BACKWARD WAVE EXCITATION AND GENERATION OF OSCILLATIONS IN ABSENCE OF FEEDBACK

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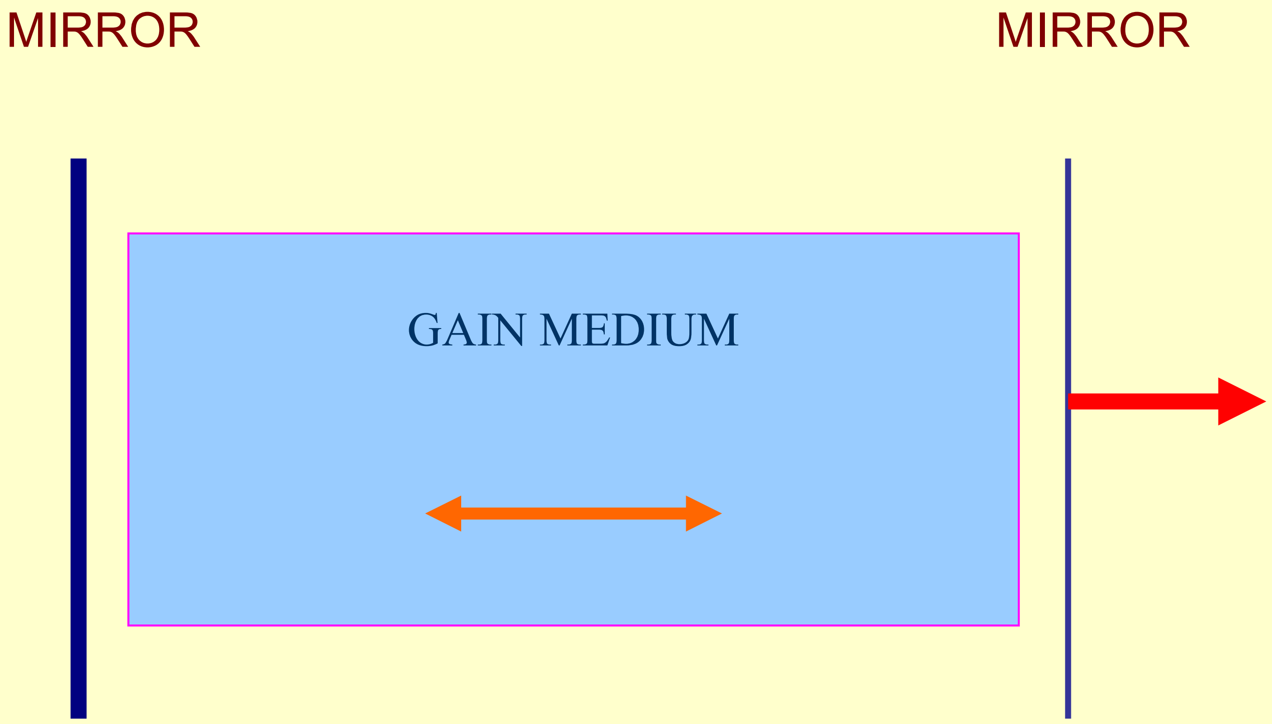
