The Effect of Various Particle Distribution Functions on the Modeling of Space Charge Effects in a High-Order Optics Code*

David L. Bruhwiler and Michael F. Reusch
Northrop Grumman Research and Development Center
4 Independence Way, Princeton NJ 08540-6620 USA

Abstract

TOPKARK is a beam optics program consisting of two Fortran codes developed in parallel: a high-order mapping code and a particle tracking code. Both codes can model the spatial distribution function as either a uniformly-filled or a Gaussian 3-D ellipsoid for the purpose of calculating space charge effects. We emphasize the tracking code, which can also model the spatial distribution function of a continuous beam as either a uniformly-filled or a Gaussian cylinder of elliptical cross-section. Furthermore, the tracking code employs more general space charge models, in both 2-D and 3-D, which assume only ellipsoidal symmetry of the spatial distribution. The evolution of initial particle distributions under the influence of various space charge models is considered in detail for beam lines in which high-order optics and space charge effects both play significant dynamical roles.

1. OVERVIEW OF THE TOPKARK CODE

TOPKARK has evolved from an earlier code, which was developed during a collaboration between Grumman, LBL and BNL [1]. The mapping version is a useful design tool, while the tracking version is useful both as a diagnostic which resolves any ambiguities regarding *very* high order effects and as a design tool itself. In addition, the tracking version incorporates more general space charge models. We discuss only the tracking version in this paper.

1.1 General Features of the Code

In the zero-current limit, TOPKARK integrates the full equations of motion, using a Hamiltonian formalism and an explicitly symplectic 4th-order integration scheme [2]. The code can generate a 6-D phase space ellipsoid of initial conditions, which yields the desired Twiss parameters in each of the 2-D phase planes. Distributions currently supported include a) uniformly-filled ellipsoid in space with Gaussian distribution in momentum and b) Gaussian distribution in space and momentum. The code can also read in a file of initial conditions for tracking.

TOPKARK currently implements a number of "hard edge" or uniform-field magnet elements, including a dipole (with arbitrary entrance and exit angles) and quadrupole through duodecapole. Also available are "thin fringe" elements for dipole and quadrupole magnets. RMS Twiss parameters and current particle coordinates can be output as desired, and the RMS or FWHM beam envelopes can be output throughout a simulation for subsequent plotting.

* This work supported by Oakridge National Laboratory Contract L/SC15X-SN293C and by Northrop Grumman Corporation.

The code uses MKS units, with all momenta normalized to the longitudinal design momentum p_0. When space charge effects are included, the Hamiltonian formulation and symplectic integration schemes are abandoned in favor of simple and direct integration of the equations of motion with a 4th-order adaptive-step-size Runge-Kutta integrator. The detailed formulation of the equations of motion for straight and bending elements have been given elsewhere [3].

1.2 Space Charge Models

We consider only models with ellipsoidal symmetry, meaning that the spatial density distribution has the form

$$\rho(x, y, \delta z) = \rho_0 f(u) , \qquad (1a)$$

where the function $u_s(x,y,\delta z; s)$ is defined by the equation

$$u_s^2(x,y,\delta z; s) \equiv \frac{x^2}{a^2+s} + \frac{y^2}{b^2+s} + \frac{\delta z^2}{c^2+s} , \qquad (1b)$$

with $a^2=<x^2>$, etc., and $u \equiv u_s(x,y,\delta z; s=0)$ The family of 3-D ellipsoids defined by Eq. (1b) are isodensity contours.

Such models yield electric fields of the following form (for all points *within* the distribution) [4,5,6]:

$$E_x = \frac{q\rho_0}{2\varepsilon_0} abc\, x \int_0^\infty ds \frac{f[u_s(x,y,\delta z; s)]}{(a^2+s)^{3/2}(b^2+s)^{1/2}(c^2+s)^{1/2}} , \qquad (2)$$

with analogous results for E_y and E_z. These electric fields are calculated in the *bunch* frame, then relativistically transformed to the laboratory frame [3].

