DYNAC: A MULTI-PARTICLE BEAM DYNAMICS CODE FOR LEPTONS AND HADRONS IN COMPLEX ACCELERATING ELEMENTS*

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Abstract

A new set of very accurate quasi-Liouvillian beam dynamics equations has been introduced by the authors in the computer code DYNAC in 1994, applicable to protons, heavy ions and non-relativistic electrons. DYNAC also has several space charge routines, in particular the new 3D space charge routine HERSC presented at this conference. The most recent version of the code is described in this article, with a focus on simulations for the Spallation Neutron Source (SNS).

1 INTRODUCTION

Initially, particle acceleration through the gaps of linear accelerators was computed with the so-called Panofsky equations, introducing the concept of transit time factors. In 1965 a full set of more accurate quasi-Liouvillian equations was derived from the Panofsky equations [1]. These equations were later completed with second order corrections in longitudinal motion [2]. Modern linear accelerators may pose new kinds of problems, which cannot be solved by the previous set of equations, only applicable under restrictive constraints to single elements. Indeed, particles may be accelerated through long accelerating elements with complex electromagnetic fields where their transit time is of the order of 10π , their velocities vary by 10% or more, or where their relativistic γ varies by a factor 3 or 4. For this, an approach including new concepts as the *equivalent accelerating field* has been introduced in DYNAC in 1994 giving a full set of quasi-Liouvillian equations accurate to the second order [3]. These equations, available for all kinds of particles, result in a convenient matrix formalism, allowing checking of the Liouvillian character of longitudinal and transverse motion computations and can be used for expansions around a central particle. Due to the type of analytical approach, each of the 6D coordinates of the particles are known in any position in the element, thus also permitting multi-step space charge computations. Dynamics computations both for non-relativistic electrons with significant acceleration and for heavy ions undergoing large velocity variations can be made with this one and the same code.

2 DYNAMICS EQUATIONS IN DYNAC

For the accelerating effect, TM modes in axial symmetrical cases are considered for which it is common practice to use cylindrical coordinates.

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The transverse motion can be derived from an integration of the equation:

$$\frac{d(mv_r)}{dt} = q(E_r - v_z B_\theta) \tag{1}$$

After integration over the gap, the transverse momentum is changed by the amount $\Delta(mv_r)$. The variation in slope r' becomes:

$$\Delta r' = \frac{\Delta(mv_r)}{mv_z} - \frac{mv_r}{(mv_z)^2}\Delta(mv_z) + ... \tag{2}$$

The extra terms in Eq.2 are due to the fact that r and r' are not canonically conjugate variables (the conjugate of r is mv_r). As a consequence, computations are complicated and developing second order corrections to improve the transverse motion computation is hardly possible [2]. This difficulty has been resolved with the Picht transformation [3]:

$$R = r\sqrt{\beta\gamma} = r\left(\gamma^2 - 1\right)^{1/4} \tag{3}$$

where R refers to the 'reduced radius'. From this transformation, Eq.1 can be rewritten:

$$\frac{d^2R}{dz^2} - R\frac{q}{2m_0c^3\beta^3\gamma^3}\frac{\partial E_z}{\partial t} + R\left(\frac{q}{2m_0c^2}\right)^2\frac{\gamma^2+2}{\beta^4\gamma^4}E_z^2 = 0 \tag{4}$$

The equation is now completed with a term in E_z^2, always focusing. The original concept of transit time factor based on a Fourier analysis of the axial field amplitude is still used:

$$E_z(z,r) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[T(k_z)\cos k_z z + S(k_z)\sin k_z z\right] \times I_0(k_r r)dk_z \tag{5}$$

with:

$$T(k_z) = \int_0^L E_z(z,0)\cos k_z z dz,\ S(k_z) = \int_0^L E_z(z,0)\sin k_z z dz \tag{6}$$

and $$k_r^2 = k_z^2 - \left(\frac{\omega}{c}\right)^2 = \frac{\omega^2}{c^2\beta^2\gamma^2}$$

The axial field is periodic:

$$E_z(z,r,t) = E_z(z,r)\cos(\omega t + \varphi_0) \tag{7}$$

For the longitudinal motion, one can keep the usual canonically conjugate variables energy and phase. The phase motion across an accelerating element can be expressed in terms of a phase jump $\Delta\varphi$, which at the input is given by integrating:

$$d(\Delta\varphi)=\left(\frac{q}{m_0c^2}\right)\frac{\omega}{c}\frac{zE_z}{\beta^3\gamma^3}dz \qquad (8)$$

