SPECIFYING HOM-POWER EXTRACTION EFFICIENCY IN A HIGH AVERAGE CURRENT, SHORT BUNCH LENGTH SRF ENVIRONMENT

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Abstract

High average current, short bunch length beams in superconducting cavities can excite significant amounts of higher-order mode (HOM) power. The fraction that is dissipated in the cavity walls is of primary relevance as it can potentially limit the peak and average current due to the finite cryogenic capacity. A model has been developed which estimates the amount of power dissipated on the walls based on the dependence of the cavity's quality factor Q on frequency, due to BCS losses, and an analytic expression for the cavity impedance in the high frequency limit. Specifications for the HOM power extraction efficiency are derived so that the cryogenic load due to the HOM excitation is of similar magnitude as the load due to the accelerating fields.

1 INTRODUCTION

In the recent months, a strong interest has developed in exploring the parameter regime over which energy-recovery superconducting rf (srf) linacs can be used either as FEL/light source drivers or as colliders [1,2,3]. Average currents of the order of a few hundred mA are considered with charge per bunch in the nC range and ~ psec long bunches. These high average current, short bunch length beams excite HOMs in the rf cavities which, in addition to beam stability consequences, in an srf environment present the challenge of increased cryogenic load due to power dissipation in the cavity walls. Unless these modes are sufficiently extracted, the additional refrigeration load may be prohibitive.

For the CEBAF 5-cell cavity, the loss factor for a psec long bunch is of order 10 V/pC, therefore 100 mA average current would result in HOM power per cavity in the kW level, and twice as much during energy recovery. Although the magnitude of the total HOM power is amazing, it is the amount dissipated on the walls, which could present a true limitation on the peak and average current in an srf environment, due to the finite capacity of refrigerators. To determine the power dissipated on the walls we invoke two separate models: For modes below the beam-pipe cut-off, where mode characteristics are quite accurately known both from numerical codes and measurements, powers are calculated as sums over individual modes. For modes above cut-off, an analytic model for the impedance in the high-frequency limit is used, which agrees quite accurately with URMEL calculations of the loss factor for CEBAF cavities. Furthermore, the degradation of the cavity's quality factor Q with frequency, $Q \propto f^{-2}$, due to BCS surface resistance is taken into account and the power dissipated on the walls P_c, is calculated assuming that the rest of the power is dissipated into a load coupled to a mode ω with coupling strength $\beta = \beta(\omega)$.

The expression for P_c depends on Q_{ext}, thereby allowing us to derive specifications on the magnitude of Q_{ext}, in order for the cryogenic load due to HOM losses not to exceed the load due to the accelerating fields.

Finally, a "multiple reflection model" has been developed, valid in the geometric optics limit, where much of the spectrum considered here belongs, and is compared with the "high-frequency behaviour model."

2 MODES BELOW BEAM-PIPE CUTOFF

For modes below the cut-off of the beam pipe detailed data exist both from URMEL and measurements on the CEBAF 5-cell cavity [4] for each mode. To calculate the power dissipated by the beam in exciting these modes, we first calculate the power in each mode n, and then sum up over all the higher order modes.

Assume an infinitely long train of bunches each with charge q spaced in time by $T_b = 1/f_{\text{bun}}$. If $T_d = 2Q_L/\omega_n$ is the time constant of the decay of the fields in a given mode n, then the power dissipated by the beam in exciting this mode is [5]

$$P_b = \frac{I_0^2 (r/Q)_n Q_{0,n}}{(1+\beta_n)} F_r \tag{1}$$

where

$$F_r = \frac{\frac{T_b}{T_d}\left[1-\exp\left(-2\frac{T_b}{T_d}\right)\right]}{2\left[1-2\exp\left(-\frac{T_b}{T_d}\right)\cos\left(\Delta\omega T_b\right)+\exp\left(-2\frac{T_b}{T_d}\right)\right]}$$

and the bunches are assumed short enough to be considered as point charges. Here $\Delta\omega = \omega_n - \omega_{rf}$ and $I_0 = |q| f_{\text{bun}}$ is the average current. This expression allows for external coupling with coupling strength β_n between the mode n and a load with $Q_L = Q_0/(1+\beta)$. The power dissipated on the cavity walls by the mode n is $P_{c,n} = P_{b,n}/(1+\beta_n)$.

