Computer Design of Coupler Cavities for a Travelling-Wave-Type Buncher

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Abstract

A method used to design a coupler cavity for a travelling-wave structure was proposed and applied to a new buncher. Although this method is based on an equivalent-circuit technique, it depends on MAFIA to determine the resonant frequencies of the system. The field asymmetry in the coupler was also investigated.

1. INTRODUCTION

A coupler for an accelerator guide can be simply treated as a two-port resonant circuit. A waveguide and an accelerator guide after the coupler correspond to an L-coupled transmission line and a C-coupled periodic structure, respectively. It generally can be assumed that both the waveguide and accelerator guide have no wall loss and no reflection, and that power loss of the resonant circuit can be made negligibly small compared to the power flowing outside through the ports. Then, impedance matching means that the external Q value for a waveguide is equal to the other external Q, representing an accelerator guide of infinite length, at the resonant frequency of the coupler circuit. That is, the problem of obtaining impedance matching can be solved by knowing the two external Q values, even if the input impedance of the coupler is not directly obtainable.

It has recently been reported [1],[2] through application of a three-dimensional code, such as MAFIA [3], that it is possible to know the external Q of a waveguide-coupled cavity. In this report, however, we propose another method to compute the external Q's of a coupled system, and introduce an application to a newly planed prebuncher and buncher.

Both bunchers are of the disk-loaded type and were designed to have rather large beam irises on the disks. The design would result the low external Q's, i.e., unexperienced large coupling slots of the couplers, which may cause considerable asymmetrical fields. This asymmetry was investigated in order to estimate its effects on a beam orbit.

2. EQUIVALENT CIRCUIT APPROACH

As the first step we consider a waveguide-coupled cavity. Let the input port of the waveguide be of length l and terminate short-circuited, as shown in Fig. 1. Then, the lossless waveguide is seen as a pure imaginary impedance Z_l. The other simplified equivalent circuit represents this shorted waveguide as a series reactance Z_g in the coupler circuit, as in Fig. 1. In the case that $Z_l >> j\omega L_g$, for example, $l \approx \lambda_g/4$, Z_g is simply

$$Z_g = \frac{(\omega M)^2}{Z_l} = -j\frac{(\omega M)^2}{Z_{ch}}\cot\beta_g l\ , \tag{1}$$

where M, Z_{ch} and β_g are the mutual inductance of the coupled L's, the characteristic impedance of the waveguide and the phase constant, respectively.

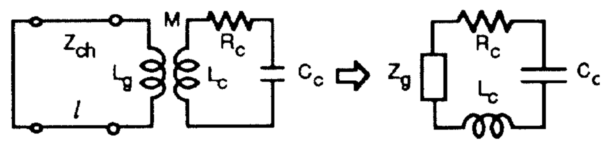

Fig. 1. Equivalent circuit for a waveguide-coupled cavity.

The circuit equation of this coupled system is

$$j\omega L_c + \frac{1}{j\omega C_c} - j\frac{(\omega M)^2}{Z_{ch}}\cot\beta_g l = 0\ . \tag{2}$$

In a practical system, the coupling slot is not small and tight magnetic coupling may cause an effective length of the waveguide to be slightly longer than the real one. Therefore, l in eq. (2) should be changed to $l + d$ in order to represent this effect.

When the waveguide has an infinite length or a matched termination of Z_{ch}, Z_g is observed as a pure resistor, $(\omega M)^2/Z_{ch}$. The external Q value for the waveguide $Q_{ext,g}$ can be represented as

$$Q_{ext,g} = \frac{\omega L_c}{Z_g} = \frac{L_c Z_{ch}}{\omega M^2}\ . \tag{3}$$

Finally, equation (2) becomes a form which includes $Q_{ext,g}$,

$$1 - \left(\frac{\omega_c}{\omega}\right)^2 - \frac{\omega}{\omega_c}\frac{1}{Q_{ext,g}(\omega_c)}\cot\beta_g(l+d) = 0\ , \tag{4}$$

where $\omega_c{}^2 = 1/L_cC_c$ and $Q_{ext,g}(\omega_c) = L_cZ_{ch}\ /\ \omega_cM^2$.

