Diffraction Radiation of Charges Moving past Conducting Screens

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Abstract

The time-dependent diffraction radiation is treated for highly relativistic charges moving past conducting obstacles. In two dimensions, where a charge sheet or a line charge passes by a wedge, the diffracted fields are known in form of a closed analytical expression. Three-dimensional cases are not yet solved. In order to understand better the fundamental differences, the time-dependent Green's function in free space is used and a Green's function for a line and ring excitation is derived.

I Introduction

The electromagnetic interaction between moving charges and environment is a complicated mathematical problem. Some simple cases can be solved in the frequency domain, but there a physical understanding is difficult to be gained. A better, but only partly, understanding follows from high frequency diffraction models. In these approaches the co-travelling fields of the charges are considered as waves scattered at an inhomogeneity of the surroundings. Thus, at least, energy losses can be calculated and a simple model of the energy depletion of the original fields can be derived. But probably the only way to gain a real insight in the process is in time-domain. Unfortunately this is also the most difficult approach. For finite charge dimensions numerical solutions are possible with a technique developed for radar puls diffraction, called leap-frog method [1]. Analytically only very few solved problems are known. One is the diffraction radiation of fast moving charge sheets passing by wedges [2]. This is a two-dimensional problem and a closed analytical expression can be derived. In three dimensions the only solved problems known to the author are a charge passing a pill-box cavity (see e.g. [3]) or a charge crossing the gap between two infinite plates [4].

This paper shall serve to understand better the fundamental differences between a one-, two- and three-dimensional case. For that purpose the time-dependent Green's function in free space is used in order to calculate Green's functions for line sources and ring sources.

II The Two-Dimensional Problem

In two dimensions the problem is stated as a charge sheet or a line charge passing by a wedge (Fig.1).

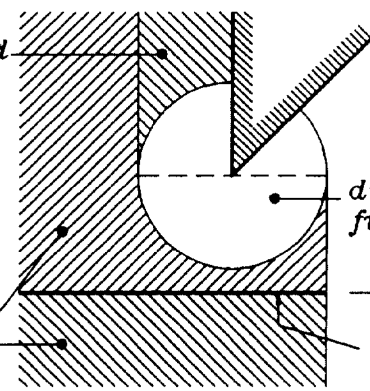

Figure 1: Charge sheet passing a conducting wedge

In that case an exact analytical solution exists due to the fact that the cylindrical diffraction field propagates undisturbed (the boundaries are coordinate surfaces) in space and time. That means, at any instant the field pattern stays constant. Therefore, the space coordinates can be normalized with respect to c_0t and the problem becomes time independent. With a suited distortion of the space coordinates one can then derive Laplace's equation with given boundary values, since there is a continuous transition between the wave front of the diffracted field, the source field, the reflected field and the field-free space (see Fig.1). The resulting fields are given in a closed analytical expression [2]. A snapshot at a fixed time instant looks like in the Fig.2. Shown are the electrical field lines for a line charge excitation. Since the electric field and the displacement current are related by a time derivative in the same way as line charge and charge sheet excitation are related, the field lines are also displacement current lines for a charge sheet excitation.

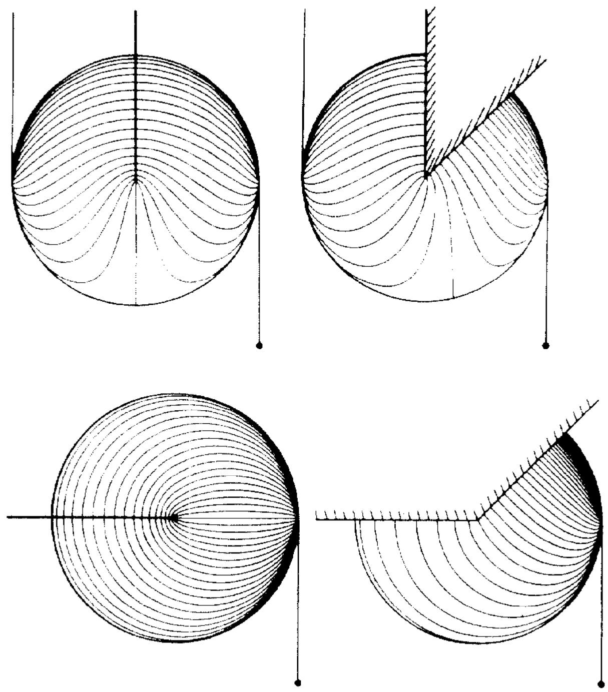

Figure 2: Lines of constant magnetic field

III The Time-Dependent Green's Function

Point Source

In three dimensions the approach we used above can no longer be used. Therefore we try at first to understand the differences by studying the time-dependent scalar Green's function which is useful in solving the source problem

$$\nabla^2\psi - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\psi = -4\pi q(\mathbf{r},t) \quad . \tag{1}$$

