A Single Bunch Compression in the Exponential Field

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Abstract

In a linear accelerator the short length of an electron bunch is required for its effective acceleration. To compress the bunch, the energy modulation in the field of travelling or standing wave of constant amplitude is used. Using an exponential growth of the field amplitude along the longitudinal coordinate, the more effective bunching and the self-consistent compensation of longitudinal space charge forces can be reached.

1 LONGITUDINAL BEAM DYNAMICS

1.1 *A Single Particle Approach for the Small Phase Oscillations*

The longitudinal motion of an electron in a sinusoidal field of travelling wave with an exponentially growing amplitude can be presented by the following system of differential equations:

$$\left\{ \begin{array}{rcl} \frac{d(\gamma\beta_e)}{dt} & = & \frac{eE_0}{mc}\sin(\omega t - kz)\exp\{\alpha z\} \\ \frac{dz}{dt} & = & \beta_e c \end{array} \right. \qquad (1)$$

where $\beta_e = v/c$ is a relative velocity of an electron,γ is a relativistic factor,α is an increment of the field growth. To transfer in (1) from coordinate to phase presentation and to perform the substitutions $\tilde{\alpha} = \alpha/k$ and $\tau = \exp\{\tilde{\alpha}\omega t\}$, the system (1) is reduced to a single second order differential equation, which is the particular case of Bessel's equation in a small phase approach:

$$\frac{d^2\psi}{d\tau^2} + \frac{1}{\tau}\cdot\frac{d\psi}{d\tau} + \frac{eE_0}{\gamma^3 mc\beta_p\omega\tilde{\alpha}^2}\cdot\frac{\psi}{\tau} = 0 \qquad (2)$$

Here β_p is a phase velocity of the wave. The solution of this equation is

$$\psi = Z_0\left(\frac{2}{\tilde{\alpha}\gamma c}\sqrt{\frac{eE_0}{\gamma m\omega\beta_p c}\tau}\right) =$$
$$Z_0\left(\frac{2}{\tilde{\alpha}\gamma c}\sqrt{\frac{eE_0}{\gamma m\omega\beta_p c}}\exp\left\{\frac{\tilde{\alpha}\omega t}{2}\right\}\right) \qquad (3)$$

where $Z_0(x) = C_1 J_0(x) + C_2 N_0(x)$ is a linear combination of Bessel's and Neumann's functions of zero order and constants C_1 and C_2 can be found from initial conditions. The asymptotic of the solution (3) leads to $1/\sqrt{x}$, so when $E(t)$ is growing exponentially, the amplitude $\psi(t)$ is damping also exponentially with the decrement equals to one forth of the increment of the increasing field.

The same result could be obtained from the simple model of one-dimensional oscillator:

$$\ddot{\psi} + \Omega^2\psi = 0 \qquad (4)$$

where the frequency $\Omega(t)$ depends on time. In our particular case

$$\Omega^2(t) \approx \frac{\omega}{\gamma^3 mc}eE(t) \qquad (5)$$

When the frequency is changing relatively slow, the adiabatic invariant is retaining:

$$I = \epsilon/\Omega = \frac{\Omega^2\psi^2}{\Omega} = \Omega\psi^2 \approx const \qquad (6)$$

then

$$\psi \sim \exp\{-\frac{\alpha t}{4}\} \qquad (7)$$

1.2 *The Equations of Motion in the Presence of the Space Charge Forces*

Let the bunch of the length σ , radius a and charge Q has the cylindrical shape and the uniform density of charge. We consider its motion in a sinusoidal field of travelling wave. The center of the bunch is moving synchronously with the phase $\psi = 0$. The presence of the strong longitudinal magnetic field provides the good focusing of the beam near the axis, so we can neglect the transverse motion of the particles in the bunch. The system of dynamic equations for the electron on the axis of the bunch located at the position with phase ψ can be written as following:

$$\left\{ \begin{array}{rcl} \frac{d(\gamma\beta_e)}{dt} & = & \frac{e}{mc}\left[E(t)\sin\psi + E'\right] \\ \frac{d\psi}{dt} & = & k\beta_e c \end{array} \right. \qquad (8)$$

where E' is a longitudinal component of the own field of the bunch at the location ψ . In the case of relatively small longitudinal size of the bunch, the approximation of the thin disk can be applied for the bunch field:

$$E' = \frac{2Q}{a^2}\left(1 - \frac{\sigma}{\sqrt{a^2+\sigma^2}}\right)\frac{\psi}{k\sigma} \qquad (9)$$

The estimation of the space charge field influence leads to the following equation for the frequency of the phase oscillations:

$$\frac{d^2\psi}{dt^2} + \Omega^2\psi = 0;$$

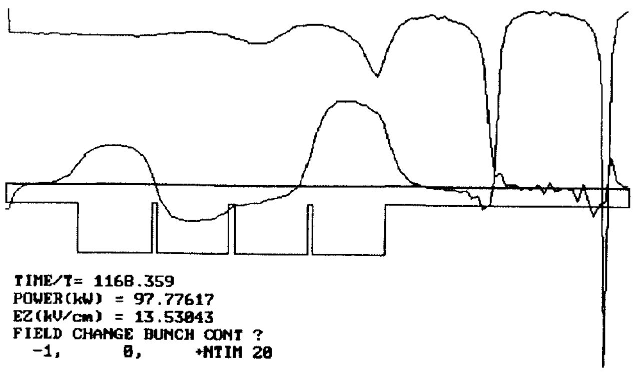

Figure 1: The geometry of the buncher cells and the distribution of the electric field and current (above)

$$\Omega^2 = \frac{ek}{\gamma^3 m}\left[E(t) - \frac{2Q}{a^2 k\sigma}\left(1 - \frac{\sigma}{\sqrt{a^2+\sigma^2}}\right)\right] \qquad (10)$$

When $E(t)$ = const the reduction of the bunch length during bunching leads to the reduction of the frequency of phase oscillations and, as a result, to the limitation of minimum length and maximum peak current of the bunch. The comprehensive study of this phenomena is presented in [1]. When $E(t) \sim \exp\{\alpha t\}$ the length of the bunch is decreasing as $\sigma(t) \sim \exp\{-\frac{\alpha t}{4}\}$, which cancels the limitation of its length and peak current.

It is necessary to note that the identical method of bunching is used in the multicavity klystrons of high efficiency.

2 THE BUNCHER DESIGN

To realize the method of bunching described above we suggest to use the buncher which is operated on the backward travelling wave. The buncher is performed as the disk-loaded structure from the material with the high surface electric resistance. It provides the strong damping of RF power inside. By this means the use of the backward travelling wave makes possible to provide the exponential field growth along the bunch motion.

For buncher design calculations in the presence of space charge forces the method of numeric solution of equations of motion was applied in combination with Maxwell's equations. The results are presented in Fig. 1 where the geometry of buncher cells and instant field and current distributions are shown.

3 REFERENCES

[1] I.M. Kapchinsky and A.S. Kronrod, "The Space Charge Influence On The Phase Oscillations Of Particles In The Ion Linear Accelerator", The Devices And Techniques Of An Experiment, vol. 3, pp. 26-31 May-June, 1964.