This model is applied to the first 5 passbands up to frequencies of 4.2 GHz, slightly above the beam-pipe cut-off at 3.27 GHz for the CEBAF cavities. The frequencies, impedances and Q_0 (for a copper cavity) of these modes are obtained from URMEL calculations. To calculate Q_0 for Nb cavities we use $Q_0 = G/R_s$ where G is a geometric factor and R_s is the surface resistance of Nb given by the sum of the BCS resistance R_{BCS}, and the residual resistance R_0, $R_s = R_{\text{BCS}} + R_0$. For each mode, G is determined by, $G = Q_0^{\text{Cu}} R_s^{\text{Cu}}$ where Q_0^{Cu} is the Q_0 value obtained from URMEL and R_s^{Cu} the surface resistance of copper equal to 10.143 mΩ. To calculate the residual resistance R_0, which is frequency-independent, we make use of the data on the fundamental accelerating mode of 1.5GHz: $Q_0 = 8\times10^9$, $G = 275\,\Omega$, $R_{\text{BCS}}^{\text{f}} = 1.455\times10^{-8}\ \Omega$ (the superscript f stands for fundamental and this value is at 2° K), and find $R_0 = 1.98\times10^{-8}\ \Omega$.

The BCS surface resistance of the higher modes is calculated using the expression

$$R_{\text{BCS}}(f) = R_{\text{BCS}}^{f}\left(\frac{f}{1.5}\right)^2 \qquad (2)$$

where f is the rf frequency in GHz. The values of Q_0 vary between 8×10^9 and 3×10^9.

For Q_{ext} we use Amato's number whenever available, else we set it equal to 1000. The four modes below the accelerating mode are treated differently. For these, Q_{ext} is scaled from Q_{ext} of the π mode which is 6.6×10^6, according to $Q_{\text{ext}}^{\text{j}} \propto 1/\Phi_{5j}^2$ where Φ_{5j} is the fields amplitude in the 5$^{\text{th}}$ cell and j mode, and accounts for the fact that in the different modes the field distributions vary resulting in variations in the coupling strength [6]. Based on this scaling, the external Q's for the first four modes are: 3.5×10^7, 9.6×10^6, 5.0×10^6, 3.6×10^6 for $\pi/5$, $2\pi/5$, $3\pi/5$, $4\pi/5$ respectively.

Using the method we just outlined, we calculated the power dissipated by the beam P_b, in exciting the first 20 longitudinal modes and the fraction of this power that ends up on the cavity walls P_c, for a train of bunches each with charge 4 nC and bunch repetition frequency of 150 MHz. We find that $P_b = 4752$ W and $P_c = 1.6$ mW. Clearly these numbers depend rather strongly on the exact frequencies of the modes, and since the actual frequencies may be shifted by up to several MHz from the calculated ones, one should perform a statistical analysis to get a more precise answer. However, the fact remains that the power on the walls is negligible compared to the power lost by the beam, and does not present a significant cryogenic load, therefore we now turn our attention to the remaining infinity of modes, above ~4.5 GHz.

3 MODES ABOVE BEAM-PIPE CUTOFF

The power dissipated by the beam in modes above the beam-pipe cutoff is given by

$$P_b = \frac{f_{\text{bun}}}{\pi}\int_{\omega_c}^{\infty} I^2(\omega)\,\text{Re}\,Z(\omega)\,d\omega \qquad (3)$$

where f_{bun} is the bunch repetition frequency, $I(\omega)$ is the Fourier component of the beam current,

$$I(\omega) = \int_{-\infty}^{+\infty} I(t)e^{i\omega t}dt$$

and

$$I(t) = \frac{Q_b}{\sqrt{2\pi}\sigma_t}\exp(-t^2/2\sigma_t^2)$$

where Q_b is the bunch charge and σ_t the rms bunch length. It follows that $I(\omega) = Q_b\exp(-\omega^2\sigma_t^2/2)$.

