THEORETICAL ANALYSIS ON THE HALO FORMATION OF THE PROTON BEAM FROM LEDA

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Abstract

From the experimental facility dedicated to beam-halo formation study at Los Alamos [1] many interesting experimental results have been obtained. The measured beam transverse charge profiles with significant halo particles due to beam envelope oscillation through the FODO channel, however, have not been understood yet by applying only particle-core model [1][2][3]. In this paper we try to reconstruct this experimentally obtained transverse charge profiles by applying another halo formation theory [4].

1 INTRODUCTION

To study beam-halo formation experimentally, a 52-quadrupole FODO beam transport channel was set up at Los Alamos by using proton beam coming from the Low Energy Demonstration Accelerator (LEDA) [1]. The 6.7 MeV proton beam is accelerated by a 350 MHz cw radio frequency quadrupole (RFQ) with a 75 KeV proton injector. The detailed descriptions of the RFQ and LEDA halo experiments are given in refs. [5]-[12]. Among others, the experimental measurements reveal a typical beam transverse charge profile with apparent halo particles due to beam envelope oscillation as shown in Fig. 1. It is

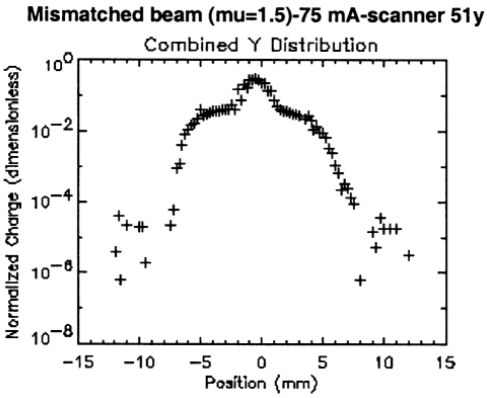

Figure 1: Joined profiles with wire and scraper scanners showing a dynamic range from core to maximum extent of nearly six orders of magnitude (the figure and the caption are from ref. [6].

seen from Fig. 1 that the profile has three distinct regions: the "head", the "shoulder", and the "feet". As stated in the summary of ref. [1], this profile shape is not understood. In this paper we try to apply the halo formation theory established in ref. [4] to explain and reconstruct it with the corresponding experimental parameters. In section 2 we first make a brief review of our theoretical bases, and in section 3 we calculate analytically the beam transverse charge profile and compare with that from experiment.

2 REVIEW OF THEORY

For a continuous round beam, its envelope can be described by the envelope equation:

$$\frac{d^2R}{dz^2} + K_0^2 R - \frac{K}{R} - \frac{\epsilon^2}{R^3} = 0 \tag{1}$$

where R is the beam envelope, $K = 2(I_b/I_0)/(\beta\gamma)^3$, $\pi\epsilon$ is the beam unnormalized transverse emittance, γ and β are the normalized particle's energy and velocity (v/c), respectively, I_b is the beam current, and $I_0 = 4\pi\epsilon_0 m_0 c^3/q$ with m_0/q being the mass charge ratio of the particle ($I_0 = 1.7 \times 10^4$ A for electron). When emittance is zero, one find the matched beam envelope $R_0 = \sqrt{K}/K_0$. According to ref. [4], the first order equilibrium transverse charge distribution for fermion follows Fermi-Dirac statistics, and the distribution function can be expressed as:

$$n(x) = \frac{F}{R_0^2\left(1 + \exp\left((x^2 - R_0^2)/\lambda_D^2\right)\right)} \tag{2}$$

where F is a normalization factor:

$$F = \frac{\left(R_0/\lambda_D\right)^2}{\ln\left(\frac{1+\exp\left(-(R_0/\lambda_D)^2\right)}{\exp\left(-(R_0/\lambda_D)^2\right)}\right)} \tag{3}$$

with the normalization condition of $\int_0^\infty I_b n(x) dx^2 = I_b$. In eq. 2 R_0 corresponds to the maximum particle excursion from the beam axis when the Debye length $\lambda_D = 0$. With finite beam envelope modulation amplitude ΔR and finite transverse beam emittance ϵ, the general expression for Debye length is expressed as:

$$\lambda_D^2 = \frac{(\delta R^2 + \Delta R^2)K_R^2}{K_{\Delta R}^2 + K_\epsilon^2} \tag{4}$$

with $\delta R = \frac{\epsilon^2}{2K_0^2 R_0^3}$, $K_R = \sqrt{2}K_0$, $K_{\Delta R} = \sqrt{2K_0^2 \Delta R/R_0}$, $K_\epsilon = \epsilon/R_0^2$ ($\delta R << R_0$), and ΔR and δR are statistically independent.

