# QUALITY FACTOR MEASUREMENTS IN CAVITIES WITH MODE OVERLAP\*

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#### Abstract

A new method of measuring quality factors in cavities is presented. This method is well suited to measure quality factors in undamped cavities as well as in heavily damped cavities, and in addition this method provides a possibility of separating modes and measuring quality factors especially in cases of overlapping modes. Measurements have been carried out on HOM-damped cavities for the DESY/THD linear collider project. Results are presented.

#### 1. INTRODUCTION

For use in future linear colliders, normal- and superconducting iris structures are proposed, operating at L-, S-, X-, or K-band. In all proposed schemes, wake field effects play an important role [e.g. 1,2,3]. According to the Panofsky theorem [4] they are mainly due to HEM<sub>11</sub>-modes (Hybrid-Electric-Magnetic-modes). To reduce their beam perturbing influence HOM-couplers are required to couple those modes into loads. The effectiveness of such a damping system has to be judged by measuring transversal shuntimpedances and Q-values of the higher order modes in damped cavities [5]. Up to now we have derived the O-values from a comparision of the square of the electrical field strength E in the undamped (index u) and the damped (index d) resonator (for measurements in the undamped case, the damping system was removed from the resonator) using the following expressions [6]:

$$Q_{1} = \frac{Q_{0}}{1+k} = \frac{E_{d}^{2}}{E_{u}^{2}}Q_{0}$$
(1)

Here k is the coupling coefficient which is defined by the ratio of power delivered to the damper and the power dissipated in the resonator.  $Q_L$  is the quality factor if the damper is attached and  $Q_0$  is the quality factor without any damper. For measuring the square of field strength in heavily damped cavities we have used a nonresonant perturbation technique since the field strength is very low. Equation (1) holds only under the premise, that the field distribution of the mode remains unchanged. But this is usually not fulfilled in heavily damped cavities. We have shown by experiment, that for symmetric damping systems always the largest number of the ratio  $E_u^2/E_d^2$  gives the correct coupling factor k which has to be used in (1) to calculate the correct loaded quality factor  $Q_L$ . Thus, to judge the effectiveness of a

HOM-damping system one has to measure the ratio  $E_u^2/E_d^2$  at many different locations in the cavity. From field measurements one is able to calculate the ratio of transversal shuntimpedances which is important to investigate the transversal particle motion inside the cavity and, as mentioned above, one has to look for the maximum number of the ratio  $R_1^u/R_1^d$  to calculate the loaded  $Q_L$ . This procedure fails in cases where mode overlap occurs. We shall show that also in these cases a proper measurement of Q-values is possible by using a new simple technique, which again is based on nonresonant perturbation theory which we have used for measurements in the past [6, 7, 8].

#### 2. THEORY

Nonresonant perturbation techniques allow the measurement of fields both electric and magnetic in an arbitrary cavity, by observing the change of the complex reflection coefficient  $\Gamma$  at the input port while a bead is pulled through the cavity. It has to be emphasized that no resonance is required. If the bead is assumed to be of arbitrary size and material one finds

$$2P_{inc}(\Gamma - \Gamma_{c}) = \mathbf{i}(\omega - \omega_{0}) \int_{V} (\mu_{0}\mathbf{H} \cdot \mathbf{H}_{0} - \varepsilon_{0}\mathbf{E} \cdot \mathbf{E}_{0}) dV$$
$$- \mathbf{i}\omega \int_{V_{B}} (\mathbf{H}_{0} \cdot \mathbf{M} - \mathbf{E}_{0} \cdot \mathbf{P}) dV$$
(2)

where 0 denotes the unperturbed case and  $P_{inc}$  is the Power arriving at the input port. **P** and **M** are the vectors of electric and magnetic polarization respectively. The first integral has to be taken over the volume of the cavity, the second integral comprises the volume of the bead. In the case of lossy walls or any lossy material inside the cavity, V means the volume which is bounded by the surface of zero electric- and magnetic field strength. If the bead is assumed to be small and of isotropic material (2) becomes

$$2\mathbf{P}_{inc}(\mathbf{\Gamma}-\mathbf{\Gamma}_0) = -i\omega \left(\varepsilon_0 \alpha_c \mathbf{E}_0^2 - \mu_0 \alpha_m \mathbf{H}_0^2\right)$$
(3)

 $\alpha_e$  and  $\alpha_m$  are form factors depending on the shape and material of the bead. In accelerator physics one usually is interested in measuring electric fields only. Thus we have

$$|\mathbf{E}_0|^2 = \mathbf{i} \frac{2P_{\text{inc}}\Delta\Gamma}{\omega\epsilon_0\alpha_e} \quad , \quad \Delta\Gamma = (\Gamma - \Gamma_0) \tag{4}$$

In the case of a resonance with a very small Q-value in the order of 10 or 20 (say), one measures a typical resonance curve if  $|\Delta\Gamma|$  is scanned over the frequency region of interest (fixed position of the bead). If we describe the resonance of