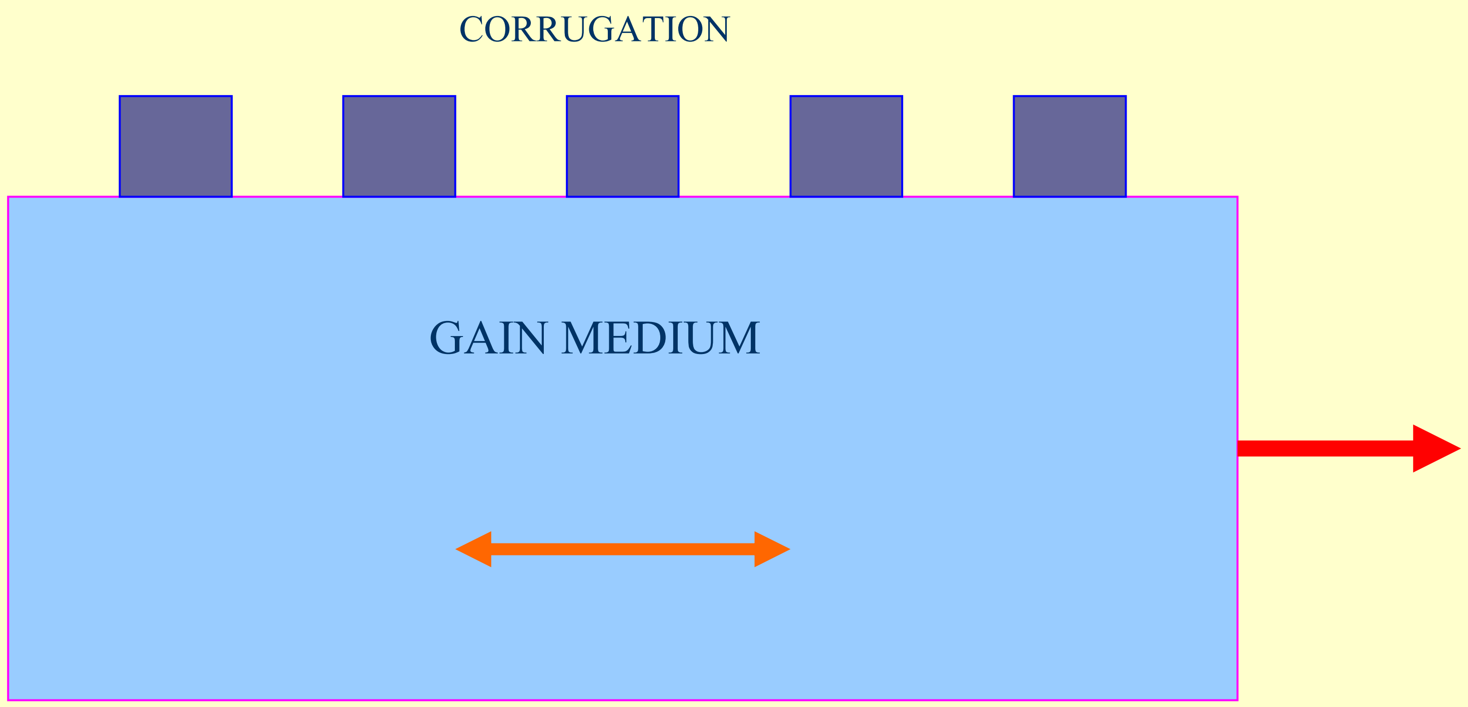
"Development of generalized theory and simulation of a wide-band interaction radiation sources and lasers"