For the CEBAF 5-cell cavities, URMEL calculations of the loss factor as function of bunch length fit to the functional form $k_{\|} \propto \sigma_t^{-0.55}$, suggesting that the $1/\sqrt{\sigma_t}$ dependence describes best the behavior of the loss factor in the high-frequency, short bunch length limit. Thus, the analytic expression used for the impedance of a cavity is

$$\text{Re}\,Z(\omega) = \frac{Z_0}{2\pi}\frac{1}{a}\sqrt{\frac{g}{\pi k}} \qquad (4)$$

where $k = \omega/c$, g is the gap length and a is the radius of the aperture. The loss factor can be calculated using

$$k_{\|} = \frac{1}{\pi}\int_0^{\infty} d\omega Z(\omega)e^{-\omega^2\sigma_t^2}$$

yielding

$$k_{\|} = \left(\frac{Z_0}{4\pi}\right)\frac{\Gamma(1/4)}{\pi}\frac{1}{a}\sqrt{\frac{gc}{\pi\sigma_t}} \qquad (5)$$

Note that this expression is valid for a single cell. The CEBAF 5-cell cavity number is 5 times larger, and for an rms bunch length = 1 psec, radius = 3.5 cm and gap=10 cm, $k_{\|}$ is 15.3 V/pC.

From eq. (3) we can now calculate the total power dissipated by the beam in the HOMs:

$$P_b = Q_b^2 f_{\text{bun}}\left(\frac{Z_0}{4\pi}\right)\frac{\Gamma\left[1/4, \omega_c^2\sigma_t^2\right]}{\pi}\frac{1}{a}\sqrt{\frac{gc}{\pi\sigma_t}}$$

For 4 nC bunch charge, 150 MHz bunch repetition frequency and 1 psec bunch length, the power dissipated

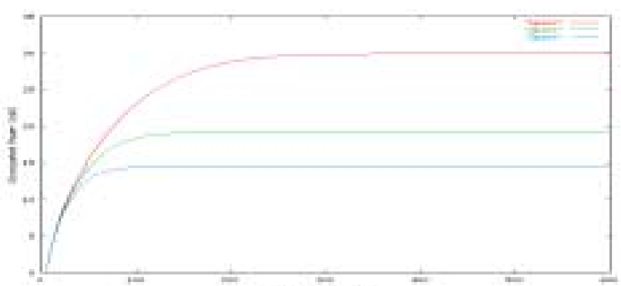

Figure 1. Frequency distribution of HOM power

by the beam is approximately 30 kW per cavity. It is interesting to see how this amount of power is distributed in frequency. Figure 1 is a plot of the power lost in frequencies between ω_c and ω_l, as function of ω_l, for three different values of the bunch length, 1, 2 and 3 psec. Note that in all cases, greater than 90% of the power is below 100 GHz, although frequencies up to 600 GHz are excited by short bunches.

The power dissipated on the cavity walls, P_c is given, from energy conservation by

$$P_c = \frac{f_{\text{bun}}}{\pi} \int_{\omega_c}^{\infty} \frac{Q_L(\omega)}{Q_0(\omega)} I^2(\omega) \operatorname{Re} Z(\omega) d\omega \qquad (6)$$

where the loaded Q is given by $1/Q_L = 1/Q_0 + 1/Q_{\text{ext}}$. In the following we assume that Q_{ext} is frequency-independent, at least over a wide frequency band. Using eq. (2) the intrinsic quality factor $Q_0(\omega)$ is written as

$$\frac{1}{Q_0(\omega)} = \frac{1}{Q_0^{\text{BCS}}} \left(\frac{\omega}{\omega_f} \right)^2 + \frac{1}{Q_0^{\text{res}}} \qquad (7)$$

where $Q_0^{\text{BCS}} = 1.89 \times 10^{10}$, and $Q_0^{\text{res}} = 1.34 \times 10^{10}$. Since even at the highest frequencies Q_0 is still a few orders of magnitude larger than Q_{ext}, we will approximate $Q_L \approx Q_{\text{ext}}$. Combining eqs (5), (6) and (7) we can now calculate P_c,

$$P_c = (Q_b^2 f_{\text{bun}}) Q_{\text{ext}} \left(\frac{Z_0}{4\pi} \right) \frac{1}{\pi a} \sqrt{\frac{gc}{\pi}} \times$$
$$\left\{ \frac{1}{Q_0^{\text{BCS}} \omega_f^2} \frac{\Gamma\left[\frac{5}{4}, \omega_c^2 \sigma_t^2\right]}{\sigma_t^{5/2}} + \frac{1}{Q_0^{\text{res}}} \frac{\Gamma\left[\frac{1}{4}, \omega_c^2 \sigma_t^2\right]}{\sigma_t^{1/2}} \right\}$$