MAFIA can compute three resonant frequencies ω of the coupled system, which correspond to the different l's. By putting these ω's and l's into equation (4), one obtains simultaneous equations. Thus, the unknown values ω_c, $Q_{ext,g}(\omega_c)$ and d can be obtained by solving these equations numerically.

In practical computation, there is one problem of how to set the boundary conditions in order to simulate actual oscillation at a given mode in the coupler cavity. In our case, in which the disk has a 5 mm thickness, a good approximation for the $2\pi/3$ mode is achieved by applying a magnetic-short boundary at the end of the disk iris opening to the next cell.

We next assume that a disk-loaded-type accelerator guide can be treated as a well-known equivalent circuit for a periodic structure (Fig. 2) which accounts for only the TM_{01}-mode propagation. That is, a cylinder section of the guide is represented as a transmission line having the characteristic admittance Y_0, and a disk hole acts as a shunt susceptance $j\omega C_s = jbY_0$. Notice that C_0 ($> C_c$) is the capacitance which gives the so-called 0-mode resonance $\omega_0 = 1/(L_c C_0)^{1/2}$.

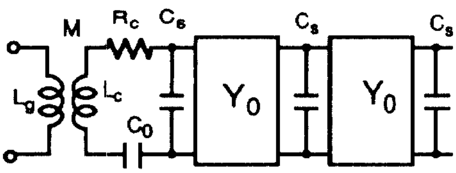

Fig. 2. Equivalent circuit for a disk-loaded structure.

An equation which gives a dispersion curve of this periodic structure is approximately [4]

$$\cos k'L = \cos k(L - t) - \frac{b}{2} \sin k(L - t) , \tag{5}$$

where k is the wave number of the TM_{01} mode in the unit cylinder of length L, k' represents propagation in the accelerator guide. The disk thickness t in this equation is introduced in order to take account of the actual length of the unit cavity. The normalized susceptance b cannot be expressed theoretically if the disk iris is large. This equation, however, gives the value of b when the cylinder radius and the phase shift $k'L$ are determined.

In the case that the periodic structure has no reflection, the equivalent circuit given in Fig. 2 is reduced to a simple series resonator presented in Fig. 3. The additional series impedance R_a and C_a are

$$R_a = \frac{Z_a}{1 + (Z_a\, b\, Y_0 / 2)^2} \tag{6}$$

and

$$\omega_c C_a = \frac{1}{2} b\, Y_0 \left(1 + \frac{1}{1 + (Z_a\, b\, Y_0 / 2)^2}\right), \tag{7}$$

where

$$Z_a = \frac{1}{Y_0} \frac{\sin k(L - t)}{\sin k'L} . \tag{8}$$

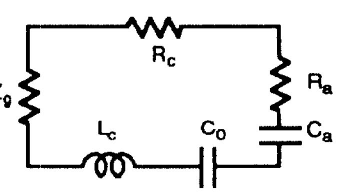

Fig. 3. Periodic structure represented as R_a and C_a.

Then, the external Q for the accelerator guide ($Q_{ext,a}$) becomes

$$Q_{ext,a} = \frac{\omega L_c}{R_a} = \frac{\omega}{\omega_0^2\, C_0\, R_a} . \tag{9}$$

The resonant frequency ω_c of the resonator is approximately, neglecting R_c, Z_g and R_a,

$$\omega_c \approx \omega_0 \sqrt{1 + C_0 / C_a} . \tag{10}$$

We let the two couplers be combined together, without the accelerator guide, so that they can have the disk in common, thus producing 0- and π-mode oscillations. These frequencies result in a ratio of C_0/C_s, giving actual values of $Q_{ext,a}$ and ω_c. Here, it is supposed that the disk iris has the same shunt capacitance C_s of the accelerator guide mentioned above. The frequencies ω_0 and ω_π are respectively

$$\omega_0 = \frac{1}{\sqrt{L_c C_0}} \quad \text{and} \quad \omega_\pi = \frac{1}{\sqrt{L_c\{C_0 C_s / (2C_0 + C_s)\}}} . \tag{11}$$