Three distinct space charge models have been implemented in the tracking version of TOPKARK. The first assumes a uniformly-filled ellipsoid in space, for which $f(u)=1$, while the second assumes a gaussian ellipsoid in space, for which $f(u)=\exp(-u^2/2)$. The third is a more general scheme developed by Garnett and Wangler [6] in which f(u) is Fourier expanded.

For particles within the bounds of the assumed 3-D uniformly-filled ellipsoid, the purely linear space charge forces can be found analytically in terms of complete elliptic integrals [4]. In the Gaussian model, which has been used previously [5], the integrals cannot be evaluated in closed form. In this model, the space charge forces will have strong nonlinear components. For the more general scheme of Garnett and Wangler [6], f(u) is left arbitrary. Here, one must Fourier expand f(u), obtaining the expansion coefficients directly from the particle positions.

2. HIGH-ORDER OPTICS WITH SPACE CHARGE

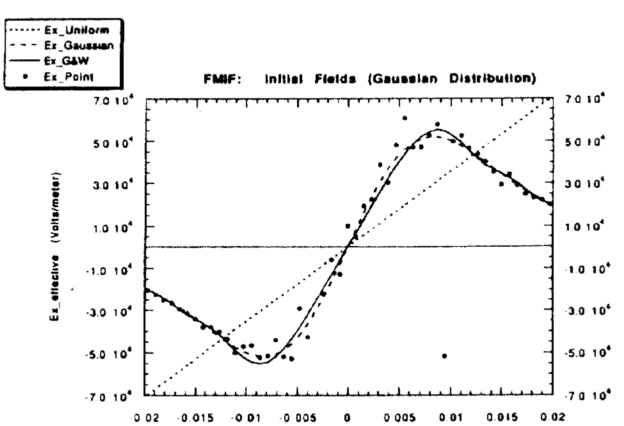
As an example, we consider the proposed Fusion Materials Irradiation Facility (FMIF), a D-Li (D^+ beam / flowing Li target) system that seeks to mimic the 14 MeV D-T fusion neutron spectrum. Similarly, Accelerator Transmutation of Waste is an example of various large scale accelerators, generically denoted as AXY, which function as neutron spallation sources. These applications require high current and high-order beam manipulation to generate the desired beam uniformity profile and shape on target. A code like TOPKARK, which combines high-order optics with space charge models, is required for the design of such systems.

In Table 1, we present a preliminary accelerator lattice for both the zero-current limit and the proposed 125 mA design. The quadrupole field strengths are given in T/m, the octupole field strengths in T/m^3, and the duodecapole field strengths in T/m^5. Given a beam pipe radius of 5 cm, the maximum pole tip field for the quadrupoles is ~0.6 T, and ~0.06 T for the higher multipoles. Table 1 shows that the lattice parameters must be significantly altered in order to accomodate the effects of space charge; thus, space charge plays a significant dynamical role in this nonlinear optics design.

Table 1
Preliminary Lattice Parameters for Zero-Current and 125 mA Designs of FMIF Final Focus

	Element Type	Length	Field Strength		
			0 mA Design	125 mA Design	
#1	quadrupole	20.0 cm	-9.978730	-10.16958	
#2	drift	40.0 cm	-.-	-.-	
#3	combined-function	20.0 cm	11.41190	11.65657	(quadrupole)
			-323.8069	150.0	(octupole)
			28000.0	-30000.0	(duodecapole)
#4	drift	40.0 cm	-.-	-.-	
#5	quadrupole	10.0 cm	1.210256	0.9161311	
#6	drift	40.0 cm	-.-	-.-	
#7	combined-function	20.0 cm	-11.52224	-11.72983	(quadrupole)
			511.4759	300.0	(octupole)
			95000.0	200000.0	(duodecapole)
#8	drift	40.0 cm	-.-	-.-	
#9	quadrupole	20.0 cm	9.420227	10.44962	
#10	drift	40.0 cm	-.-	-.-	
#11	quadrupole	20.0 cm	3.039918	0.0	
#12	drift	1690.0 cm	-.-	-.-	

