Analysis of the Effects of Superconducting Wiggler on Beam Dynamics in Storage Ring*

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Abstract

Superconducting wiggler is planned to be installed in the Pohang Light Source (PLS) storage ring. Its strong magnetic field is likely to disturb the motion of the circulating electron beam. Even though the wiggler field shape does not allow the analytic approach, a computer-dependent semi-analytic method can be adopted. Due to rectangular geometry of the wiggler, the horizontal focusing vanishes leaving only the vertical focusing. The linear and nonlinear effects are classified and discussed. Results indicate that nonlinearities of the superconducting wiggler are not as big as one might expect. Therefore the dynamic aperture is still big enough for operation. Finally the modification of the electron beam parameters is presented.

1. INTRODUCTION

The Pohang Light Source is the third generation synchrotron light source of 2 GeV energy. In order to give users a variety of wavelength photons, a few insertion devices will be installed in PLS storage ring. Among the insertion devices, the most interesting is 7.5 Tesla, one period superconducting wiggler (or wavelength shifter) designed by Institute for Nuclear Physics, Novosibirsk [1,2]. Main parameters of this superconducting wiggler is given in Table 1 Due to the strong peak magnetic field of this wiggler, it is expected to disturb the electron beam motion of the storage ring. The effects of the wiggler can be divided into two parts, the betatron motion of an electron, and properties of the circulating beam. The effects on the betatron motion can be further divided to linear optics change, and non-linear effects which have impacts on the dynamic aperture. The linear optics change can be almost repaired by adjusting quadrupoles adjacent to the superconducting wiggler. The non-linear effects of the wiggler on the betatron motion have to be evaluated numerically. Electron beam properties such as natural beam emittance and energy spread are also affected by the wiggler.

2. EFFECTS ON BETATRON MOTION

The field shape of superconducting wiggler is shown in Fig. 1. This magnetic field deflects 2 GeV electrons in the horizontal plane by the amount of $\pm 2°$ approximately. Even though this magnetic field is not possible to express analytically, we can obtain much information by numerical analysis if we concentrate on the integrated effects

Table 1 Main parameters of the wiggler

Peak field on beam axis (T)	7.5
Length (m)	0.9
Photon critical energy (keV)	19.95
Total radiation power (kW)	2.9

over the wiggler period. We will use (x_0, s_0, y_0) to denote right-handed outside coordinates with s_0-axis pointing the direction of the wiggler's longitudinal axis. On the other hand, (x, s, y) will denote the local coordinates moving with electrons on the trajectory. Due to the planar and rectangular pole faces of the wiggler, $y = y_0$ holds. The wiggler fields, $B(x_0, s_0, y)$ and $B(x, s, y)$ are same fields expressed in terms of the two different coordinates. Since the transversal betatron motion of electrons are described by local coordinates, we have to transfer from $B(x_0, s_0, y)$ to $B(x, s, y)$ to analyze betatron motion.

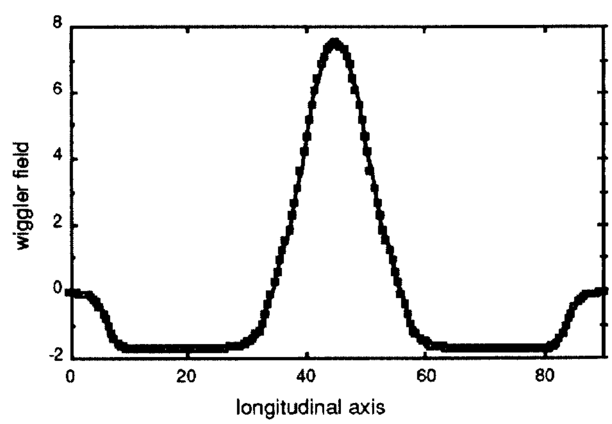

Fig. 1 Field shape of superconducting wiggler

Since $B_y(x_0, s_0, y)$ is actually independent of x_0, the local multipole components are computed from $B_y(x_0, s_0, y_0)$ as [1,3]

$$B_n(s) = \left(\sin\theta \frac{\partial}{\partial s_0}\right)^n B_y \Big|_{x_0 = g(s_0), y=0}, \qquad n = 1, 2, 3, \cdots \tag{1}$$

