Implementation and Validation of Space Charge and Impedance Kicks in the Code Patric for Studies of Transverse Coherent Instabilities in the Fair Rings*

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Abstract

Simulation studies of the transverse stability of the FAIR synchrotrons have been started. The simulation code PATRIC has been developed in order to predict coherent instability thresholds with space charge and different impedance sources. Some examples of code validation using the numerical Schottky noise and analytical stability boundaries will be discussed.

INTRODUCTION

The FAIR synchrotrons will be operated with medium energy, intense heavy ion beams of low momentum spread. The range of bunch lengths and bunch profiles during a typical cycle covers dc beams, long dc-like bunches in barrier buckets, long bunches in single and in dual rf waves [1]. Before extraction the bunches are converted into a single, short (50 ns) bunch. The required accumulation times are of the order of 1 s. Because of the high intensities together with low momentum spreads transverse instabilities driven e.g. by resistive wall or kickers impedances [2] are of concern. In addition the bunches will experience large transverse space charge tune shifts, compared e.g. to the effective tune spread for Landau damping. Space charge itself will not drive coherent instabilities, but it can greatly effective tune spread for Landau damping. Space charge of concern. In addition the bunches will experience large transverse space charge kicks every

\[ \left( \begin{array}{c} x_{s+\Delta s} \\ y_{s+\Delta s} \\ z_{s+\Delta s} \end{array} \right) = M(\Delta s) \left( \begin{array}{c} x_s \\ y_s + \Delta y_{imp} \\ z_s \end{array} \right) \]

(1)

\[ \Delta y_{imp}(x, y, z) = \frac{qE(x, y, z)\Delta s}{mv_0^2\gamma_0^2} \]

(2)

with the relativistic parameter \( \gamma_0 \), the velocity of the center particle \( v_0 \), mass \( m \), charge \( q \) and the transverse electric field \( E_x \) obtained from the 2D Poisson equation

\[ \frac{\partial E_x(z_i)}{\partial x} + \frac{\partial E_y(z_i)}{\partial y} = \frac{\rho(x, y, z_i)}{\epsilon_0} \]

(3)

with the beam density \( \rho \). The horizontal space charge kicks are calculated every \( \Delta s \) from a MAD-X sectormap [7] objects obtained from a MAD-X output file. The length \( \Delta s \) of a sectormap is determined by the variation of the beam envelope and the required space charge kicks per betatron wave length. The horizontal space charge kicks are calculated every \( \Delta s \) from

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with the beam density \( \rho \) interpolated on a longitudinal grid defined between the bunch ends (see Fig. 1) with \( N_b \) grid points, grid spacing \( \Delta l \ll l_b \) (bunch length \( l_b \)) and grid point positions \( z_i \). At every \( z_i \) a fast 2D Poisson solver is used to obtain \( E_x(x, y, z_i) \). This approach is often called ‘2.5D space charge’ or ‘sliced space charge calculation’, with the slice position \( z_i \) and slice length \( \Delta l \). It is assumed that the bunch is long relative to the transverse beam and pipe dimensions. Another important simplification is that we assume the same position \( s_0 \) in the lattice for all bunch slices, not just for the center slice. Therefore the fast variation of the transverse bunch envelope in alternating gradient focusing lattices is neglected in the space charge calculation.

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For the parallelization of the tracking step and the space charge calculation the bunch is divided into \( N_m \leq N_b \) macro-slices, with slice center positions \( z_{m} \) and slice length \( \Delta Z \geq \Delta l \). For efficient load-balancing the macro-slice length is adjusted automatically to the bunch profile during a simulation run. Particles migrate slowly between macro-slices due to synchrotron motion. After each tracking step the particles are exchanged between neighboring macro-slices using MPI [8] routines. Each macro-slice keeps \( \Delta Z/\Delta l \) 2D transverse grids and performs \( \Delta Z/\Delta l \) serial 2D Poisson solver calls and bilinear grid-particle/particle-grid interpolations. The calculation forms macro-slices, with slice center positions \( Z \).

The time between the kicks is then \( \Delta t = T \) and \( t_k = kT \). \( \psi(z_j, t_k) \) is the dipole moment at the position \( z_l \) and turn of the transverse space charge grid-particle/particle-grid interpolations. The calculation of the transverse space charge field is greatly simplified if we use the approximation of rigid dipole oscillations. If we take the example of a parabolic beam profile the resulting electric space charge field is given through

\[
E_x(x, y, z) \approx \frac{I(z)(x - \bar{x})}{2\pi\varepsilon_0\varrho_0a^2} \left( 2 - \frac{1}{a^2}(x - \bar{x})^2 + y^2 \right) \tag{4}
\]

with the local current \( I(z) \) and the beam radius \( a \). In this case the transverse grids are not needed. However, careful comparisons of stability boundaries obtained with self-consistent space charge are required before the 'frozen' space charge calculation can be used in the required 3D studies for the FAIR rings.