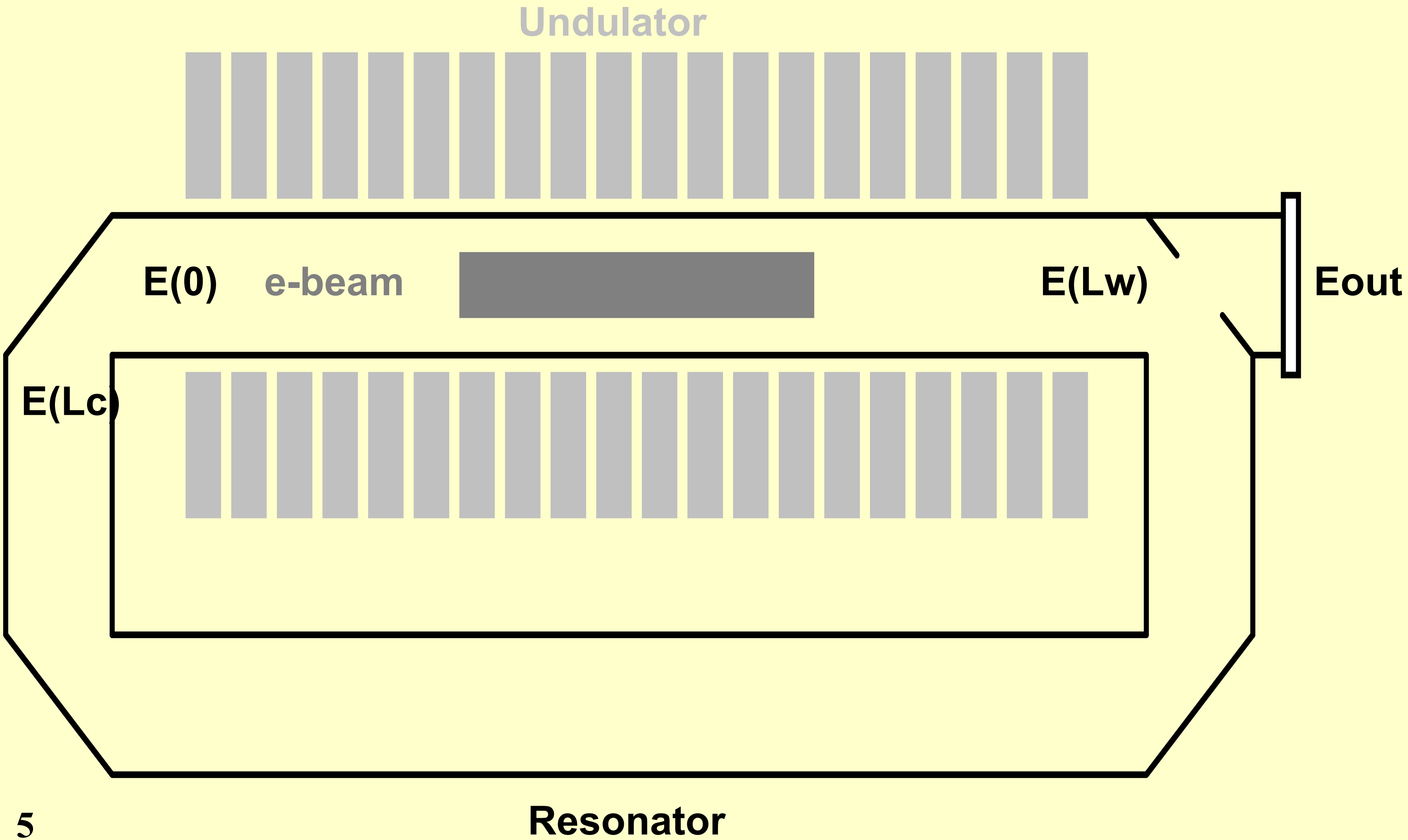
Laser Oscillator



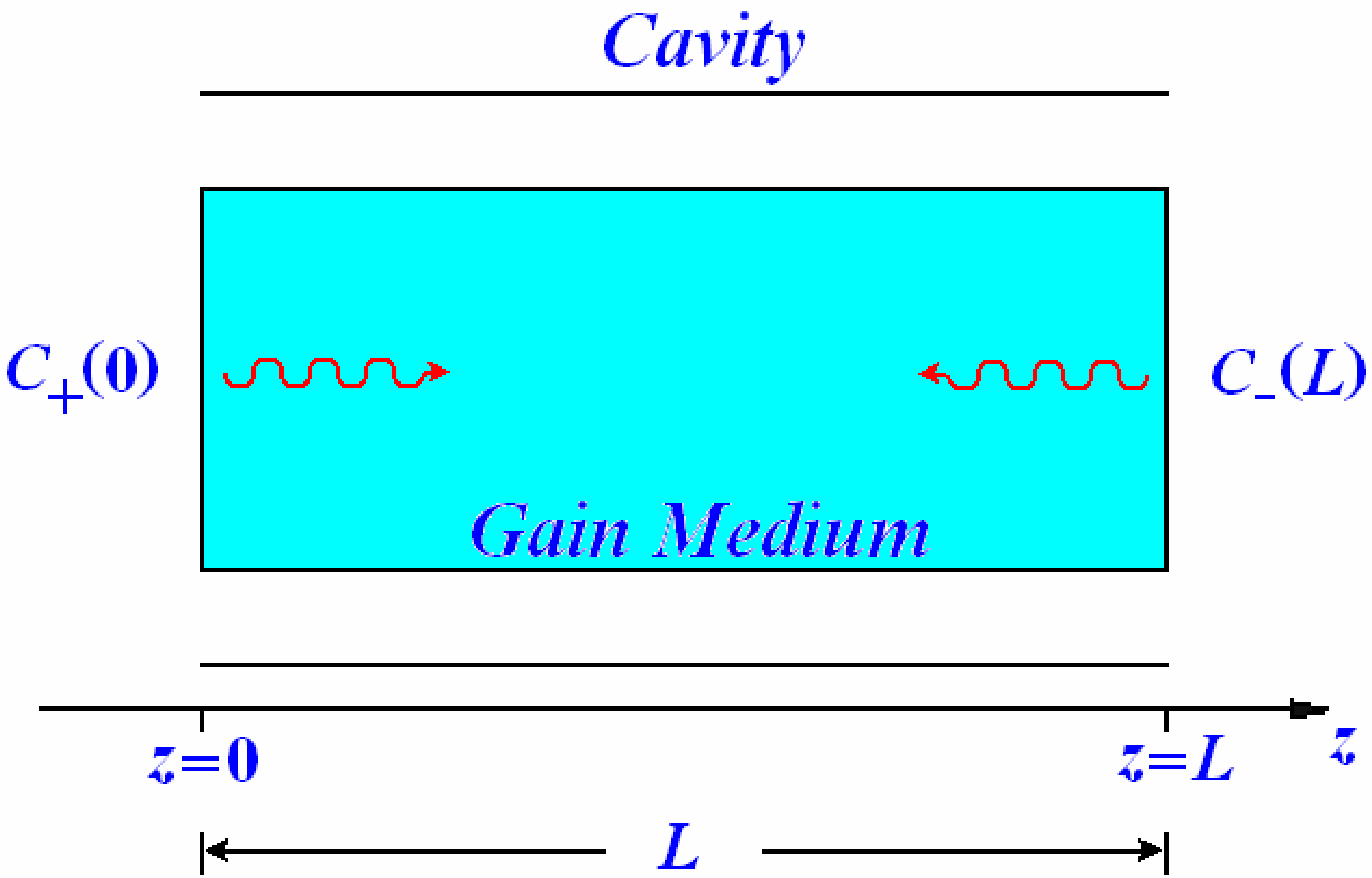
Distributed Feedback - DFB



FEL OSCILLATOR



Distributed Gain Medium



Excitation of Forward and Backward Modes

$$\mathbf{E}(\mathbf{r},t)=\mathrm{Re}\left\{\widetilde{\mathbf{E}}(\mathbf{r})\cdot e^{-j\omega t}\right\}$$

$$\widetilde{\mathbf{E}}(\mathbf{r})=\left[C_+(z)e^{+jk_z z}+C_-(z)e^{-jk_z z}\right]\cdot\boldsymbol{\varepsilon}(x,y)$$

$$\frac{d}{dz}C_\pm(z)=\mp\frac{1}{2N}e^{\mp jk_z z}\iint\widetilde{\mathbf{J}}(r,f)\cdot\boldsymbol{\varepsilon}^*(x,y)dxdy$$

$$P(z)=\frac{1}{2}\left[\left|C_+(z)\right|^2-\left|C_-(z)\right|^2\right]\mathrm{Re}\{N\}$$

Induced polarization

$$\widetilde{\mathbf{J}}(r) = -j\omega\widetilde{\mathbf{P}}(r) = -j\omega\varepsilon_0\boldsymbol{\chi}(r,\omega)\cdot\widetilde{\mathbf{E}}(r)$$