where $x_0 = g(s_0)$ is the electron trajectory and θ is the

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angle between s_0 and s. Then the numerical value of the integrated quadrupole component is given by

$$\int ds \frac{B_1(s)}{(B\rho)} = 0.147\mathrm{m}^{-1}, \tag{2}$$

while

$$\int ds \frac{1}{\rho^2} = 0.147\mathrm{m}^{-1}. \tag{3}$$

Since these two cancel each other in the integrated equation for x leaving only the vertical focusing, the integrated linear betatron equations are given by

$$\begin{aligned} \int ds x'' &= 0, \\ \int ds\left(y'' + \frac{B_1}{(B\rho)}y\right) &= 0, \end{aligned} \tag{4}$$

which can be also expected from the rectangular geometry of the wiggler. Hence the horizontal tune does not change, while vertically tune changes by the amount of

$$\Delta\nu_y = \frac{1}{4\pi}\bar{\beta}_y \int ds \frac{B_1(s)}{(B\rho)} \approx 0.05, \tag{5}$$

where $\bar{\beta}_y$ is a mean value over the wiggler region. The most serious linear optics distortion is the breaking of $\alpha_x = 0, \alpha_y = 0$ conditions. However these linear changes can be minimized by adjusting quadrupoles adjacent to the superconducting wiggler. The α matching condition is fully recovered.

On the other hand higher order multipoles are evaluated as

$$\begin{aligned} \int ds \frac{B_2(s)}{(B\rho)} &= 0.10\mathrm{m}^{-2}, \\ \int ds \frac{B_3(s)}{(B\rho)} &= 1.6\mathrm{m}^{-3}, \end{aligned} \tag{6}$$

and so on. Although these multipoles can not be eliminated or even minimized, their influence on the beam motion is not decisive at all, because their strengths are very small compared to that of linear focusing. Computer simulation verifies that the effect of these multipoles on the dynamic aperture is almost negligible. Dynamic aperture is obtained using computer code DIMAD [4], and the result is shown in Fig. 2. Reduction of the dynamic aperture from the case of no wiggler should be attributed to the lattice symmetry breaking.

3. EFFECTS ON EMITTANCE AND ENERGY SPREAD

The effect of the wiggler on the electron beam (and photon beam) can be measured by two parameters, natural beam emittance and rms energy spread, defined by

$$\begin{aligned} \epsilon &= \frac{55}{32\sqrt{3}} \frac{\hbar\gamma^2}{\mathrm{mc}} \frac{I_5}{I_2 - I_4}, & (7) \\ \sigma_\epsilon^2 &= \frac{55\hbar\gamma^2}{32\sqrt{3}\mathrm{mc}} \frac{I_3}{2I_2 + I_4}. & (8) \end{aligned}$$

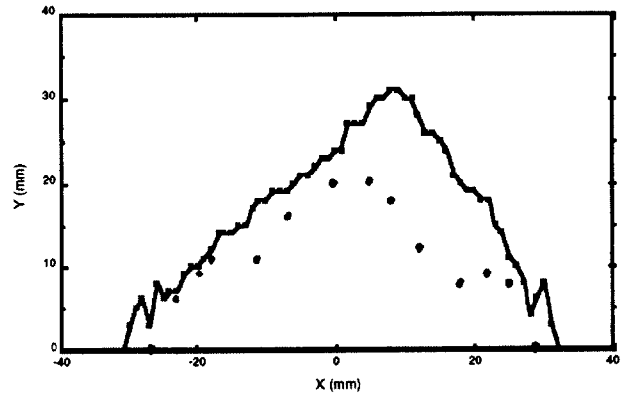

Fig. 2 Comparison of dynamic apertures. Dotts denote the dynamic aperture with the wiggler

