Calibration of the X-Ray Ring Quadrupoles, BPMs, and Orbit Correctors Using the Measured Orbit Response Matrix *

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Abstract

The quadrupole strengths, beam position monitor (BPM) gains, and orbit correction magnet strengths were adjusted in a computer model of the NSLS X-Ray ring in order to best fit the model orbit response matrix to the measured matrix. The model matrix was fit to the 4320 data points in the measured matrix with an rms difference of only 2 to 3 microns, which is due primarily to noise in the BPM measurements. The strengths of the 56 individual quadrupoles in the X-Ray ring were determined to an accuracy of about 0.1%. Small variations of a few parts in a thousand in the strengths of the quadrupoles within an individual family were resolved. The BPM and orbit corrector calibrations were also accurately determined. A thorough analysis of both random and systematic errors is included.

1 INTRODUCTION

The high accuracy BPMs at the NSLS yield very precise information about the ring optics.[4] In the X-Ray ring there are 51 horizontal correctors and 39 vertical correctors, and the closed orbit can be measured in both planes at 48 BPMs. When we measure the change in orbit at each BPM for a change in each corrector magnet, we have $(51 + 39)48 = 4320$ very accurate pieces of data describing the magnetic field gradient around the ring. With this data we are able to find all the quadrupole strengths in the ring as well as the BPM and corrector calibrations.[1]

The work we have done builds upon and borrows ideas from the computer codes CALIF [2] and RESOLVE [3].

2 METHOD

We used the COMFORT [5] accelerator optics modeling program to calculate the model response matrix. The quadrupole, BPM, and corrector calibrations were varied in order to best fit the model matrix to the measured one. The orbit response matrix is defined by

$$\begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} = M \begin{pmatrix} \boldsymbol{\theta}_x \\ \boldsymbol{\theta}_y \end{pmatrix}$$

where M is either the model or the measured matrix which gives the change in orbit $\mathbf{x}, \mathbf{y}$ with a change in corrector strengths $\boldsymbol{\theta}_{x,y}$. To minimize the difference between the model and measured matrices, we made a vector, $\mathbf{V}$, with the elements of $\mathbf{V}$ equal to the difference between the measured and model response matrices. $\mathbf{V}$ has 4320 elements, which is the number of horizontal and vertical correctors times the number of BPMs. Then the equation

$$\mathbf{V} = \frac{d\mathbf{V}}{dK_j}\Delta K_j + \frac{d\mathbf{V}}{d\theta_j}\Delta\theta_j + \frac{d\mathbf{V}}{dG_j}\Delta G_j + \frac{d\mathbf{V}}{d(\Delta p/p)_j}\Delta(\Delta p/p)_j \tag{1}$$

was solved for changes in quadrupole strengths (K_j), corrector strengths (θ_j), the BPM gains (G_j), and $(\Delta p/p)_j$ in order to best fit the measured to model response matrices. The parameter $(\Delta p/p)_j$ is the electron energy shift that occurs when the j^{th} horizontal corrector strength is changed by θ_j. This energy shift causes a shift in orbit proportional to the dispersion that is just large enough to keep the total path length of the electron trajectory fixed. The elements of $\frac{d\mathbf{V}}{d(\Delta p/p)_j}$ are equal to the horizontal dispersion.

In equation 1 we varied 57 K_j's for the 56 individual quadrupoles in the X-Ray ring plus the gradient in the dipoles. We varied 51 $(\Delta p/p)_j$'s for the 51 horizontal correctors, and we varied 96 G_j's for the 48 horizontal BPMs and the 48 vertical BPMs. We could not independently vary all the BPM G_j's and all the corrector θ_j's because there would be a degeneracy in the solution. All the BPM gains could be increased while all the corrector θ_j's were decreased, and the model matrix would stay constant. To avoid this degeneracy, we assumed one horizontal corrector and one vertical corrector were calibrated correctly. We fixed these two corrector strengths, and calibrated all the other correctors and BPMs relative to these two correctors. Thus we varied 50 θ_j's for the 51 horizontal correctors and 38 θ_j's for the 39 vertical correctors. This gave us a total of 292 varied parameters to fit the 4320 measured data points. Equation 1 can be written as

$$V_i = \frac{dV_i}{dx_j}\Delta x_j \tag{2}$$

with the 292 parameters denoted by x_j's.