$$\frac{d}{dz}C_{\pm}(z) = \pm j\frac{\omega\varepsilon_0}{2N}e^{\mp jk_z z}\iint \boldsymbol{\varepsilon}(x,y)\cdot\boldsymbol{\chi}(r,\omega)\cdot\boldsymbol{\varepsilon}^*(x,y)dxdy$$

$$\frac{d}{dz}\begin{bmatrix} C_+(z) \\ C_-(z) \end{bmatrix} = \begin{bmatrix} +\kappa(z) & +\kappa(z)e^{-j2k_z z} \\ -\kappa(z)e^{+j2k_z z} & -\kappa(z) \end{bmatrix}\begin{bmatrix} C_+(z) \\ C_-(z) \end{bmatrix}$$

$$\kappa(z,\omega) = j\frac{\omega\varepsilon_0}{2N}\iint \boldsymbol{\varepsilon}(x,y)\cdot\boldsymbol{\chi}(r,\omega)\cdot\boldsymbol{\varepsilon}^*(x,y)dxdy$$

Quantum LASER

$$\frac{d}{dz}\begin{bmatrix} C_+(z) \\ C_-(z) \end{bmatrix} = \begin{bmatrix} +\kappa & +\kappa e^{-j2k_z z} \\ -\kappa e^{+j2k_z z} & -\kappa \end{bmatrix}\begin{bmatrix} C_+(z) \\ C_-(z) \end{bmatrix}$$