Then,

$$C_0 = \frac{C_s}{2} \left\{\left(\frac{\omega_\pi}{\omega_0}\right)^2 - 1\right\} = \frac{b Y_0}{2\omega_c} \left\{\left(\frac{\omega_\pi}{\omega_0}\right)^2 - 1\right\} . \tag{12}$$

We can therefore express C_0 in terms of Y_0 and b, so that equations (9) and (10) give actual values.

The power transparency $T(\omega)$ is described as

$$T(\omega) = \frac{4\, Q_L^2}{Q_{ext,g} Q_{ext,a}} \frac{1}{1 + Q_L^2 \left(\frac{\omega}{\omega_c} - \frac{\omega_c}{\omega}\right)^2} , \tag{13}$$

where Q_L is the loaded Q of the system.

If the unloaded quality factor Q_c is much smaller than the external Q values, VSWR can be expressed simply as

$$VSWR = \frac{1 + \sqrt{1 - T}}{1 - \sqrt{1 - T}} . \tag{14}$$

The following relation is a good approximation to obtain the minimum VSWR:

$$Q_{ext,q} = Q_{ext,a} , \quad \omega = \omega_c . \tag{15}$$

3. EXAMPLE

In order to verify this method, we computed the slot widths for the present buncher and accelerator guide of the KEK electron linac, and compared them with the actual dimensions, which were determined by a cut-and-measure method. Here, though one example, the input coupler for the buncher, is given below, all of the results from comparison were satisfactory.

The general way to perform the calculation is as follows:

i) Prepare the initial dimensions of a coupler with a waveguide.
ii) Run MAFIA and obtain three ω's according to the procedure mentioned above.
iii) Obtain $Q_{ext,g}$ and ω_c by solving equation (4)
iv) Calculate the susceptance b from equation (5).
v) Make combined couplers and apply MAFIA in order to know ω_0 and ω_π.
vi) Combining equations (6), (9) and (12) results in $Q_{ext,a}$.
vii) Adjust the coupler radius or the slot width, and then repeat the computations until finding relation (15).

In this case, Q_c (> 10000) was ignored. Figure 4 shows $Q_{ext,q}$ and $Q_{ext,a}$ as a function of the slot width. The crossing point of the two lines, where the slot width = 33.5 mm, may give a good matching of the waveguide and the accelerator guide. Since the actual value is 32.5 mm at VSWR = 1.08, the error in the result is less than 3%.

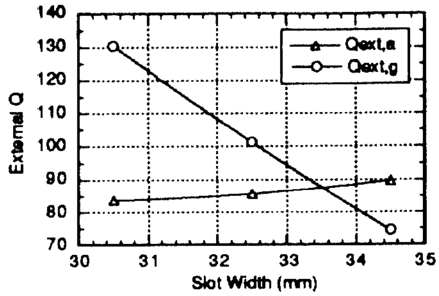

Fig. 4. External Q's plotted as a function of the slot width.

4. APPLICATION TO BUNCHER COUPLER

4.1 Dimensions of coupler

The new S-band buncher under development has large disk irises in order to reduce a wake field. The cavity radius and slot width for the new buncher were searched according to the method described in the previous section. Figure 5 is a contour plot of VSWR as a function of both parameters. The central circle represents the area which satisfies VSWR < 1.1. The external Q for the accelerator guide was about 25.

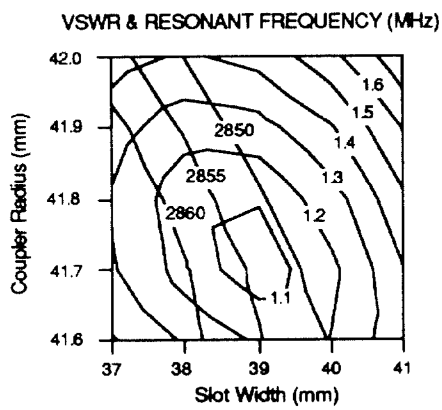

Fig. 5. Contour diagram of VSWR for the new buncher.