The function q describes the source density. Let us assume vanishing boundary and initial conditions. Then the solution of (1) is given by

$$\psi(\mathbf{r},t) = \int_0^t \int_{V'} G(\mathbf{r},t;\mathbf{r}',t')q(\mathbf{r}',t')\,\mathrm{d}\mathbf{r}'\,\mathrm{d}t' \tag{2}$$

with G being the Green's function, i.e. the solution of (1) with a δ-function source $\delta(\mathbf{r}-\mathbf{r}')\delta(t-t')$.

In free space the Green's function is well known (see for instance [5])

$$G(\mathbf{r},t;\mathbf{r}',t') = \begin{cases} 0 & \text{for} \quad R/c_0 > t-t' \\ \dfrac{1}{R}\delta(R/c_0 - [t-t']) & \text{for} \quad R/c_0 < t-t' \end{cases} \tag{3}$$

where $R = |\mathbf{r}-\mathbf{r}'| \geq 0$, $t \geq t'$. It is a δ-function spherical shell around the source point expanding with a radial velocity c_0. Substituting (3) into (2) yields

$$\psi(\mathbf{r},t) = \int_{V'} q(\mathbf{r}',t-R/c_0)\frac{\mathrm{d}\mathbf{r}'}{R} \quad . \tag{4}$$

The effect at $\mathbf{r}$ and at a time t is caused by the value of the source function q at $\mathbf{r}'$ and at a time $t - R/c_0$ the *retarded* time.

Line Source

In two-dimensional problems the situation is very different. As source we take a uniform line source extending from $z' = -\infty$ to $z' = +\infty$ along a line parallel to the z-axis. The Green's function may be easily found by integrating the three-dimensional point source, eqn.(3), over all z'

$$G(\varrho,t;\varrho',t') = \int_{-\infty}^{\infty} \frac{\delta[R/c_0 - (t-t')]}{R}\,\mathrm{d}z'$$

with $R^2 = |\varrho - \varrho'|^2 + (z-z')^2$, which gives

$$G(\varrho,t;\varrho',t') = \begin{cases} 0 & \text{for} \quad p > c_0(t-t') \\ \dfrac{2c_0}{\sqrt{c_0^2(t-t')^2 - p^2}} & \text{for} \quad p < c_0(t-t') \end{cases} \tag{5}$$

with $p = |\varrho - \varrho'|$, $\varrho = x\mathbf{e}_x + y\mathbf{e}_y$. As can be seen the impulse is no longer concentrated in a δ-function like front but spread out over the entire region $p < c_0(t-t')$. Again, there is a singularity at $p = c_0(t-t')$, but it is very weak compared to the δ-function singularity. The reason is that each point of the line source emitts spherical δ-function waves which result in a wake trailing behind the wave front. Lines of constant G are cylindrical surfaces (Fig.3).

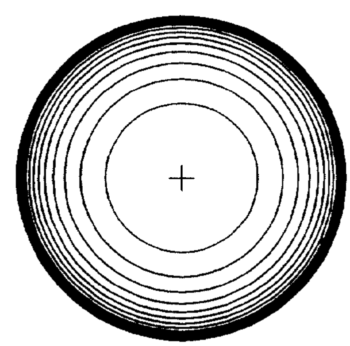

Figure 3: Lines of constant G for a line source

Ring Source

A third very important case is related to a relativistic point charge or a line charge travelling in longitudinal direction. The fields are radial, electrostatic fields. When they are intercepted by an axissymmetric structure, the diffracted fields are constituted from ring-shaped Green's functions. In order to make live easier we restrict ourselves to such ring sources in free space. Again they can be gained from eqn.(3) by integrating over $0 \le \varphi' \le 2\pi$ at ϱ'

$$G(\mathbf{r},t;\mathbf{r}',t') = 2\int_0^{\pi} \delta(R/c_0 - [t-t'])\frac{\varrho'\,\mathrm{d}\varphi'}{R} \qquad (6)$$

with $R^2 = r^2 + \varrho'^2 - 2r\varrho'\cos\gamma$, r being the distance from the origin and γ is the angle between $\mathbf{r}$ and $\boldsymbol{\varrho}'$. Using the relation

$$\mathbf{r}\cdot\boldsymbol{\varrho}' = r\varrho'\cos\gamma = x\varrho'\cos\varphi'$$