$$C_+(0)$$

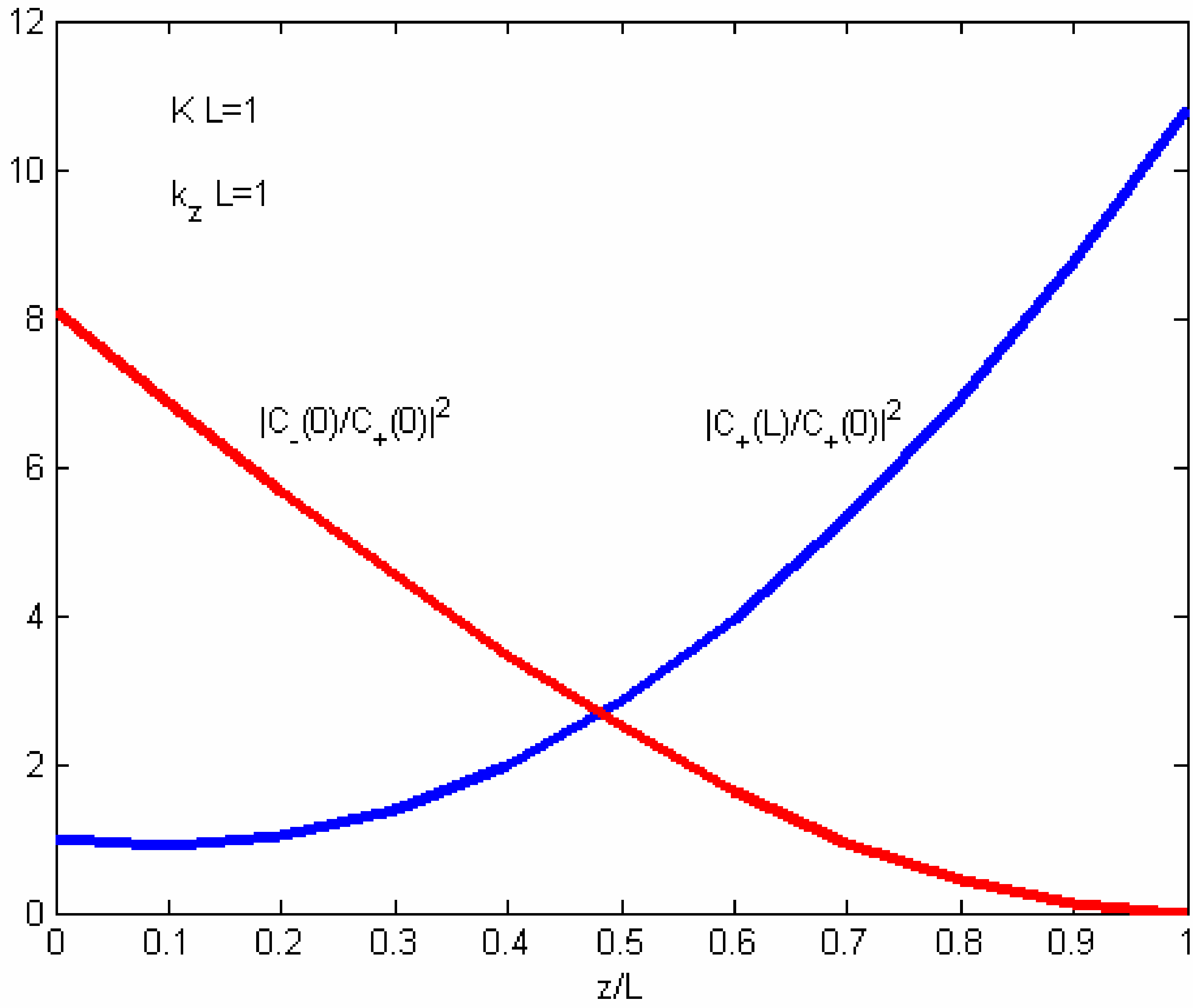
$$C_-(L) = 0$$

Incident and Reflected waves

$$\frac{C_+(z)}{C_+(0)} = \frac{(\kappa + jk_z)\sinh[S(L-z)] - S\cosh[S(L-z)]}{(\kappa + jk_z)\sinh(SL) - S\cosh(SL)} \cdot e^{-jk_z z}$$

$$\frac{C_-(z)}{C_+(0)} = \frac{-\kappa\sinh[S(L-z)]}{(\kappa + jk_z)\sinh(SL) - S\cosh(SL)} \cdot e^{+jk_z z}$$