Under the influence of the periodic envelope oscillation, it is found that the particles transversely located at $x \geq R_0 + \Delta x_{max}$ will execute stochastic motions, and Δx_{max} can be calculated analytically as follows [4]:

$$\frac{\Delta x_{max}(z)}{R_0} = \frac{2R_0^3\sqrt{\beta(z)}}{\sqrt{27}LK\Delta R_0 \beta(z_i)^{3/2}} \tag{5}$$

where $\beta(z)$ is the beta function of the focusing channel of the zero space charge effect, and L is the envelope oscillation period. If one takes $\beta(z) = \beta(z_i) = \beta_{av}$, eq. 5 can be further simplified as:

$$\frac{\Delta x_{max}}{R_0} = \frac{2R_0^3}{\sqrt{27}LK\Delta R_0\beta_{av}} \tag{6}$$

where $\beta_{av} = R_0/\sqrt{K}$. The current corresponding to the particles executing stochastic motions can be easily estimated:

$$I_{h1} = \frac{I_b F}{R_0^2}\int_{x=R_0+\Delta x_{max}}^{x=\infty} \frac{1}{1+\exp\left(\frac{x^2-R_0^2}{\lambda_D^2}\right)}dx^2$$

$$= I_b F\frac{\lambda_D^2}{R_0^2} ln\left(\frac{\exp\left((x^2-R_0^2)/\lambda_D^2\right)}{1+\exp\left((x^2-R_0^2)/\lambda_D^2\right)}\right)|_{x=R_0+\Delta x_{max}}^{x=\infty} \tag{7}$$

Due to the stochastic motion the particles located at $x \geq R_0 + \Delta x_{max}$ will suffer from diffusion process towards outside and follow different distribution function from that expressed in eq. 2. Before giving the new charge distribution function we remind the reader an important finding from numerical simulations. It is found *numerically* that there exists a maximum halo amplitude which can be expressed *empirically* as [2][3]:

$$x_{max} = a(A + B|ln(\mu)|) \tag{8}$$

where a is the matched core rms size, A and B are week weak functions of the tune-depression ratio, approximately given by $A = B = 4$, and μ is the initial mismatch parameter defined as $\mu = R_{initial}/R_0$. In this paper we will define x_{max} as the "virtue" boundary, and the beam pipe dimension, R_m, as "hard" boundary. We distinguish now two possible cases. Firstly, when $R_m \leq x_{max}$, the particles in the region $R_0 + \Delta x_{max} \leq x \leq R_m$ follow the charge distribution function expressed as:

$$h(x) = \frac{2}{R_m^2 - (R_0+\Delta x_{max})^2}\times$$

$$\left(1 - \frac{x^2-(R_0+\Delta x_{max})^2}{R_m^2-(R_0+\Delta x_{max})^2}\right) \tag{9}$$

where $\int_{x=R_0+\Delta x_{max}}^{x=R_m} I_{h1}h(x)dx^2 = I_{h1}$. Secondly, when $R_m > x_{max}$ one has to find first the charge distribution within $R_0 + \Delta x_{max} \leq x \leq x_{max}$. Similar to eq. 9 one has:

$$h1(x) = \frac{2}{x_{max}^2 - (R_0+\Delta x_{max})^2}\times$$

$$\left(1 - \frac{x^2-(R_0+\Delta x_{max})^2}{x_{max}^2-(R_0+\Delta x_{max})^2}\right) \tag{10}$$

Now we estimate the particle populations beyond the virtue boundary due to finite beam emittance. The quantity δR is the uncertain measure for the beam envelope with finte emittance. From eq. 10 we can calculate the total current, I_{h2} beyond the the virtue boundary:

$$I_{h2} = I_{h1}\int_{x=x_{max}-\delta R}^{x=x_{max}} h1(x)dx^2 \approx 4\left(\frac{\delta R}{x_{max}}\right)^2 I_{h1} \tag{11}$$

The particles beyond the virtue boundary will follow a similar distribution to that expressed in eq. 10 in the region $x_{max} \leq x \leq R_m$.

$$h2(x) = \frac{2}{R_m^2 - x_{max}^2}\left(1 - \frac{x^2-x_{max}^2}{R_m^2-x_{max}^2}\right) \tag{12}$$

Now we will explain briefly the stationary distribution functions, h, $h1$, and $h2$. Since they are due to diffusion processes (the physical description can be consulted in ref. [13]), they can be obtained from diffusion equations [14]. In eqs. 9, 10, and 12, we have used a simpler function, such as $f(x) = \left(1 - \frac{x^2}{R}\right)$, to replace zero order Bessel function, $J_0\left(\frac{x}{R}u_{01}\right)$, where $u_{01} = 2.405$.

