$ modes in adjacent cells $\gamma^{(i,j)}_{(m,m\pm1)}$ as:

$$\gamma^{(i,j)}_{(m,m\pm1)} = -A_s H^c_{m\pm1,i}(r_s)H^c_{m,j}(r_s)/_{S^{m\pm1}_m}, \quad (6)$$

and between the $i-th$ and the $j-th$ modes in one $(m-th)$ cell as:

$$\gamma^{(i,j)}_{(m,m)} = -A_s(H^c_{m,i}(r_s)H^c_{m,j}(r_s))/_{S^{m-1}_m} + +(H^c_{m,i}(r_s)H^c_{m,j}(r_s))/_{S^{m+1}_m}), \quad (7)$$

and rewrite the system of equations as:

$$h^c_{mn}\frac{(\omega^2 + \gamma^{(n,n)}_{(m,m)}\omega^2 - \omega^{c2}_{mn})}{\omega^2} +$$
$$+\sum_i \gamma^{(n,i)}_{(m,m-1)}h^c_{m-1,i} + \sum_{i\neq n}\gamma^{(n,i)}_{(m,m)}h^c_{m,i} +$$
$$+\sum_i \gamma^{(n,i)}_{(m,m+1)}h^c_{m+1,i} = 0. \quad (8)$$

3 DISCUSSION

This multi-mode approach has been developed in INR during study of the Rectangular Directly Coupled Bridge cavities (RDCB), proposed in [8], for the DAW structure in the INR linac. Several own RDCB TE_{10n} modes are in the wide passband of the DAW structure and to describe the field in the 'coupling cell' (RDCB) multi mode approximation is required.

The equations (8) describe a typical behavior of coupled system. In general slots couple all modes in the $m-th$ cell with all modes in $(m-1)-th$ and $(m+1)-th$ cells. Also slots provide a mutual coupling between all modes of the $m-th$ cell.

As a role, the slots are short enough and the own slot frequency $\omega_s \gg \omega$. This case we can simplify expressions for $\gamma^{(i,j)}_{(m,m\pm1)}$ and $\gamma^{(i,j)}_{(m,m)}$ by assumption $\omega^2 - \omega^2_s \approx -\omega^2_s = -(\frac{\pi c}{l_s})^2$ as :

$$\gamma^{(i,j)}_{(m,m\pm1)} = \frac{4l^3_s h_s}{\pi^4 c^2\epsilon_0 t_s W_0}H^c_{m\pm1,i}(r_s)H^c_{m,j}(r_s)/_{S^{m\pm1}_m},$$
$$\gamma^{(i,j)}_{(m,m)} = \frac{4l^3_s h_s}{\pi^4 c^2\epsilon_0 t_s W_0}(H^c_{m,i}(r_s)H^c_{m,j}(r_s)/_{S^{m-1}_m} + +H^c_{m,i}(r_s)H^c_{m,j}(r_s)/_{S^{m+1}_m}).$$

The value of the coupling constant between adjacent cells $\gamma^{(i,j)}_{(m,m\pm1)}$ is proportional to the relative strength of the magnetic field $\frac{H^c_{m\pm1,i}(r_s)H^c_{m,j}(r_s)}{W_0}$ and the slot height h_s, inverse proportional to the slot depth t_s and proportional to the l_s^3. The cubic dependence $\gamma^{(i,j)}_{(m,m\pm1)}$ on l_s^3 is confirmed by direct MAFIA simulations and in experiments in lot of investigations.

All time the slots influence leads to the frequency reduction of coupled modes - the term $\gamma^{(n,n)}_{(m,m)}\omega^2$ in (8). If we change the dimensions of slots, this frequency reduction is proportional to the coupling coefficient between modes in adjacent cells.

By using the general system (8) we can investigate the mode mixing problem in the slot coupled structures.

For example, one conclusion is straightforward for the Annular Coupled Structure (ACS). It is known, that in this structure the TM_{110} and TM_{210} modes in coupling cells are enough close to the structure passband. From (2) it directly follows - for four slot ACS option [9] these modes are not coupled with the operating TM_{010} mode in accelerating cell - the total integral over all slots in (2) is zero.