$$S \equiv \sqrt{(\kappa + jk_z)^2 - \kappa^2}$$



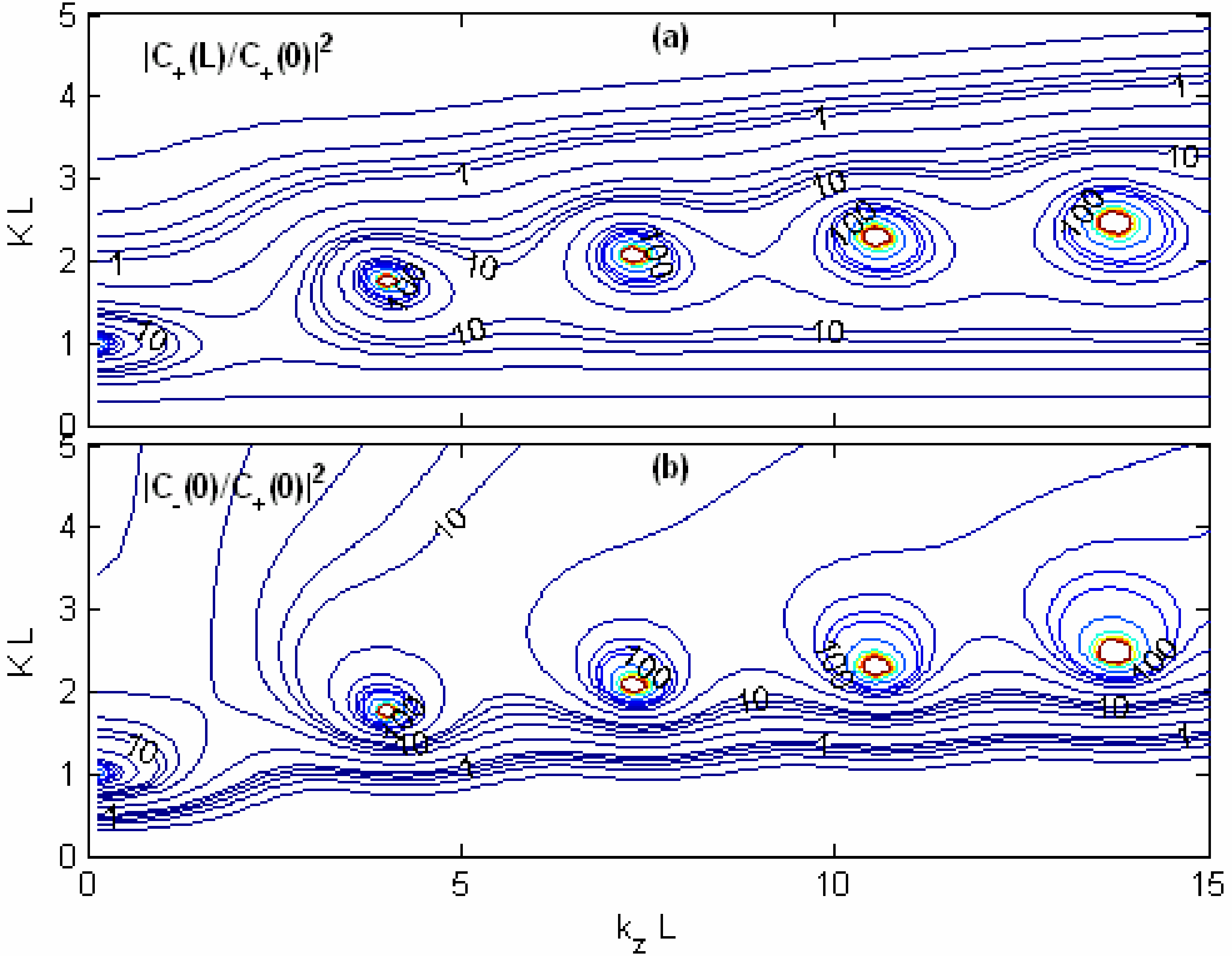
K L=1
k_z L=1
$|C_-(0)/C_+(0)|^2$
$|C_+(L)/C_+(0)|^2$
0
2
4
6
8
10
12
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
z/L

Transmission and Reflection gain

$$\frac{C_+(L)}{C_+(0)} = \frac{-SL}{(\kappa + jk_z)L\sinh(SL) - SL\cosh(SL)} \cdot e^{-jk_zL}$$

$$\frac{C_-(0)}{C_+(0)} = \frac{-\kappa L\sinh(SL)}{(\kappa + jk_z)L\sinh(SL) - SL\cosh(SL)}$$

$$\tanh(SL) = \frac{SL}{\kappa L + jk_zL}$$


(a)
$|C_+(L)/C_+(0)|^2$
(b)
$|C_-(0)/C_+(0)|^2$
K L
k_z L
0
5
10
15
1
2
3
4
10
70
100

High gain Free-Electron Laser

$$\frac{d^3}{dz^3}\begin{bmatrix} C_+(z) \\ C_-(z) \end{bmatrix} = \begin{bmatrix} +\kappa & +\kappa e^{-j2k_z z} \\ -\kappa e^{+j2k_z z} & -\kappa \end{bmatrix}\begin{bmatrix} C_+(z) \\ C_-(z) \end{bmatrix}$$

$$\kappa = j\frac{\varepsilon_0 \varsigma}{2N}\frac{\omega_p{}^2}{v_{z0}{}^2}(k_z + k_w)\iint f(x,y)\varepsilon_{pm}(x,y)\mathrm{V}_\perp{}^w \cdot \boldsymbol{\varepsilon}^*(x,y)dxdy$$