Defining plasma angular frequency $\omega_p = (nq^2/\epsilon_0 m_0\gamma)^{1/2}$ (where n is the charge density) and plasma wave number $k_p = \omega_p/\beta c$, one gets the plasma wavelength $\lambda_p = \sqrt{\frac{2\pi}{K}}\frac{R_0}{\gamma}$. If the particle re-distribution distance, or the so-called relaxation distance $\lambda_p/4$ [3], is shorter than the envelope oscillation period, L, the lost beam current due to envelope oscillation during one oscillation period can be estimated as:

$$I_{loss} = I_{h2}\int_{x=R_m-\Delta R_0}^{x=R_m} h2(x)dx^2 \approx 4\left(\frac{\Delta R}{R_m}\right)^2 I_{h2} \tag{13}$$

The beam current loss rate, $\mathcal{R}_{loss}$ (A/m), can be obtained by $\mathcal{R}_{loss} = I_{loss}/L$.

3 ANALYSIS ON LEDA BEAM HALO EXPERIMENTS

In this section we apply our theoretical model to a set of parameters similar to those in the experiments at Los Alamos and compare our analytical result with that observed experimentally. Before going on it is necessary to make two assumptions. Firstly, the our round beam model is applicable to a beam transported in a FODO channel, and secondly, the continuous beam model is applicable to a long bunched beam and the beam current I_b in the analytical formulae should be replaced by the bunch current. Now, we take a proton beam of 6.7 MeV with bunch current $I_b = 0.42$ A and average beam current 75 mA. The rms beam size from the RFQ is assumed to be $\sigma_x = 0.0013$ m. To apply our theoretical model we chose $R_0 = 0.0013$ m, $\Delta R = R_0/2$, $\mu = 1.5$, $L = 1.1$ m, $\epsilon_x = 2$ mm·mrad, and the FODO channel beam pipe inner radius $R_m = 0.01393$ m. In this specific case one has R_m larger than x_{max}. Before going on, we should stress stronly that the beam parameters correspond to those at the beginning of the FODO

channel, instead of at the end of FODO channel. By applying eqs. 2, 10, and 12, one obtains theoretically the equilibrium normalized transverse charge profile as shown in Fig. 2 with $\Delta x_{max} = 0.3$ mm and $x_{max} = 7.3$ mm. The average halo current loss rate at the beam pipe over beam pulse is $\mathcal{R}_{loss,av} = I_{loss} f/L = 1.6$ nA/m, where f is the ratio of the average beam current with respect to the peak bunch current. Comparing Fig. 1 with Fig. 2, it is obvious that the theoretical model reconstruct rather well the experimentally observed transverse charge profile at the end of FODO channel.

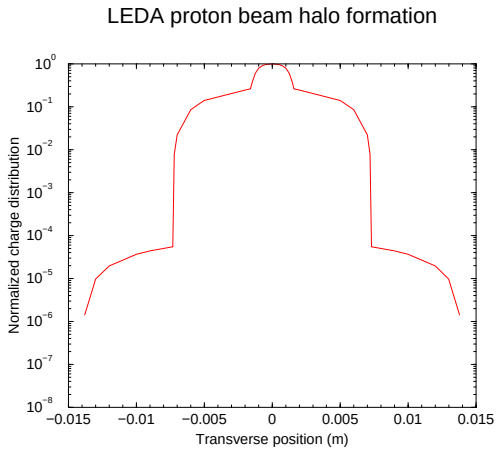

Figure 2: The theoretical transverse charge profile with beam envelope modulation $\mu = 1.5$, $W = 6.7$ MeV, $R_0 = 0.0013$ m, $\Delta R_0 = 0.00065$ m, $R_m = 0.01393$ m, $L = 1.1$ m, $I_b = 0.41$ A (proton), $f = 0.18$, and $\epsilon = 2$ mm·mrad.

4 CONCLUSION

In this paper we have applied the halo formation theory developed in ref. [4] to explain the experimentally observed transverse charge profile obtained at Los Alamos. The agreement between the experimental and analytical results is rather satisfactory.

5 ACKNOWLEDGEMENTS

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