Notice the strong dependence of P_c on the bunch length. The fraction of the total power that is dissipated on cavity walls in each frequency band from ω_1 to ω_2 is given by

$$\frac{\Delta P_c}{\Delta P_b} = Q_{\text{ext}} \left\{ \frac{1}{Q_0^{\text{BCS}} \omega_f^2 \sigma_t^2} \frac{\Gamma\left[\frac{5}{4}, \omega_1^2 \sigma_t^2\right] - \Gamma\left[\frac{5}{4}, \omega_2^2 \sigma_t^2\right]}{\Gamma\left[\frac{1}{4}, \omega_1^2 \sigma_t^2\right] - \Gamma\left[\frac{1}{4}, \omega_2^2 \sigma_t^2\right]} + \frac{1}{Q_0^{\text{res}}} \right\}$$

and varies as σ_t^{-2}. For the parameters used earlier we compute the power dissipated by the beam and the fraction that is lost on the walls, for three frequency bands, 4.5 to 10 GHz, 10 to 100 GHz and above 100 GHz, and show the results on Table 1.

Table 1: Power calculations and Q_{ext} specs ($\sigma_t = 1$psec)

Frequency Range [GHz]	ΔP_b [kW]	$\frac{\Delta P_c}{\Delta P_b}$ [$\times Q_{\text{ext}}$]	Q_{ext} spec	ΔP_c [W]
4.5 - 10	3.33	1.2×10^{-9}	10^5	0.4
10 – 100	19.66	6.2×10^{-8}	2000	2.4
>100	6.88	6.2×10^{-7}	2000	8.5

Note that although relatively little power is generated in modes above 100 GHz, a larger fraction of it is deposited on the cavity walls, therefore adequate extraction of these modes is of importance, whereas the extraction requirements can be more relaxed at lower frequencies. One may consider using couplers which act independently on different parts of the spectrum with different means. As an example, if the coupling is such that $Q_{\text{ext}} \leq 2000$ for frequencies above 10 GHz and $Q_{\text{ext}} \leq 10^5$ for modes between 4.5 and 10 GHz, then the total power dissipated on the walls will be approximately equal to the losses due to the fields of the fundamental $P_c = V^2/(r/Q)Q_0$, which is ~10W for $Q_0 = 8 \times 10^9$ and V=12.5 MV/m.

4 MULTIPLE REFLECTIONS MODEL

In the geometric optics limit, the fraction of power that ultimately goes out the various openings, of area α, of a cavity with surface reflectivity R (function of ω) is $\alpha/[1-(1-\alpha)R]$. Then the fraction going into the walls is ε/α, where $R = 1-\varepsilon$. If ε is expressed in terms of the surface resistance which is allowed to vary $\propto \omega^2$ according to BCS theory, then for the CEBAF 5-cell cavity dimensions, the effective coupling due to beam-pipe openings has a Q_{ext} of order 100. This implies that most of the power is coupled out of the beam-pipe and a smaller fraction (than the one corresponding to Q_{ext} of 2000) is dissipated on the walls, assuming that the beam-pipe power is dissipated into a load.

5 CONCLUSIONS

High average current, short bunch length beams in srf environments can give rise to unprecedented amounts of HOM power, as high as tens of kW per cavity, and twice as much during energy recovery. We have asked the question: “Where have all the HOM losses gone?” Using measured values of frequencies and Q’s for modes below the beam-pipe cut-off, and a simple model which determines the Q’s based on BCS scaling with frequency for modes above cut-off, and an analytic expression for the cavity impedance, we conclude that: a) Most of the power lost by the beam is in modes below 100 GHz b) the amount that is dissipated on the walls is a strong function of bunch length c) we specified Q_{ext} in order for the fraction that is dissipated on the walls to be of the order of the losses due to the fundamental accelerating fields in the cavity. We note that the derived estimates assume gaussian distributions and may be different (within factors of 2) for distributions with sharper edges than gaussian. Finally, the power flow issue, implicit in the Q_{ext} of the beam pipe, must be carefully thought through and possibly properly-placed cooled absorbers will be needed in the warm section of the beam line. Future plans include numerical studies of the problem as well as experiments.

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