we get

$$R^2 = r^2 + \varrho'^2 - 2x\varrho'\cos\varphi'$$

$$2R\,\mathrm{d}R = 2x\varrho'\sin\varphi'\,\mathrm{d}\varphi' = \sqrt{(2x\varrho')^2 - (r^2+\varrho'^2-R^2)}\,\mathrm{d}\varphi'$$

and eqn.(6) becomes after performing the integration

$$G(\mathbf{r},t;\varrho',t') = \begin{cases} 0 \quad \text{for} \quad \begin{array}{l} c_0(t-t') < R(\varphi'=0) \quad \text{or} \\ c_0(t-t') > R(\varphi'=\pi) \end{array} \\ \dfrac{4c_0\varrho'}{\sqrt{(2x\varrho')^2 - (r^2+\varrho'^2-c_0^2[t-t']^2)}} \\ \quad \text{elsewhere} \quad . \end{cases} \qquad (7)$$

Inspecting Fig.4 we can rewrite the radical of eqn.(7)

$$\begin{aligned} &(2x\varrho')^2 - (r^2+\varrho'^2-c_0^2[t-t']^2)^2 = \\ &= \left[2x\varrho' + r^2 + \varrho'^2 - c_0^2(t-t')^2\right]\left[2x\varrho' - r^2 - \varrho'^2 + c_0^2(t-t')^2\right] = \\ &= \left[(x+\varrho')^2 + z^2 - c_0^2(t-t')^2\right]\left[c_0^2(t-t')^2 - (x-\varrho')^2 - z^2\right] = \\ &= \left[r_+^2 - c_0^2(t-t')^2\right]\left[c_0^2(t-t')^2 - r_-^2\right] \end{aligned}$$

and obtain

$$G(\mathbf{r},t;\mathbf{r}',t') = \begin{cases} 0 \quad \text{elsewhere} \\ \dfrac{4c_0\varrho'}{\sqrt{[c_0^2(t-t')^2 - r_-^2][r_+^2 - c_0^2(t-t')^2]}} \\ \text{for} \quad \begin{array}{l} r_- < c_0(t-t') < r_+ \quad \text{or} \\ r_+ < c_0(t-t') < r_- \end{array} \end{cases} \qquad (8)$$

where the quantities are defined by Fig.4.

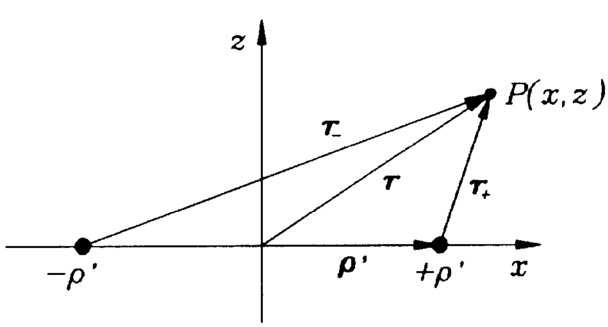

Figure 4: Defining the quantities in eqn.(8)

Now we find still another situation. Neither is there a δ-function field impulse like for a point source nor an infinitely long trailing wake like for the line source. Every point experiences a field pulse of finite duration determined by $2\varrho'/c_0$. The fields are spread over a lapse of time of $r_-/c_0 < t-t' < r_+/c_0$, where $\pm$ can be interchanged. This is due to the travelling times between observation point and different source points of the exciting ring. Note, that, once the wave fronts have overlapped a field free region is left behind (Fig.5).

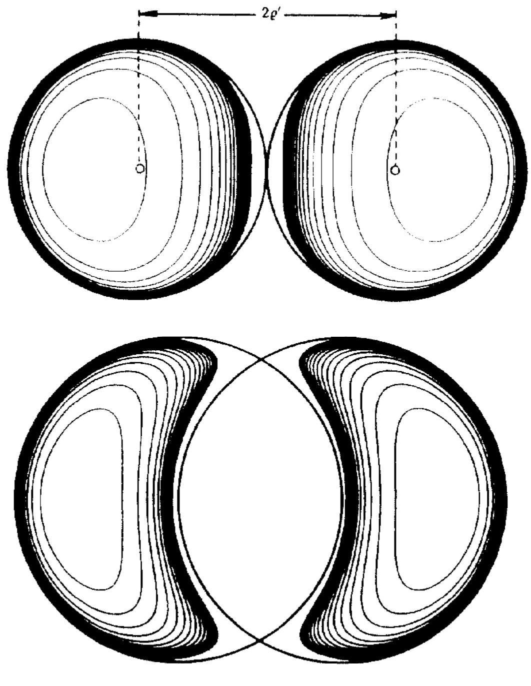

Figure 5: Lines of constant G for a ring-shaped source for two time instants

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