Actually the equation we solved was a slightly modified version of equation 2. Different BPMs in the ring have different noise levels associated with their orbit measurements. We measured the noise level for each BPM

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by measuring the orbit many times in succession without changing any corrector magnet strengths. The rms orbit shift between successive orbits for the k^{th} BPM, σ_k, gave the noise level associated with that BPM. The rms noise levels ranged from 1.1 μm to 5.1 μm, with a typical noise level of about 2 μm. We gave greater weight to those BPMs with lower noise by solving

$$V_i/\sigma_k = \frac{dV_i/\sigma_k}{dx_j}\Delta x_j. \quad (3)$$

In this way we were minimizing the χ^2 deviation of the model from the measurements, where

$$\chi^2 = \sum_{i=1}^{4320} \frac{V_i^2}{\sigma_k^2}. \quad (4)$$

The change in the model matrix with quadrupole strengths is nonlinear, so equation 3 was solved iteratively until the solution converged to the minimum χ^2. After convergence, the rms difference between the model and measured matrices was 2.7 μm which is very close to the BPM noise level of 2.0 μm. Figure 1 shows the very good agreement between the measured and model response from one of the vertical correctors.

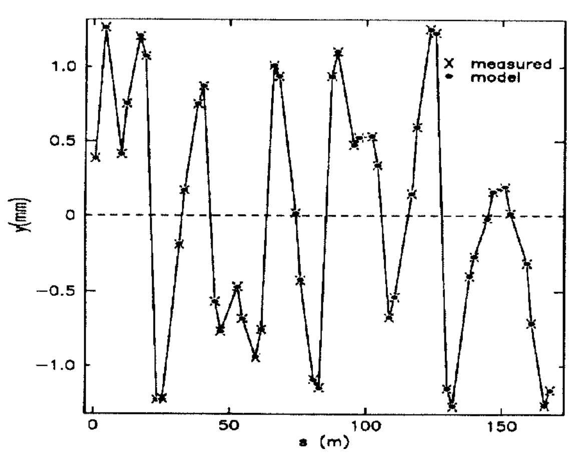

Figure 1: The measured and model response from one of the vertical correctors at the 48 BPMs. The 2.5 μm rms difference between the measured and model orbit shifts is too small to see on the graph.

3 ERROR ANALYSIS

Once the algorithm had converged to minimize χ^2, we had calibrations for the quadrupoles, BPMs, and correctors that gave a very good fit to the measured data. Then we had to determine if these magnet strengths and BPM calibrations were really the ones in the ring. We had to address both random and systematic errors.

3.1 random errors

The easiest way to determine how much the fitted parameters vary due to random errors in the measurements is simply to take many data sets, analyze each one separately, and see how much variation there is between fitted parameters for the different data sets. We measured the response matrix ten times, and fitted a model to each response matrix. Then for each of the parameters we took the average over the ten data sets and calculated the rms difference from this average. For example, table 1 shows the average quadrupole gradient over the ten data sets for the 8 quadrupoles in the QD family of the X-Ray ring: The rms deviations from the average are about .05% which means that the solution is unique to within .05%. There is only one model that fits the data. As far as random errors are concerned, we can be confident that the model we have calculated gives the true quadrupole gradients of the ring. The rms deviations over the ten models for the calibrations of the BPMs and correctors were .17%, so random errors give a contribution of .17% to the error bars on our model BPM and corrector calibrations.

Table 1: Calculated gradients averaged over ten data sets

Quadrupole	$<K>(m^{-2})$	rms deviation (%)
QD1	1.3374	.06
QD2	1.3371	.05
QD3	1.3378	.04
QD4	1.3381	.03
QD5	1.3385	.04
QD6	1.3404	.07
QD7	1.3343	.06
QD8	1.3374	.05

3.2 systematic errors

We found that random errors in the orbit measurements propagated to put .05% error bars on our fit quadrupole gradients. We needed to determine if systematic errors increased these error bars. To do so we needed to look at χ^2_{min} which is the value of χ^2 for the best fit model. If the only errors in the fitting are normally distributed random errors, then χ^2_{min} should be about equal to the number of degrees of freedom, $N - M$, where N is the number of data points (4320), and M is the number of fit parameters (292). More precisely, if there were only normally distributed random errors, and we took many data sets, solving for χ^2_{min} for each data set, then the distribution of χ^2_{min}'s would be centered at $N - M = 4028$ and would have a standard deviation of $\sqrt{2(N-M)} = 90$ [6].

For the ten data sets we fit, we found χ^2_{min} averaged about 7500, which is many standard deviations above 4028. A value of χ^2_{min} that is one or two standard deviations above 4028 could be explained by the fact that our orbit measurement errors were not normally distributed, but a χ^2_{min} of 7500 can only mean that the systematic errors, though small, are not small enough to be neglected in determining the error bars on our fit parameters.