4.2 Field asymmetry

The coupling iris causes a field asymmetry in the coupler cavity. We assume that the actual driven fields are a superposition of the undriven and drive fields. When the drive frequency is exactly equal to the resonant frequency of the coupler, the drive fields from the waveguide have a $\pi/2$ phase shift to the undriven magnetic field in the coupler, since the coupler has a large power loss corresponding to the power flow into the accelerator guide. On MAFIA v. 3.0, however, the magnetic field in the waveguide and that in the coupler oscillate in the same or opposite phase, since the power flowing out cannot be taken into account.

The asymmetry of undriven fields can be easily compensated for by offsetting the coupler cylinder, or by making a counter slot on the cylinder wall just facing the waveguide slot. The amount of compensation of both methods can be found by using MAFIA. For example, the field asymmetry of the buncher coupler can be cancelled by offsetting the cylinder wall by 2.4 mm.

The asymmetric drive fields from the waveguide may yield a transverse force which would not be negligible if $Q_{ext,g}$ is small. We first tried to obtain approximate field distributions of the drive fields as follows:

- Set the waveguide length of the coupled system to about $\lambda_g/2$ and run MAFIA.
- This results in two oscillations ω_- and ω_+ at around ω_c. The oscillations in the waveguide and the coupler are in phase at ω_- or out of phase at ω_+.
- Multiply some factor on the fields for ω_- and subtract the result from the fields for ω_+ in order to cancel the excited oscillation of the coupler. Thus, the residual fields may be approximately the drive fields.

Suppose that a particle e is passing straight through the cavity parallel to its axis. The transverse momentum kick $\mathbf{p}_\perp$ to the particle, given by the drive fields, can be calculated by the following well-known relation:

$$\mathbf{p}_\perp = -j\frac{e}{\omega}\{\mathbf{E}_\perp(0) - \mathbf{E}_\perp(d) + \nabla_\perp \int_0^d E_z\, dz\}\ , \qquad (16)$$

where $\mathbf{E}_\perp$ and E_z contain time-depending terms. Although $\mathbf{E}_\perp(0)$ and $\mathbf{E}_\perp(d)$ would vanish if one extends the integral range, but the integral term will change only slightly. We calculated p_y for an electron passing in the yz plane, y mm away from the central beam axis. The obtained results are plotted on the polar plane in Fig. 6. Note that the phase angles of the values depend on the starting point of integration.

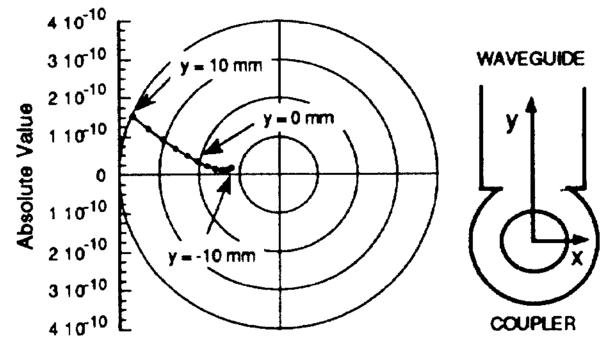

Fig. 6. Normalized complex momentum kick $p_y / e\sqrt{P_{wg}}$. P_{wg} is microwave power fed to the coupler.

In the case of an electron beam of 5 mm diameter, passing straight in the input coupler at y = 0 mm with a velocity 0.695c, the corresponding emittance growth at P_{wg} = 3 MW was estimated to be $3\times10^{-5}\cdot\pi$ MeV/c·cm, which is much smaller than the acceptance of the linac. It, however, may not be negligibly small to realize a low emittance beam in the future.

5. REFERENCES

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