In Fig. 1, we show E_X due to space charge for the initial and final particle distributions from our 125 mA simulation. The dotted line shows the fields calculated by the uniformly-filled ellipsoid model, the dashed line those of the Gaussian ellipsoid model, the solid line those of the Garnett & Wangler model (with 11 Fourier modes), and the dots those of a simple coulomb model. The coulomb model is too slow and noisy for simulations, but is used as an impartial arbiter here. These plots show that the space charge fields are initially Gaussian and finally linear, so neither the uniformly-filled ellipsoid nor the Gaussian ellipsoid models are adequate. More importantly, third-order aberrations due to space charge are important in this application. The uniformly-filled ellipsoid model, with its explicitly linear fields, would miss this effect entirely, while the Gaussian model would exagerrate the aberrations. A more general model like that of Garnett & Wangler is required.



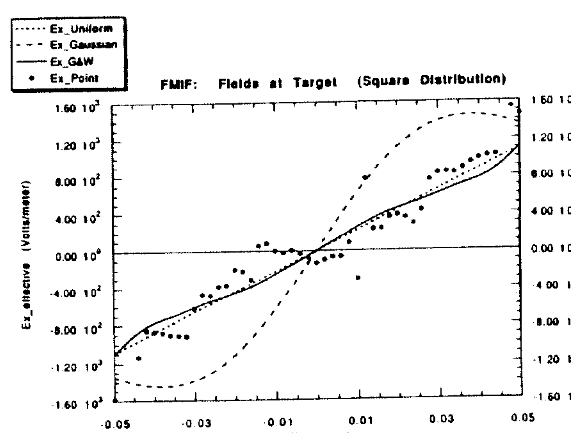


Figure 1. Effective Space Charge Field E_X in V/m for Initial and Final Particle Distributions.

In Fig. 2, we show puncture plots of the final beam. The zero-current simulations are above, while the 125 mA results are below. The x-y plots show how the distribution has been made square and roughly uniform through the use of high-order multipoles. A combined-function octupole/duodecapole magnet is required for both x and y: these are placed at extreme beam waists in order to minimize coupling.

The upper right plot in Fig. 2 shows how the x-Px phase space has been folded over by the high-order multipoles in order to obtain the requisite square/uniform beam. This sort of design has been discussed previously [7,8]. The lower right plot shows the results obtained with space charge. Here, third-order space charge aberrations, which greatly increase the beam emittance, largely did our work for us by yielding a nearly-uniform distribution: the high-order magnets were only used to square off the beam slightly. This is an example where both high-order optics and space charge are dynamically important and must be treated with some accuracy.

