# NUMERICAL SIMULATION OF SPACE CHARGE COMPENSATION IN LOW ENERGY PROTON BEAMS

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#### Abstract

Self consistent beam dynamics calculations are described that account for space charge compensation in low energy, high intensity proton beams (100 mA, 100 keV) which propagate through a gaseous medium. For this purpose we have used a plasma description of the beam. Kinetics equations which govern the secondary particles behavior are derived in a 1D model. From their expressions we have looked into and discussed the existence of stationary solutions. We have also done numerical work to build up a solution where no assumptions are made on the thermalization of the created neutralizing electrons. The calculation technique consists of an explicit computation code using a PIC method. The diagnostics consist in snapshots in the phase space. They enable to identify the different steps of the space charge compensation mechanism. The energetic spectra of the particles reaching the wall have been obtained in order to make comparisons with experimental results.

## **1** INTRODUCTION

When an ion beam is extracted from a plasma chamber it propagates through a residual gas that comes mainly from the source. Ionization takes place inside the bulk of the beam due to collisions between the ions and the residual gas molecules. This creates a plasma which spreads around the beam. To understand how the beam propagates in such a medium we have to get a precise knowledge of the medium itself, our goal is then to modelize and simulate the behavior of the plasma.

In this paper we are interested in proton beams generated continuously. We seek for a stationary state of the plasma. The work presented here relates a modelization and a numerical resolution that account for the evolution of the plasma towards a stationary state and find out a necessary condition of existence of such a state. Diagnostics of this simulation can be compared to experimental measurements realized on the same kind of beam for the project APT at Los Alamos, USA [1] and at Francfort University for heavy ion beams [2]. This work is a part of theoretical and numerical researches which are to be applied to the project IPHI led at CEA-SACLAY, France.

### 2 SPACE CHARGE COMPENSATION

In the low energy section of a linear proton accelerator facility the residual gas is mostly hydrogen, it is at the room temperature T = 300 K and its pressure is around  $P = 10^{-5}$  torr. For the type of beam considered, with transverse section  $S = 1 \ cm^2$  the gas density  $n_g \simeq 3.10^{11} \ cm^{-3}$  is 300 times greater than the beam density  $n_b$ . The plasma created reaches a density of the order of  $n_b$  after a time short enough to neglect the decreasing of gas density due to ionization.

Ionization is one of the most important microscopic process between beam ions and gas molecules and it is the only binary interaction that we will take into account in our model. The rate of production of electrons and residual gas ions depends on its cross section,  $\sigma_i$ :

$$\frac{\partial n_{e,i}}{\partial t} = \sigma_i v_b n_b n_g = \frac{n_b}{\tau_i}$$

where  $n_e$ ,  $n_i$  denotes the electron and ion density,  $v_b$ is the beam velocity and  $\tau_i$  is called the characteristic time of ionization; after  $t = \tau_i$  as much pair of electron and ion have been produced as there are beam ions in a given volume. For a beam energy  $W = 100 \ keV$ ,  $\sigma_i = 2.10^{-16} \ cm^2$  and  $\tau_i \simeq 30 \ \mu s$ . On this time scale recombinations of ions and electrons can be neglected.

In the beam potential, residual gas ions fall freely to the walls of the accelerator where they are absorbed.

Electrons are trapped in this potential. Since the production rate of ions and electrons is the same the total electric charge in the beam starts decreasing as soon as the beam enters the gaseous medium.

The self-consistent potential will decrease with the charge density. The ions are accelerated by a lower potential and will reach the walls slower and slower, consequently their density will increase, as electron density will do. Some electrons produced with high kinetic energy will be able to escape from the potential well and reach the walls where they are absorbed.

This monotonic evolution will continue but will not reach a state of negative charge density. Such a state is unstable because all electrons would escape fastly from the beam core whereas the ions staid in the beam. We can define the neutralization rate as follows:

$$\zeta = 1 - \frac{\rho_{total}}{\rho_b} = 1 - \frac{\langle n_b \rangle + \langle n_i \rangle - \langle n_e \rangle}{\langle n_b \rangle}$$



Figure 1: Physical system.

where  $\rho_{total}$  and  $\rho_b$  are respectively the total and the beam charge densities,  $\langle n \rangle$  denotes the mean value of the particle density n in the beam. According to the previous description, during the plasma generation  $\zeta$ will tend to 100 %, this means that the space charge of the beam will be fully compensated or neutralized.

Experimental measurements have shown that  $\zeta$  reaches a value standing between 95 % and 99 % and remains stable. This observation leads us to look for stationary states of the plasma responsible for this partial space charge compensation. To achieve this we will modelize its evolution and find out the stationary limits of the model derived.

#### **3 MODELIZATION**

**Geometry.** Let us consider a cold, continuous and non divergent beam. We modelize the propagating medium as a longitudinally infinite canal of gas, with uniform density  $n_g$  confined transversally between two infinite plane walls distant of 2R. The beam thickness is  $2a \simeq 1 \ cm$ ; the beam is symmetric with respect to the symmetry plane of the canal (figure ??). The system is then longitudinally invariant, and we will derive a 1D transverse model.