$$C_+(0)$$

$$C_+'(0) = C_+''(0) = 0$$

$$C_-(L) = C_-'(L) = C_-''(L) = 0$$

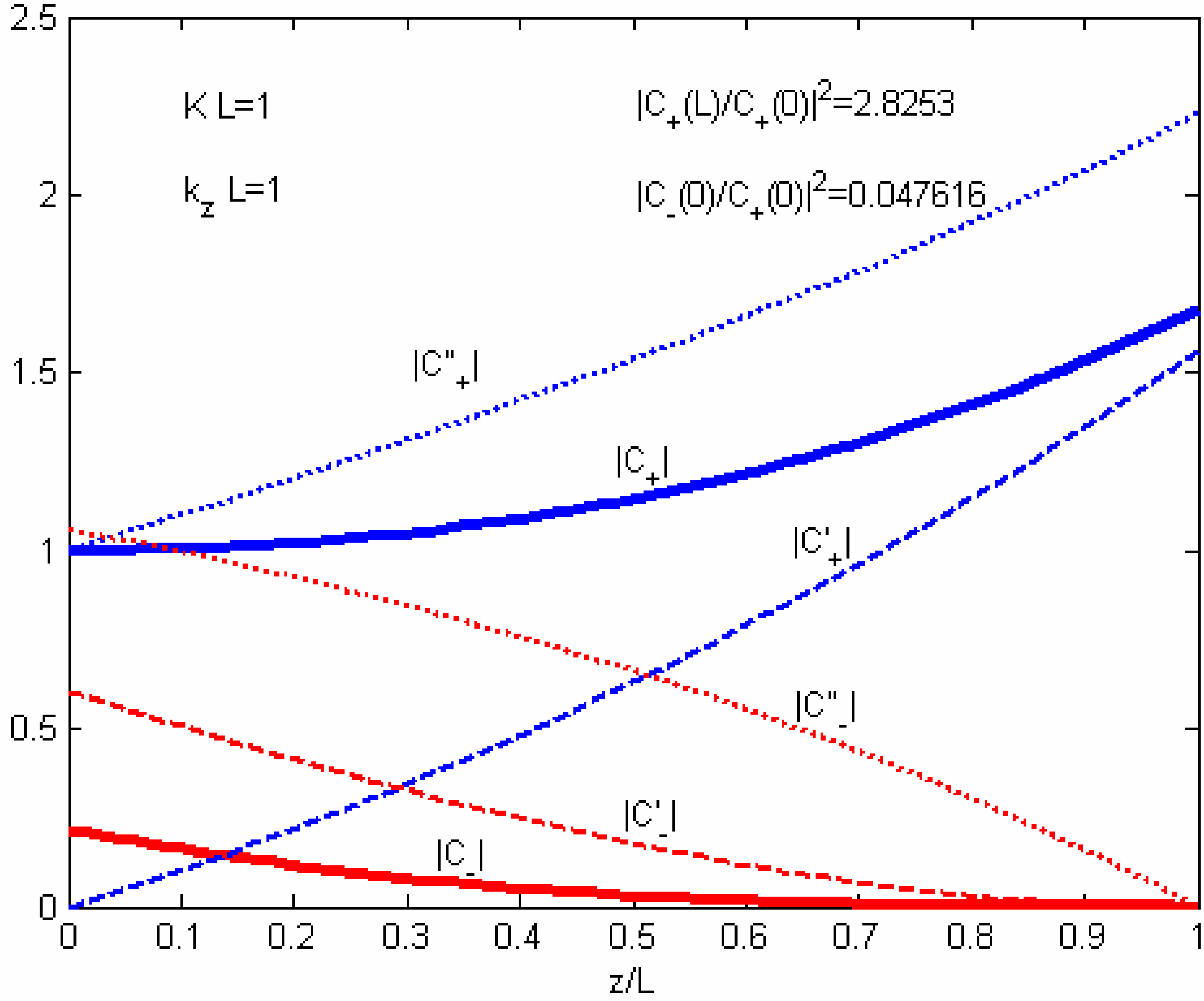
The analytical solution

$$\frac{d^3}{dz^3}C_-(z) = -e^{+j2k_z z}\,\frac{d^3}{dz^3}C_+(z)$$

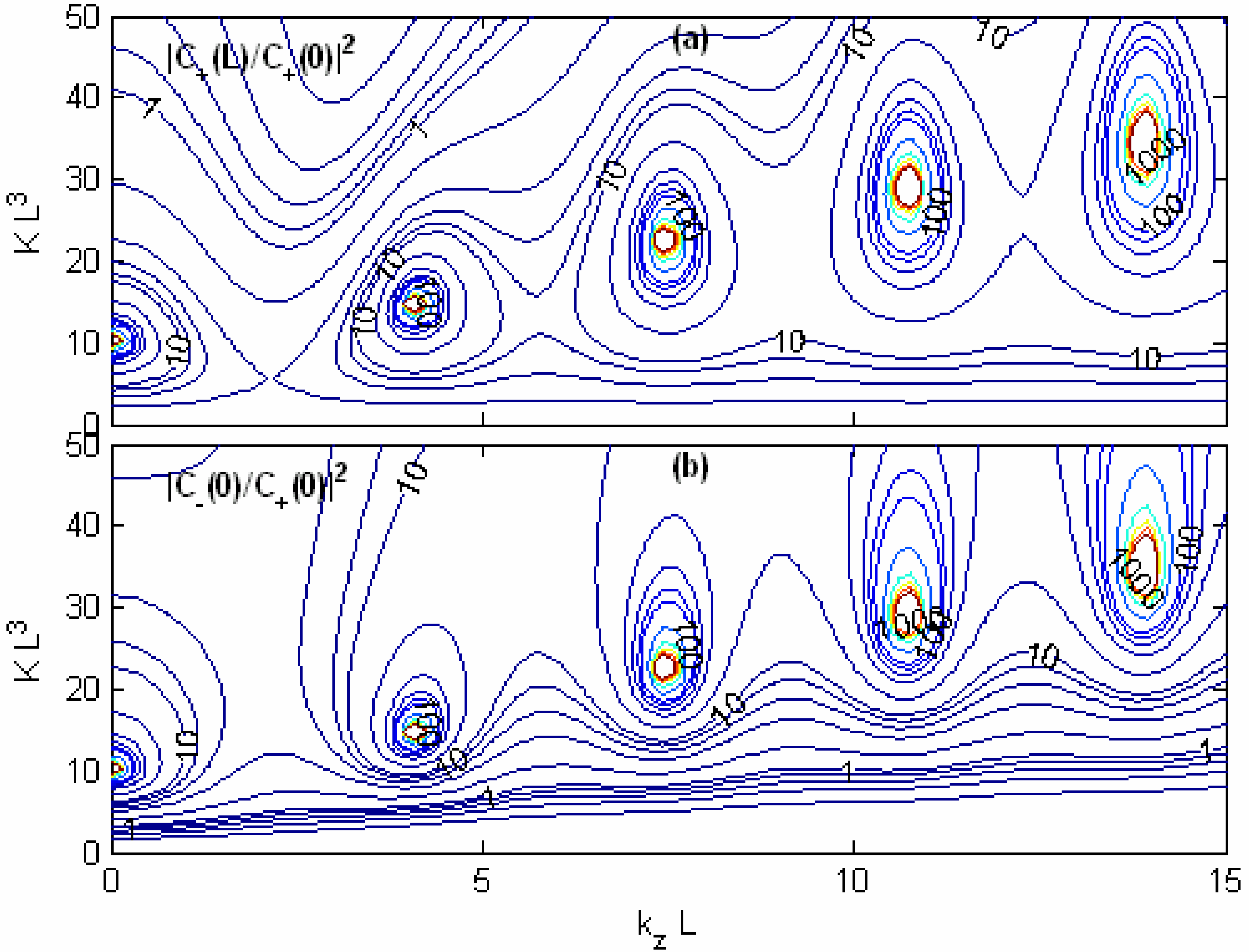
$$C_+(z) = e^{-jk_z z}\sum_{i=1}^{6} c_i \cdot e^{\lambda_i \cdot k_z z}$$

$$C_-(z) = e^{+jk_z z}\sum_{i=1}^{6} \frac{\left(j-\lambda_i\right)^3}{\left(j+\lambda_i\right)^3}\cdot c_i \cdot e^{\lambda_i \cdot k_z z}$$

$$\left(\lambda^2\right)^3 + 3\cdot\left(\lambda^2\right)^2 + 3\cdot\left(\lambda^2\right)\cdot\left(1 - j\frac{2\kappa}{k_z^{\,3}}\right) + \left(1 + j\frac{2\kappa}{k_z^{\,3}}\right) = 0$$



K L=1
k_z L=1
$|C_+(L)/C_+(0)|^2$=2.8253
$|C_-(0)/C_+(0)|^2$=0.047616
$|C''_+|$
$|C_+|$
$|C'_+|$
$|C''_-|$
$|C'_-|$
$|C_-|$
2.5
2
1.5
1
0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
z/L


(a)
$|C_+(L)/C_+(0)|^2$
(b)
$|C_-(0)/C_+(0)|^2$
$K L^3$
$k_z L$

Phase Matching

$$\theta_+ L = \left[\frac{\omega}{V_{z0}} - \left(+k_z + k_w \right) \right] \cdot L \qquad \theta\_L = \left[\frac{\omega}{V_{z0}} - \left(-k_z + k_w \right) \right] \cdot L$$

$$\left| \theta_+ L - \theta\_L \right| = 2 \cdot k_z(\omega) \cdot L < 2\pi$$

$$k_z(\omega) \cdot L = \frac{L}{c} \sqrt{\omega^2 - \omega_{co}{}^2} < \pi$$

Waveguide FEL