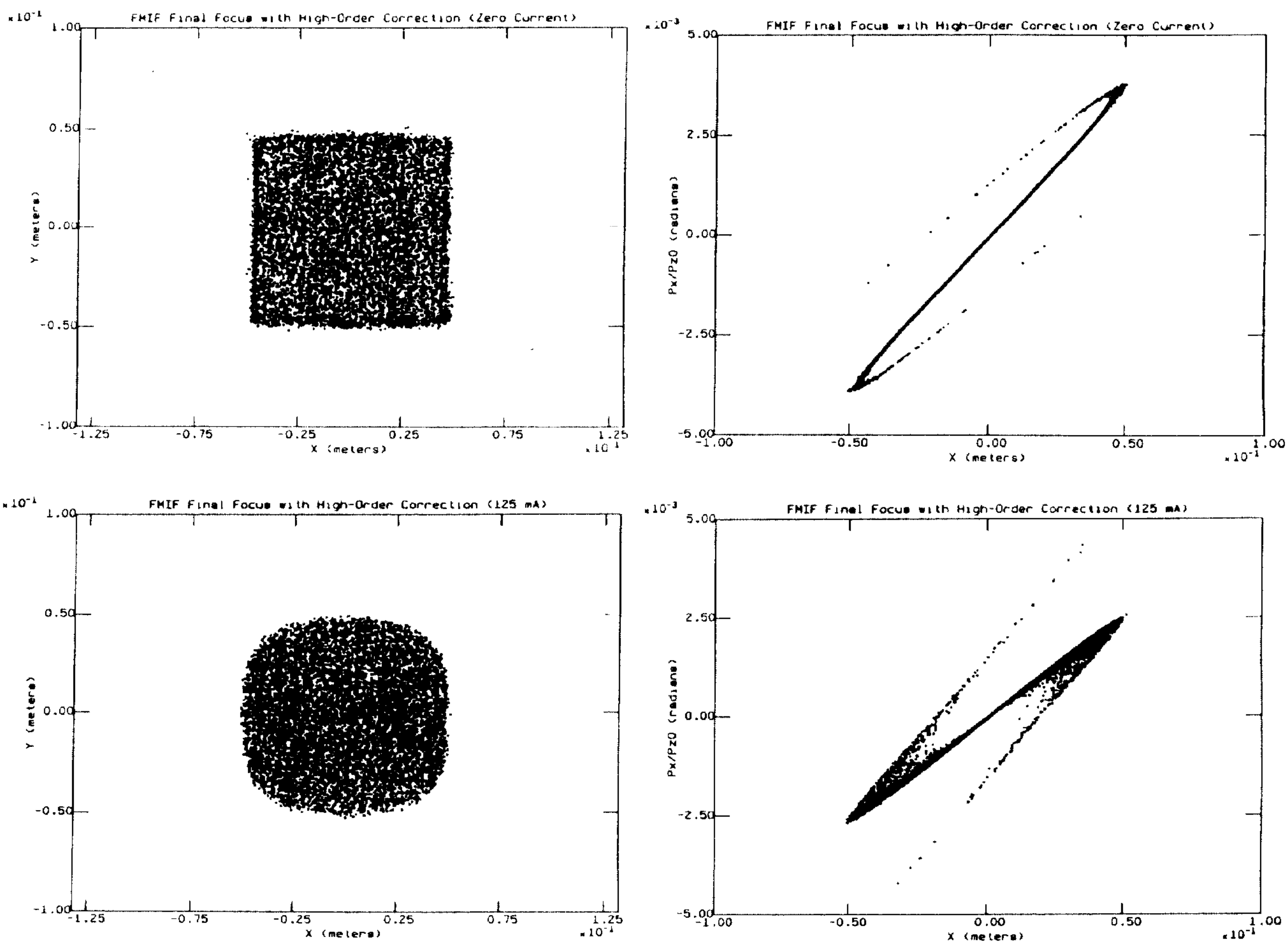


Figure 2. Puncture Plots of the Final Beam: Zero-Current limit shown above; 125 mA case shown below.

3. CONCLUSIONS

Large scale accelerators for neutron spallation sources, such as FMIF and AXY, represent an application with demanding design requirements. The final focus for such devices requires correct treatment of both high-order optics and space charge.

4. REFERENCES

[1] M. Reusch, E. Forest and J. Murphy, "Particle Tracking and Map Analysis for Compact Storage Rings", in IEEE Part. Accel. Conf. Proc., San Francisco, U.S.A., May 1991, pp. 1651-1653.

[2] E. Forest and R. Ruth, Physica D, vol. 43, p. 105, 1990. H. Yoshida, "Construction of Higher Order Symplectic Integrators", Phys. Lett. A, vol. 150, pp. 262-8 Nov., 1990.

[3] D. L. Bruhwiler and M. F. Reusch, "High-Order Optics with Space Charge: The TOPKARK Code", in AIP Conf. Proc. 297, R. Ryne, ed., New York: AIP, 1994, pp. 524-531.

[4] O. Kellogg, Foundations of Potential Theory, New York: Dover Publications, 1954, pp. 192-196.

[5] M. Martini and M. Promé, Part. Accel., vol. 2, p. 289, 1971.

[6] R. Garnett and T. Wangler, "Space-Charge Calculation for Bunched Beams with 3-D Ellipsoidal Symmetry", in IEEE Part. Accel. Conf. Proc., San Francisco, U.S.A., May 1991, pp. 330-332.

[7] B. Blind, "Production of uniform and well-confined beams by nonlinear optics", Nucl. Instr. and Meth. B, vol. 56/57, pp. 1099-1102, 1991.

[8] Y. K. Batygin, "Beam intensity redistribution in a nonlinear optics channel", Nucl. Instr. and Meth. B, vol. 79, pp. 770-772, 1993.