**Kinetic description.** Both populations of ions and electrons are described with their distribution function  $f_i(t, x, v)$  and  $f_e(t, x, v)$  in the phase space (x, v), where x is the transverse position and v is the transverse velocity. We assume that the plasma is a non collisionnal medium. Under this assumption the distribution functions obey the Vlasov equations. Because of ionization there will be a source of particles inside the phase space, this will be modelized with a source term which will be the rate of production of particle per unit of phase space volume; thus the kinetic equations for each species will be written:

$$\frac{\partial f_e}{\partial t} + v \frac{\partial f_e}{\partial x} + \frac{e}{m_e} \frac{\partial \phi}{\partial x} \frac{\partial f_e}{\partial v} = \left(\frac{\partial f_e}{\partial t}\right)_{ioniz} \quad (1)$$

$$\frac{\partial f_i}{\partial t} + v \frac{\partial f_i}{\partial x} - \frac{e}{m_i} \frac{\partial \phi}{\partial x} \frac{\partial f_i}{\partial v} = \left(\frac{\partial f_i}{\partial t}\right)_{ioniz} \tag{2}$$

where e is the proton charge,  $m_e$ , (resp.  $m_i$ ) is the mass of one electron (resp. one ion), and  $\phi$  is the electric self-consistent potential. Since there is no particle emission on the walls, the limit conditions for the kinetic equations can be written as:

$$f_{e,i}(t, R, v) = 0, v < 0, \text{ and } f_{e,i}(t, -R, v) = 0, v > 0$$

Ionization rate in the phase space is obtained by weighting the ionization rate in the real space with the probability density of initial transverse velocity communicated to a particle immediatly after its birth. The source term then reads:

$$\left(\frac{\partial f_{e,i}}{\partial t}\right)_{ioniz} = \sigma_i n_g n_b v_b S_{e,i}(v)$$

where  $S_{e,i}(v)$  are these normalized probability densities. They can be computed from the measurement of Nagy and Vegh [4] or approximated with usual functions. We usually assume that an ion is emitted with a random velocity corresponding to the thermal motion of the molecules and that an electron receives a random kinetic energy of a few eVs.

Finally these two kinetic equations are coupled through the potential which obeys the Poisson equation:

$$-\frac{\partial^2 \phi}{\partial x^2} = \frac{e}{\varepsilon_0} (n_b(x) + \int f_i dv - \int f_e dv)$$

we impose  $\phi(0) = 0$  and the symmetry of the system involves  $\frac{\partial \phi}{\partial x}(0) = 0$  as a limit condition.

This mathematical formulation modelizes the evolution of the plasma from the initial time where it is taken in an arbitrary state represented by two initial distribution functions  $f_i^0$  and  $f_e^0$ . We are interested in the invariants of this model, they will represent the stationary states of the plasma.

### **4 NUMERICAL RESOLUTION**

**Principle.** We use a particle method to solve the model. At each time step macroparticles of each species are added to the particle lists to simulate ionization. They are moved in the self-consistent electric field by using a leap-frog scheme. The time step is fixed in such a way that the sampling of electron oscillation remains stable:

$$\Delta t = \frac{\Delta x}{VMAX}$$

where VMAX is the estimated value of the maximum velocity of electrons in the system. The simulation will then be run on a typical number of time step  $N_t = \frac{\tau_i}{\Delta t}$  in order to show a significant evolution. To limit the time of computation we will decrease  $\tau_i$  by imposing a higher gas pressure. We will also choose other beam parameters to decrease the beam potential so that VMAX which is reached by electrons accelerated in the uncompensated beam will not be too high.



Figure 2: A stationary state of the neutralizing plasma. The radius of the beam is delimited with dashed lines. The beam potential difference between the core and the edge is  $\Delta \phi \simeq 50V$ . Then the neutralization rate here is 80%.

**Necessary condition of existence of a stationary state.** To reach a stationary state it is necessary that electrons be emitted with a minimum kinetic energy:

$$G = \frac{1}{2}m_e v_{thresh}^2 > 0,$$

where  $v_{thresh} = inf_{\{v:S_e(v)>0\}}|v|$ . We call G the gap. In this case, once the potential barrier is smaller than G every emitted electron will be able to reach the wall and their density will stabilize. Otherwise some electrons will always be trapped in the potential well and their density will tend to infinity. The mathematical proof of this necessary condition has been obtained in [3] and is based on the resolution of the stationary Vlasov equation with the method of characteristic curves in phase space.

**Stationary State.** We have run the simulation with the following parameters: beam energy  $W = 200 \ keV$ , beam current  $I = 20 \ mA$ , gas pressure  $P = 2.10^{-3} torr$  and with  $S_e(v)$  choosen in such a way that  $G = 10 \ eV$ . The system tends to an equilibrium presented in figure **??**. There was no particles at the beginning of this simulation  $(f_i^0 = f_e^0 = 0)$ .

In the electronic phase space one can identify two different populations of electrons: the more numerous electrons are trapped in the potential well and fill the low energy levels, the others have just been emitted, they are slowed down until they reach the wall.

**Energetic spectra** Diagnostics of the distribution function versus kinetic energy have been obtained (fig-



Figure 3: Energetic spectra of ions and electrons

ure ??). As for ions this diagnostic is computed at the wall and it can be directly compared to the derivative of their current measured in a four grid analyser with respect to their energy ([1]).

As for electrons this curve is computed at the center and shows how trapped electrons are distributed in the low energy levels.

## 5 CONCLUSION

A simulation of space charge compensation evolution has been realized which enable to understand the neutralization and the trapping of electrons. A description of stationary states has been obtained.

Further works underway concern the investigation of physical reasons for stationary states in three different ways. The resolution of a cylindrical model will give information on the real importance of low energy electrons. A computation of longitudinal electron fluxes within a 2D model will permit to get a more precise idea of the distribution of secondary particles along the beam. At last a collisionnal model should explain the energy transfer that enables electrons to reach the wall.

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