The computation of energy gain requires integration of the field in Eq.7. Assuming a constant velocity $v=\omega/k$, it can be directly obtained with Eq.5 and 7 through the *T* and *S* coefficients. Apart from the terms in $(\beta\gamma)$ computation of the phase term gives a similar integral. It is then interesting to replace the exact phase evolution along the structure by an approximate linear law according to a kind of average velocity. In codes such as PARMILA [4], derived from the modified Panofsky equations, computations are made around the middle of the element and the exact phase evolution is replaced by its tangent at the middle. A better estimate is a line parallel to the one joining the entrance and the exit phases and going through the "center of gravity" of the real phase curve. A similar approach can be applied to the evolution of the reduced radius *R* [3]. Determination of the output values necessary for these average estimates implies, of course, the use of an iterative calculation as was the case in the previous method to get the values at the middle. Given these previous remarks, the dynamics through an accelerating element are obtained from the following integrals:

$$\Delta W = q\int_0^L E_z(z,0,t)\left[1+F\right]dz \qquad (9)$$

$$\Delta\varphi=\frac{q}{m_0c^2}\frac{\omega}{c}\int_0^L\frac{1}{\beta^3\gamma^3}E_z(z,0,t)\left[1+F\right]zdz \qquad (10)$$

with: $F=\dfrac{\omega^2\left(R^2+2RR'z\right)}{4c^2\beta^3\gamma^3}$

Analogous relations exist for ΔR and $\Delta R'$. The purpose now is to replace the complex real field distribution acting on the particle by one giving equivalent dynamics but simpler in form, thus permitting an easier computation of the integrals above. For this one notices that elements having the same *T, S* and derivatives with respect to the *k* of the particle yield identical results. Therefore, computing an *'equivalent element'* simple enough to enable an easier computation of the integrals gives results equal to those obtained with the dynamics in the real element. The *'equivalent element'* is chosen in such a way that its accelerating field has a constant amplitude E_0 over a gap of '*equivalent length'* L_e. Its transit time is given by:

$$T_0(k)=E_0L_e\frac{\sin(kL_e/2)}{kL_e/2} \qquad (11)$$

It is clear that for a long multi-gap structure, the behavior of its transit time factor $T(k)$ is different. However, if one considers the useful part from k_1 to k_2 and taking into account that this useful part is centered around a finite value $\hat{k}$ to be defined, the difference is negligible[3]. One can then say that the *'equivalent field'* is a traveling wave with a constant amplitude E_0 and a phase slip equal to $\theta=\left(k-\hat{k}\right)L_e$ over the *'equivalent length'* L_e. Such a field distribution being symmetrical to the middle of L_e can be expressed with *T* coefficients. If the origin is displaced by a length l, for instance in front of the structure, one has: $T(\hat{k})=T_0(\hat{k})\cos\hat{k}l$ and $S(\hat{k})=T_0(\hat{k})\sin\hat{k}l$.

3 SPACE CHARGE ROUTINES IN DYNAC

Three space charge routines have been introduced in DYNAC: The 2D (in *r-z*) SCHEFF routine developed at LANL [5], the SCHERM routine [6], in which it is assumed that in the longitudinal direction the bunch is constituted of several ellipsoids, whereas in the transverse directions it keeps a simpler symmetrical shape respecting the transverse RMS, and the new 3D routine HERSC [7], presented at this conference, where the analytical set of beam self-field equations is found without any sort of restriction or basic hypothesis.

4 FACILITIES IN DYNAC

The first order formalism of the usual optical lenses is incorporated in the current version of the code.

The electromagnetic fields of the cavities can be read in the form of the experimental coordinates $(z,E(z))$, transit time factors or Fourier series expansions. IH structures may be simulated (here the design particle typically lies outside the beam).

The input particles can be generated from various hit-or-miss Monte Carlo processes or they can be read directly from a file.

Misalignments and systematic or random defects in the matching parameters of cavities are possible. A physical acceptance of the machine can be defined.

Routines allowing to follow halo formation through a structure or beam line are available. Space charge computations can be made in any position or in several positions in the accelerating element.

5 EXAMPLE OF COMPARATIVE COMPUTATIONS

DYNAC allows comparative simulations in identical informatics context with different space charge routines. An example is shown in Fig.1 where one represents along the MEBT and DTL of the SNS the evolution of the quantity: $R=\sqrt{E_x^2+E_y^2}$ (E_x and E_y are the horizontal and the vertical emittance). The density projection along the x axis at the end of the SNS DTL obtained from SCHEFF and HERSC is shown in Fig. 2. From the closeness of the curves (2) and (3) in Fig.1 one can conclude that the differences betweens the tails of the curves in Fig.2 result from halo, which can be seen with HERSC.