**IMPEDANCE KICKS**

The 'dipole moment times current' at a fixed ring position \( s \) is \( \psi(t) \). The resulting horizontal kick per turn (see e.g. [9]) is

\[
\Delta x'_{imp} = \frac{\int F_x ds}{\beta_0^2 E_0} \tag{5}
\]

with the frequency spectrum \( \psi(\Omega_j) \) at the coherent dipole oscillation frequencies (bare machine tune \( Q \))

\[
\Omega_j = (n \pm Q)\omega_0 \tag{6}
\]

The horizontal dipole moment along the longitudinal particle position \( z \) is defined as

\[
\psi(z, t) = \beta_0 c \int x \rho(x, y, z, t) dx dy \tag{7}
\]

and the Fourier spectrum is (ring circumference \( L \))

\[
\psi_n(t) = \exp(\mp iQ\omega_0 t) \int_0^L \psi(z, t) \exp(-inz/R) dz \tag{8}
\]

The amplitude \( \psi_n(t) \) varies slowly with time. For lumped impedances the resulting kick is applied every \( \Delta s \)

\[
\Delta x_{imp}'(z, t) = \frac{\Delta s}{L} \frac{q}{\beta_0 E_0} \times \Re \left( i \exp(\pm iQ\omega_0 t) \sum_n \psi_n(t) Z_\perp [(n \pm Q)\omega_0] \exp(inz/R) \right) \tag{9}
\]

For a localized impedance source we set \( \Delta s = L \) and the kick is applied every turn.

The numerical implementation of impedance kicks in PATRIC employs a grid along the \( z \)-axis with periodic boundary conditions at \( z = 0 \) and \( L \). Let \( z_j = j\Delta z \) be the grid points along the \( z \)-direction. For the transverse kick due to a localized impedance source we set \( \Delta s = L \). The time between the kicks is then \( \Delta t = T \) and \( t_k = kT \). \( \psi(z_j, t_k) \) is the dipole moment at the position \( z_j \) and turn

![Figure 1: Sketch of the different longitudinal grids used in PATRIC for tracking, space charge, impedance kicks and parallelization.](image)
The horizontal kick is applied to all particles every turn using a Fast Fourier Transformation (FFT)
\[
\Delta x'(z_j, t_k) = \frac{q}{\beta_0 E_0} \text{Re} \left( \text{FFT}^{-1} \left[ i \psi_n(t_k) Z_{\perp} \left( (n \pm Q) \omega_0 \right) \right] \right)
\]
with the dipole amplitudes
\[
\psi_n(t_k) = \text{FFT} \left[ \psi(z_j, t_k) \right]
\]

VALIDATION AND APPLICATION EXAMPLES

PATRIC is presently being validated against the available analytic examples. Here we discuss some of the more 'non-standard' examples of 3D code validation using coasting beams. In the examples a simplified accelerator model with constant focusing and reduced ring circumference is chosen. Finally transverse stability studies in bunched beams are briefly discussed.

Noise spectrum and stability boundary

Transverse stability boundaries in long bunches depend critically on the longitudinal momentum spread and the associated Landau damping of coherent dipole modes. In a coasting, stable beam the transverse 'Schottky' noise is one option to validate whether a 3D simulation code, like PATRIC, can correctly resolve this coupling effect. Because of the 'granularity' of the computer beam, which is much coarser than in a real beam, the noise spectrum contains a wealth of information on incoherent and coherent effects, but also on artificial effects. Here we focus on the noise frequency spectrum of offset oscillations obtained from the Fourier transformation of the dipole current \( \psi(t) \). In the simulation example chromaticity is set to zero and the rms horizontal tune spread is

\[
S_x = \left| \eta \left( n \pm Q_x \right) \omega_0 \delta_{\text{rms}} \right| \tag{12}
\]

with the slip factor \( \eta \), the bare tune \( Q_x \) and the rms momentum spread \( \delta_{\text{rms}} \). The impedance spectrum used in the example has a large negative imaginary contribution \( Z_I \) (e.g. due to wall image currents). The resulting coherent tune shift and the space charge induced incoherent tune shift are adjusted to the same value (\(-0.01\) in the example). Under these conditions Landau damping is maximized. A broadband oscillator is added to the impedance spectrum