Together with a mode mixing (or as a mode mixing), the description presented allows to investigate the distortions of the shape of operating passband. In frame of this description the passband distortion can be explained by modes interaction, more naturally as compared with the neighbor coupling coefficients introdused in the well known papers [1].

If we neglect the mode-mixing in coupled cells due to slots influence, we can use the simplified single mode approximation for the description.

4 SINGLE MODE APPROXIMATION

The single mode approximation has been developed earlier [7]. Let suppose the cavity chain has two types of cells - accelerating and coupling ones. In each cell we take into account only one mode - fundamental TM_{010} mode with mode frequencies for closed cells (without slots) ω_1, ω_2 and relative amplitudes h^a_m in the $m-th$ (with frequency ω_1) cell and $h^c_{m\pm1}$ in the $(m\pm1)-th$ cell assuming the $m-th$ cells as an accelerating one and $(m\pm1)-th$ cell as a coupling. Let define for the simplicity $\gamma^{(1,1)}_{(m,m-1)} = \gamma^{(1,1)}_{(m,m+1)} = k^s_{ac}, \gamma^{(1,1)}_{(m,m)} = \gamma_a, \gamma^{(1,1)}_{(m\pm1,m\pm1)} = \gamma_c$. The equations (8) transform to:

$$k^s_{ac}h^c_{m-1} + h^a_m\frac{(\omega^2+\gamma_a\omega^2-\omega_1^2)}{\omega^2} + k^s_{ac}h^c_{m+1} = 0. \quad (9)$$
$$k^s_{ac}h^a_m + h^c_{m+1}\frac{(\omega^2+\gamma_c\omega^2-\omega_2^2)}{\omega^2} + k^s_{ac}h^a_{m+2} = 0.$$

Considering the infinite cavity chain and solution in the form $h_{am} = h_a\exp(im\phi), h_{c,m\pm1} = h_c\exp(i(m\pm1)\phi)$ one get the dispersion equation for the bi-periodical chain:

$$(2k^s_{ac}\cos\phi)^2 = (1+\gamma_a-\frac{\omega_a^2}{\omega^2})(1+\gamma_c-\frac{\omega_c^2}{\omega^2}). \quad (10)$$

If we define the frequencies of accelerating and coupling cells as:

$$\omega_a = \frac{\omega_1}{\sqrt{1+\gamma_a}}, \qquad \omega_c = \frac{\omega_2}{\sqrt{1+\gamma_c}},$$

and the coupling coefficient k^{sc}_{ac} as:

$$k^{sc}_{ac} = \frac{2k^s_{ac}}{(1+\gamma_a)(1+\gamma_c)},$$

the dispersion equation (10) transforms to

$$(k^{sc}_{ac}\cos\phi)^2 = (1-\frac{\omega_a^2}{\omega^2})(1-\frac{\omega_c^2}{\omega^2}), \quad (11)$$

and the system of equations (9) transforms to

$$\frac{k^{sc}_{ac}}{2(1+\gamma_a)}h^c_{m-1} + h^a_m(1-\frac{\omega_a^2}{\omega^2}) + \frac{k^{sc}_{ac}}{2(1+\gamma_a)}h^c_{m+1} = 0.$$
$$\frac{k^{sc}_{ac}}{2(1+\gamma_c)}h^a_m + h^c_{m+1}(1-\frac{\omega_c^2}{\omega^2}) + \frac{k^{sc}_{ac}}{2(1+\gamma_a)}h^a_{m+2} = 0.$$

Dispersion equation (11) coincides with the well known Knapp's equation [1] for the coupled cavities for the case of zero neighbor coupling coefficients in [1]. In terms of coupled modes the dispersion curve distortions can not be described in the single mode approximation and require the multi-mode approximation [5].

5 CONCLUSION

The approach, described above, leads to the coupled circuit analysis for the slot coupled structure. This approach explains and describes fine qualitatively the slot influence in the structure, providing relations between slot dimensions, cells characteristics and coupled circuit parameters. The equivalent coupled circuit can be constructed easy with all parameters defined in the single way. The main value of this approach is in our ability to treat mode-mixing problems in the slot coupled cavities with clear physical definition of parameters.

6 REFERENCES

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