<sup>\*</sup> work supported by DESY

the cavity simply by using a lumped parallel circuit, we find the following expression for the electric Field.

$$\left|\mathbf{E}_{0}(\boldsymbol{\omega})\right|^{2} = \frac{\left|\mathbf{E}_{0}\right|_{\max}^{2}}{1 + Q^{2}\left(\frac{\boldsymbol{\omega}}{\boldsymbol{\omega}_{0}} - \frac{\boldsymbol{\omega}_{0}}{\boldsymbol{\omega}}\right)^{2}} = \frac{2P_{inc}}{\varepsilon_{0}\alpha_{e}} \frac{\frac{\left|\Delta\Pi\right|_{\max}}{\boldsymbol{\omega}_{0}}}{1 + Q^{2}\left(\frac{\boldsymbol{\omega}}{\boldsymbol{\omega}_{0}} - \frac{\boldsymbol{\omega}_{0}}{\boldsymbol{\omega}}\right)^{2}} \quad (5)$$

The index max denotes the values at maximum of the resonance curve. The quality factor Q is here defined by

$$Q = \omega_0 \frac{U}{W} = \omega_0 C \frac{R_\perp}{2}$$
 (6)

U is the total energy stored in the cavity at resonance, W is the average power dissipated in the resistor  $R = R_{\perp}/2$  and  $R_{\perp}$  is the transversal Shuntimpedance of the higher order mode of interest. In the case of an usual accelerating mode  $R_{\perp}$  has to be replaced by the longitudinal shuntimpedance. The important feature of equation (5) is that one is able to measure the Q-value simply by inserting an appropriate dielectric bead at an arbitrary position in the cavity and measuring  $|\Delta\Gamma|$  at different frequencies to obtain the resonance curve. From the resonance curve one finds the Q-value with the aid of the half maximum frequencies using

$$Q\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = 1 \tag{7}$$

What can be done to determine the Q-values and shuntimpedances of overlapping modes inside a multicell accelerator cavity? One way to separate this modes is to couple selectively to a mode. If this is possible one may insert the bead at any position inside the cavity and observe the resonance curve under the condition that there is sufficient field strength. A measurement of shuntimpedance now provides the correct value for this mode.

If this is not possible the second way of separating these modes is to insert the bead at a position within the cavity, where the mode of interest has field strength but the other has not. In this case one measures the correct value of Q but a measurement of shuntimpedance must fail since the field strength along a path parallel to the resonator axis is a summation of all fields present.

#### **3. EXPERIMENTAL RESULTS**

We shall demonstrate the applicability of the measurement technique in the case of a three-cell structure equipped with a strong damping system which is attached to the middle cell (Fig. 1).

The three-cell structure has a cell-geometry identical to the one which was chosen for the DESY/THD-collider prototype. This structure is the simplest one that shows mode overlap in the first dipole passband when being HOM-damped. With the parameters chosen the structure has a frequency of 3 GHz for the  $TM_{01}$ - $2\pi/3$ -acceleration mode. First we measured the transversal shuntimpedances and quality factors for the first three dipole modes (Table 1) of the undamped system. Afterwards we mounted the damping system and tuned the structure such that the field pattern and of course the frequency of the acceleration mode was restored.

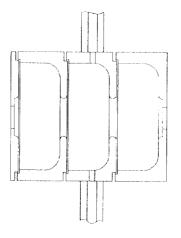


Fig. 1 The three cell structure with the dampers attached to the middle cell. This sketch has the cell geometry chosen for the DESY/THD linear collider project.

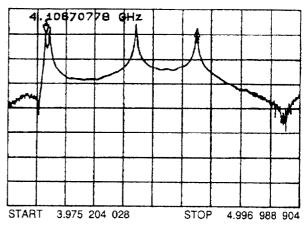


Fig. 2 Spectrum of the undamped 3-cell cavity. Modes from left to right:  $\text{HEM}_{11}$ - $2\pi/3$ ,  $\text{HEM}_{11}$ - $\pi/3$ ,  $\text{HEM}_{11}$ -0. The fourth mode belongs to a different passband. (20dB/div. scale).

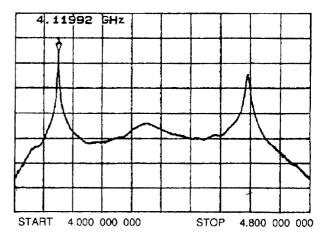


Fig. 3 Mode spectrum of the damped 3-cell cavity. Modes from left to right:  $\text{HEM}_{11}$ - $\pi/3$  and  $\text{HEM}_{11}$ -0 mode. The  $2\pi/3$ -mode has disappeared. Again the third mode belongs to a different passband. (10dB/div. scale).

We observed, that the HEM<sub>11</sub>- $\pi/3$ -mode and the HEM<sub>11</sub>- $2\pi/3$ -mode completely overlapped since the

In this situation commonly used methods for measuring Q must fail, because there is no way to separate the signals of both modes. But due to the fact that the HEM<sub>11</sub>- $\pi/3$ -mode does not couple to the damping cell one can hope to separate the HEM<sub>11</sub>- $2\pi/3$ -mode by coupling to the waveguide (Fig. 4A). But in this particular case a conventional 3 dB measurement would not work because of the weak signal observed.