\[
Z_{\perp, r} = \frac{\omega}{\omega_r} \left( 1 - i Q \left( \frac{\omega_r}{\omega_r^2 - \omega} \right) \right)
\]

with the resonance frequency \( \omega_r \) tuned to harmonic \( n = 10 \) of the simulation model. The value of \( Z_R \) is chosen well below the instability threshold obtained from the analytic dispersion relation for a Gaussian momentum distribution. The resulting noise power spectrum \( | \hat{\psi}(\omega) | \) is shown in Fig.2. For harmonics close to \( n = 10 \) one can clearly observe the relative enhancement of the 'slow' \((n \pm Q_x)\) lines relative to the 'fast' \((n + Q_x)\) lines. In the examples the circumference is 54 m or one quarter of SIS 18. The bare tune for this reduced circumference is is \( Q_x = 0.81 \). Details of the 'slow' line at \((11 - Q_x)\) are shown in Fig. 3. The location of \((11 - Q_x)\) is indicated by the vertical red dotted line. The blue dashed curve is a Gaussian with rms width \( S_x \) with the frequency shift induced exactly by the imaginary impedance component. This 'Schottky test' covers the incoherent (width of the line) and the coherent (shift of the line) beam properties relevant in beam stability studies. In Fig. 3 one can also observe that deviations from a Gaussian start at \( \approx 2 S_x \). In the present example we used only \( 5 \times 10^5 \) macro-particles. Using more particles will improve the resolution of the tails of the Gaussian. The simulation results shown were obtained with a frozen, linear space charge model. The comparison with the self-consistent space charge model and a homogenous transverse beam profile (K-V distribution) showed no significant difference in the noise power spectrum. As a second validation example the analytic stability boundary for a Gaussian momentum distribution [10] is checked. In the example we fix the space charge tune shift and vary the

\[\text{Figure 2: Noise power spectrum from the simulation.}\]

\[\text{Figure 3: Comparison of the noise power spectrum for the slow mode close to } (11 - Q) \text{ (red dotted line) with a shifted Gaussian (blue dashed curve).}\]
imaginary impedance $Z_I$ and the resonator impedance $Z_R$. Fig. 4 shows the stability boundary in the $(Z_R, Z_I)$ plane obtained from such a parameter scan. Here the result of 100 simulation runs with different $(Z_R, Z_I)$ values is automatically analyzed and displayed using Python scripts. The color code represents the obtained maximum of the dipole current. The stable area (no growth) is indicated in blue. One can see that there is a rather good agreement around the center of the stable area. These validation studies are still ongoing. The parameter scan shown in Fig. 4 has been obtained using self-consistent space charge kicks and a initial K-V distribution, leading to a linear space charge field.

**Stability boundary in bunched beams**

Besides the code validation activities 2D and 3D simulation studies with PATRIC focus on two important beam physics issues. Firstly the relevance of self-consistent space charge for transverse stability boundaries is being studied (see also Ref. [11]). With frozen space the simulation runs are faster and effects of numerical noise are minimized. The question is whether with frozen space charge one misses important instability damping or destabilization mechanisms. Secondly the transverse stability boundary in long bunches (long relatively to the resonant wavelength of the broadband oscillator) is being compared to the boundary for the corresponding coasting beam. In bunched beams the variation of the coherent and incoherent tune shifts along the bunch [12] as well the velocity of the unstable waves on a beam with finite extension are expected to increase the stability boundary. Fig. 5 shows a snapshot obtained from the simulation of the instability evolution in a beam confined between two rf barrier waves. The simulation parameters are the same as those used in the coating beam studies in the the previous section. As a preliminary result we found that the stability boundary in long barrier buckets is only slightly enlarged compared to the equivalent coating beam. However, extensive simulation studies using different beam parameters and also conventional rf buckets still have to be performed.

**CONCLUSIONS**

The simulation tool PATRIC has been developed at GSI in order to study coherent instabilities in the presence of space charge in the FAIR synchrotrons. The recently implemented 2.5D space charge and transverse impedance kicks are still being validated against various analytic examples. Preliminary results indicate that for the required 3D stability studies, at least for first estimates, a 'frozen' space charge model can be employed. Benchmarking with the HEADTAIL code [5] using FAIR and CERN PS/SPS parameters as well as experimental observation will be one of the important next steps.

**REFERENCES**

[8] see e.g. http://www-unix.mcs.anl.gov/mps/