The COMFORT model we used for the model response matrix was a linear, decoupled matrix, so it did not include the effects of sextupoles and skew quadrupole gradients on orbit shifts. To avoid the systematic errors from the sextupoles, we simply turned them off. We can store 50 mA in the X-Ray ring without any sextupoles. Magnetic measurements show a small sextupole field in the dipole magnets. This sextupole field is about the right strength to explain the larger than expected χ^2_{min}, so we believe that it is the largest systematic error. The decoupling in the X-Ray ring is very good [7], so using a decoupled model put little error in our fitting. Another smaller contribution to the systematic error comes from uncertainty in the longitudinal positions of the BPMs and the corrector magnets.

One way we can gain confidence that our fit parameters are correct despite systematic errors is to look at other measured data from the storage ring that was not used in the model fitting and see if it agrees with the model. We found that the measured tunes agreed with the model tunes to within measurement accuracy. The measured dispersion also agreed quite well with the model dispersion.

Of course the best way to show that the model quadrupole strengths are correct is to compare them to magnetic measurement data. Unfortunately magnetic measurement data for the X-Ray ring magnets is not readily available. (This is the main reason we undertook this study.) After we finished the modeling work, however, we did find some magnetic measurement data which confirmed that our model is good. One measurement was the gradient in the dipole magnets. Our fit model predicted a gradient of $.0054m^{-2}$, while magnetic measurements showed a gradient of $.0051m^{-2}$. The integrated gradient in the dipole is only 1.8% of the typical integrated gradient in the quadrupoles, so the difference between the model and measured integrated dipole gradient corresponds to only 0.1% of the integrated gradient in the quadrupoles.

We could not find measured excitation curves for the quadrupoles, but we did find a measurement of the variation in gradient from one quadrupole to another within the QC family. Before finding these measurements we were bothered that our model came up with a larger variation in gradient within the QC family than within the other three X-Ray quadrupole families. Each quadrupole gradient was varied independently when fitting the model to the measurements; nothing constrained the gradients within a family to be the same. It was encouraging to see that the fit quadrupole gradients within a family did come out close to the same—see, for example the gradients within the QD family in table 1. Table 2 shows the variation in the fitted gradients within the four quadrupole families. To get $<< K >>$ for QD for this table, we averaged the 8 gradients in table 1, and calculated the rms deviation from this average and the peak-to-peak variation of the 8 gradients. The larger variation within the QC family also showed up in magnetic measurements. Magnetic measurements showed a .34% rms and a 1.3% peak-to-peak variation in the QCs when the QCs were powered to a gradient of $1.53m^{-2}$. This shows that our modeling was able to resolve small differences between the quadrupoles in a single family.

Table 2: Calculated quadrupole gradients averaged over ten data sets, then averaged over all the quadrupoles in a family

Quad Family	$<< K >>$ (m^{-2})	rms deviation (%)	peak-to-peak variation (%)
QA	-1.5085	0.13	0.5
QB	1.3585	0.13	0.6
QC	-1.4236	0.33	1.2
QD	1.3376	0.12	0.5

We ran yet another test that confirmed that we could accurately predict quadrupole strengths. We measured the response matrix, and then we used a trim supply to add additional current to the coils of just one of the QD magnets and remeasured the response matrix. We fit the model to each of these measurements and compared the fit quadrupole strengths. The two models predicted different strengths for the one QD, while the predicted strengths for all the other quadrupoles only differed by .1% rms.

4 CONCLUSION

We have shown that it is possible to accurately determine the individual quadrupole gradients, the corrector calibrations, and the BPM calibrations of a circular storage ring by fitting the model orbit response matrix to the measured matrix. In the future we plan to use a nonlinear, coupled model response matrix to find the skew quadrupole gradient distribution as well as the normal gradient.

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6 REFERENCES

[1] J. Safranek and M.J. Lee, "Calibration of the X-Ray Ring Quadrupoles, BPMs, and Orbit Correctors Using the Measured Orbit Response Matrix," AIP Conference Proceedings, Vol 315, 1994.

[2] W.J. Corbett, M.J. Lee and V. Ziemann, "A Fast Model-Calibration Procedure for Storage Rings," SLAC-PUB-6111, May, 1993.

[3] M.J. Lee, Y. Zambre, W.J. Corbett, "Accelerator Simulation Using Computers," SLAC-PUB-5701A, 1991.

[4] R. Biscardi, J.W. Bittner, "Switched Detector for Beam Position Monitor," Proceedings 1989 IEEE Part. Accel. Conf. Chicago, IL, p. 1516.

[5] M.D. Woodley et al., "Control of Machine Functions, or Transport Systems," SLAC-PUB-3086, March, 1983.

[6] W. Press, B. Flannery, S. Teukolsky, W. Vetterling, Numerical Recipes, Cambridge, 1990.

[7] J. Safranek, S. Krinsky, "Plans to Increase Source Brightness of NSLS X-Ray Ring," Proceedings 1993 IEEE Part. Accel. Conf. Washington, D.C., p. 1491.