Now we inserted a bead into the cavity and observed the  $|\Delta\Gamma|$  curve (Fig. 4B).

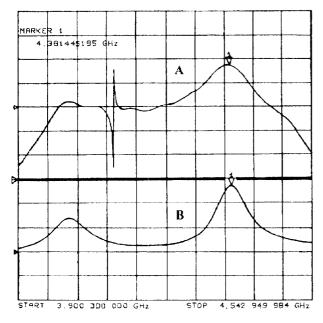


Fig. 4 (A) mode spectrum of the 3-cell structure, coupling through damper waveguide. From left to right:  $\text{HEM}_{11}$ - $2\pi/3$ ,  $\text{HEM}_{11}$ - $\pi/3$ , and  $\text{HEM}_{11}$ -0 mode. (5dB/div.). (B)  $|\Delta\Gamma|$  vs frequency. Left:  $\text{HEM}_{11}$ - $2\pi/3$ , right:  $\text{HEM}_{11}$ -0 mode. No coupling to  $\text{HEM}_{11}$ - $\pi/3$  mode observed. (5mU/div.).

### Table 1

Measured values for the undamped three cell structure. The value for the  $\text{HEM}_{11}$ -0-mode is compared to MAFIA calculations.

Mode	f [GHz]	$Q_0$	$r_{\perp}/Q_0 [k\Omega/m]$
HEM <sub>11</sub> -2 $\pi/3$	4.106365	3770	0.8
HEM <sub>11</sub> - $\pi/3$	4.121823	3030	0.6
HEM <sub>11</sub> -0	4.425976	2650	1.1
HEM <sub>11</sub> -0 mafia	4.413187	9500	1.0

Using (7) we observed the Q values given Table 2. In addition the height of the curve is a measure of the absolute value of  $E^2$  and thus for the shuntimpedance in the case of known coupling factors of the feeding antennas. If there is no one has not the chance to separate the modes by coupling selectively one still has a chance to determine the Q of the mode by inserting a bead in a position where the disturbing mode has no field or little field compared to the field of the

mode under consideration. Of course in this case one looses the information of shuntimpedance.

Table 2

Measured values for the damped three cell structure.

Mode	f [GHz]	Qo	$r_{\perp}/Q_0 [k\Omega/m]$
HEM <sub>11</sub> -2π/3	4.013031	39	0.3
HEM <sub>11</sub> -π/3	4.120056	4200	0.5
HEM <sub>11</sub> -0	4.359966	50	1.1

## 4. CONCLUSION

The HOM damper development for linear colliders has to be performed on long iris structures to judge the effectiveness of the damping system. But especially long structures show extensive mode overlap. In many cases it is possible to couple selectively to the modes under consideration. But even then the achievable coupling strength may be too weak for a precise measurement. So it is of importance to have a method allowing to measure the Q-values of the dipole modes when all other methods are no longer applicable. The bead used does not need to be calibrated, it can be inserted at any appropriate position. In addition it should be mentioned that this new method is not limited to low Q-values. Of course it is also possible to use it for measuring high Q resonances.

#### 5. REFERENCES

- BALEWSKY, K. et al., DESY and TH-Darmstadt, Status Report of a 500 GeV S-Band Linear Collider Study, DESY 91-153, D-22603 Hamburg, Notkestraße 85, pp. 1-39, 1991
- [2] RUTH, R., The Status of NLC, 3rd Int. Workshop on Linear Colliders, LC 91, vol. 1, pp. 141-175, Protvino 1991
- [3] JOHNSEN, K. et al., Report of the Advisory Panel on the Prospects e<sup>+</sup>e<sup>-</sup> Colliders in the TeV Range, CERN 87-12, May 1987
- [4] PANOFSKY, W. K. H., and WENZEL, W. A.: 'Some considerations concerning the transverse deflection of charged particles in radio-frequency fields', Rev. Sci. Instrum., 1956, 27, p. 967
- [5] HELM, R.H. and LOEW, G.A.: 'Linear Accelerators', eds. P.M. Lapostolle & A.L. Septier, North Holland, Amsterdam 1970
- [6] STEELE, C.W.: 'A Nonresonant Perturbation Theory', IEEE Trans. Theory and Technology, MTT-14, (2), pp.70-74, 1966
- [7] HÜLSMANN, P., KURZ, M. and KLEIN, H.: 'Experimental Determination of Field Strength and Quality Factor of Heavily Damped Accelerator Cavities', Electronics Letters, vol. 27, No. 19, pp. 1727-1729, September 12, 1991
- [8] HÜLSMANN, P., KURZ, M. GLOCK, H.-W. and KLEIN, H.: 'Determination of Field Strength and Quality Factor of Heavily HOM Damped Accelerator Cavities', Proceedings of the 1992 third European Particle Accelerator Conference, March 24-28, Berlin, vol. 2, pp. 1307-1309