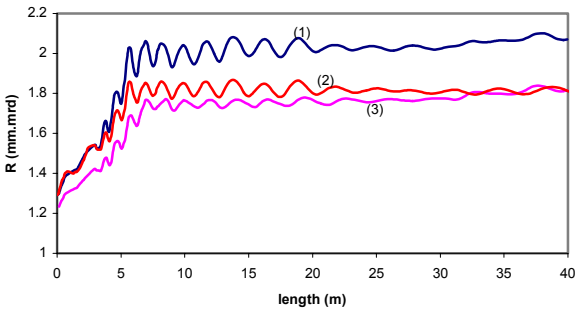

Figure 1: The evolution of *R* along the SNS MEBT and DTL. The curves (1) and (3) are computed with HERSC: curve (1) from the totality of 40000 particles in the bunch and curve (3) in removing 1% of these particles constituting a halo around the core of the bunch. Curve (2) is computed with SCHEFF in considering the total number of particles.

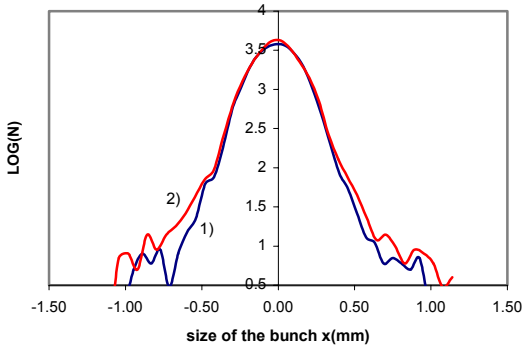

Figure 2: Density projection along the x axis at the end of the SNS DTL. Curve (1) is obtained from SCHEFF and curve (2) from HERSC.

In Fig.3 *R* is plotted for a given setting of the medium beta section of the SNS super-conducting linac (SCL). With the concept of the *'equivalent field*', the 6 cell cavities in the SCL are treated as one, properly taking into account the asymmetry of the accelerating fields in the end cells and starting from the beam coordinates at the entrance of the accelerating element. The previous set of quasi-Liouvillian equations (e.g. in PARMILA) however, deriving from the Panofsky equations, is only applicable for single elements provided that: the electric field is symmetrical with regard to the middle of the element, the transit time of the particles is of the order of π , and their velocities vary by a few percent.

When long accelerating elements are considered, the validity of the simulations with regard to the position of the space charge computation in the element is a major problem in a space charge dominated beam. As shown in Fig.3, with DYNAC it is possible to provide space charge computations in any position in the element, thus permitting to check the effects on the simulations. In the example shown, it is evidenced by the closeness of the 3 curves (corresponding to space charge computations in 3 different positions in each cavity) that the results are practically independent of to the position of the space charge computation in the element. Combining these 3 positions, the evolution of the quantity *R* becomes very close to the curve in the middle.

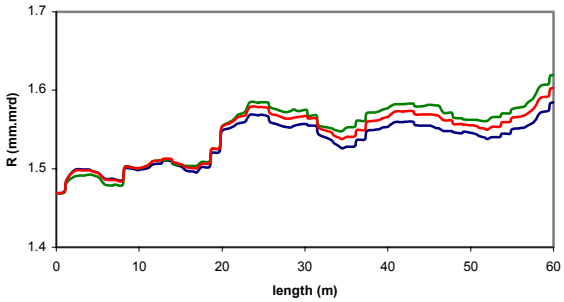

Figure 3: The evolution of the quantity *R* for a given setting of the first 33 cavities (each ~ 1m long) of the SNS SCL. The curve in the middle corresponds to the position at the middle of the cavities, the lower curve to the position at a third from the front and the higher curve at two thirds from the front of each cavity.

5 CONCLUSIONS

Apart from the new 3D space charge method, HERSC, the main particularity of the multi-particle Fortran code DYNAC is the result of the set of quasi-Liouvillian equations used in the code. It allows the second order computation of particle acceleration not only in simple gaps, but also through long structures with complex electric field (e.g., multi-gap, helix, etc.). Due to the analytical method used in DYNAC, the coordinates are available for any particle at any position in the accelerating element, thus allowing multi-step space charge calculations in the element.

6 REFERENCES

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