Here the synchrotron radiation integrals, $I_1, \cdots I_5$ are given by

$$\begin{aligned} I_1 &= \oint \frac{\eta}{\rho} ds \\ I_2 &= \oint \frac{1}{\rho^2} ds \\ I_3 &= \oint \frac{1}{|\rho|^3} ds \\ I_4 &= \oint \frac{\eta}{\rho^3} ds \\ I_5 &= \oint \frac{H}{|\rho|^3} ds, \end{aligned} \tag{9}$$

where

$$H = \frac{1}{\beta}[\eta^2 + (\beta\eta' - \frac{1}{2}\beta'\eta)^2]. \tag{10}$$

The wiggler creats its own radius of curvature and dispersion, and so contributes to the beam emittance and energy spread. Let us denote the wiggler contribution to the integrals by I_i^w. In terms of these integrals, the ratio of the new emittance to the old one is given by

$$\frac{\epsilon_x}{\epsilon_x^0} = \frac{1 + (\frac{I_5^w}{I_5})}{1 + (\frac{I_2^w - I_4^w}{I_2 - I_4})}. \tag{11}$$

This ratio is determined by the magnitude of the wiggler contributions,I_i^w, relative to the lattice contributions, I_i. In PLS, the lattice contributions are given by

$$I_1 = 0.508, \quad I_2 = 0.997\mathrm{m}^{-2}, \quad I_3 = 0.158\mathrm{m}^{-3},$$
$$I_4 = -2.538 \times 10^{-3}\mathrm{m}^{-2}, \quad I_5 = 2.052 \times 10^{-3}\mathrm{m}^{-2}.$$

The dispersion created by the wiggler is computed by the well-known formula [5],

$$\eta(s) = \int_0^s \frac{1}{\rho(\sigma)}[S(s)C(\sigma) - C(s)S(\sigma)]d\sigma, \tag{12}$$

where S(s)(C(s)) represent sine-like (cosine-like) solution of the linear lattice. To use this equation, we have to separate the wiggler into several thin pieces and treat them as linear elements. With this η and ρ, we can compute the integrals. But note that I_5^w depends on not only the wiggler property, ρ and η, but also the lattice function, β_x. Since the straight section of 6.8 m is long compared to the wiggler, it is reasonable to assume $\beta' = 0$ inside the wiggler. Therefore we can compute I_5^w as

$$I_5^w = \frac{1}{\beta_x}\int_W \frac{\eta^2}{|\rho|^3}ds + \beta_x \int_W \frac{\eta'^2}{|\rho|^3}ds. \tag{13}$$

Numerically computing these integrals, the variation of ϵ_x/ϵ_x^0 with respect to β_x is plotted in Fig. 3. The PLS lattice in which β_x =10m in the straight section gives the ratio $\epsilon/\epsilon_x = 1.37$. As one can notice from Fig. 3, smaller β_x value makes smaller ratio until $\beta_x = 0.82$, at which we get the minimum value of $\epsilon_x/\epsilon_x^0 = 1.06$. Unfortunately, we do not have much freedom to lower β_x value. Even though we can use quadrupole pairs to lower β_x value in the straight section, it is not so effective due to the strong vertical focusing of the wiggler.

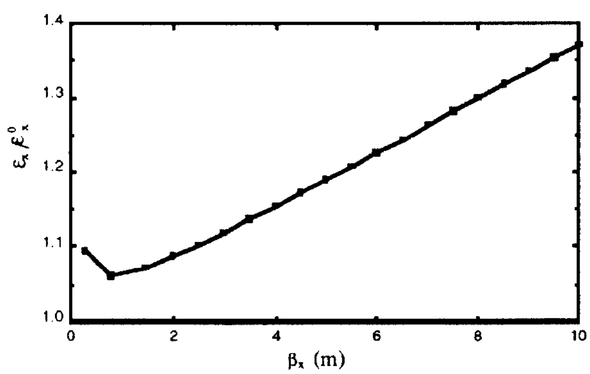

Fig. 3 Ratio of emmittance versus β_x

On the other hand, the ratio of the energy spread is given by

$$\frac{\sigma_\epsilon^2}{\sigma_{\epsilon 0}^2} = \frac{1+(\frac{I_3^w}{I_3})}{1+(\frac{2I_2^w+I_4^w}{2I_2+I_4})}. \tag{14}$$

This relative energy spread is independent of the β value. In the PLS lattice, it has the value of 1.51.

4. REFERENCES

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