Study of Injection Transient in Coupled Cell Linac Cavities

Yu.Senichev
Institute for Nuclear Research of the Russian Academy of Sciences
Moscow

1 INTRODUCTION

In high intensity proton beam linear accelerators of the Meson Factory type with final energy $\approx$ 1000 MeV and average current $n \times 100 \mu A$ there are stringent requirements for maximum losses of particles $\sim 10^{-3} \div 10^{-4}$ and momentum spread of beam at exit $< \frac{\Delta p}{p} >_{\sigma} \sim 1 \div 2 \times 10^{-3}$ [1]. The momentum spread minimization in linear accelerator injectors for high energy, high luminosity colliders defines largely all the following stages of accelerating complex.
These requirements arise stringent tolerances for stabilization of the RF accelerating field amplitude and phase in time and in space especially in Coupled Cell Linac with stepped-phase-velocity [2,3].
In case of the stationary deviations of RF parameters from design value (mistuning of cavity) the special procedure can be used with mutual compensation of RF and geometrical parameters errors [4,5,6].
To stabilize the average level of RF amplitude and phase in cavity (fundamental mode) during the injection transient the special feedback with high coefficient and synchronization of beam with compensating pulse of generator are used. However if the generator inputs power in cavity locally in some point and excites all modes while the beam radiates along cavity only the waves with phase velocity smaller than beam velocity in accordance of Cherenkov's law. So reaction of the cavity on the beam and the generator is different. In result it leads to time and space distortions of field along cavity. So a perfect compensation can not be reached.
A careful consideration of "beam loading" effect in multimode approach shows that in some accelerating structure real momentum spread is in excess of waited design value calculated in one mode approach. Especially it is very important for linac where the beam pulse length comparable with the transient time. The permanent distortion of RF field depends on characteristics of accelerating structure so as the couple coefficient, the quality factor and the length of cavity (number of cells).
In this paper the interaction of beam with the cavity is considered in multimode approach. The final expressions are presented in simplifying assumption through usual characteristics of accelerating structure. Some practical recommendations for decreasing the RF field distortion are suggested as well.

2 THE MULTIMODE APPROACH TO THE CAVITY DESCRIPTION

The most distributed structure for proton beam accelerating is the standing wave structure consisting of resonators-cells sequence. The behavior of standing wave accelerator cavity made up of chains of resonant cells is conveniently studied by considering the properties of a chain of coupled resonators with general couplings, frequencies and quality factors [8]. This coupled resonator model is useful so long as it is possible to describe the behavior using separated modes of a single cavity. Numerical calculations show that we loss insignificant part of information in this model and it may be used to analyze the radiotechnical properties of structure.
However the study of the transient is rather by the eigenfunction method. Why is it so? It is known that the problem of forced oscillation of string can be solved by the expansion string eigenfunction series (Bernoulli's method) and by analysis of running and reflected waves (d'Alembert's method). It is obviously that both methods lead to the same result. Moreover the transition from each to other is not difficult and is required sometimes for interpretation. Really at first moment of transient there is only direct and back waves running from the point of power input:

$$E = \sum_s (C_s E_s + C_{-s} E_{-s}), \tag{1}$$

where C_s and C_{-s} - functions of longitudinal coordinate z and

$$E_s = E_s^0(r,z) e^{j h_s z} \tag{2}$$
$$E_{-s} = E_s^0(r,z) e^{j h_{-s} z} \tag{3}$$

where $E_s^0(r,z)$ is distribution function of field in resonant cell, h_s is waveguide number. In the waveguides with dissipation of energy $h_s = \mu/d + j\alpha$, $0 \le \mu \le \pi$, where μ-advanced phase per period d, α is the damping coefficient of wave and equals to $1/l_d$, where l_d is the length on which the wave damps in e times. The transient in cavity can be described by summing up direct and reflected waves. As example, let's write the following expression for any $s-th$ mode through the sum of direct and k times reflected waves:

$$\begin{aligned} E_s = {} & E^0 e^{j\varphi} e^{-z\alpha} e^{j\mu z/d} \\ & (1 + (1 + ... + (1 + e^{j2\mu L_c/d} e^{-2\alpha L_c} \dot{G}_1 \dot{G}_2) \\ & e^{j2\mu L_c/d} e^{-2\alpha L_c} \dot{G}_1 \dot{G}_2)...) e^{j2\mu L_c/d} e^{-2\alpha L_c} \dot{G}_1 \dot{G}_2) \end{aligned} \tag{4}$$

where L_c is length of cavity and $\dot{G}_1, \dot{G}_2$ are the complex coefficients of reflection at ends of cavity. If for $k-th$ mode the resonance condition $\dot{G}_1 * \dot{G}_2 = 1$ and $\mu L_c/d = \pi m, (m = 1, 2, 3...)$ is fulfilled itself then

$$E_s = \frac{1 - e^{-2\alpha L_c k}}{1 - e^{-2\alpha L_c}} E^0 e^{j\phi} e^{-\alpha z} e^{j\mu z/d}, \tag{5}$$

The propagation time of wave front along cavity is $\tau_{space} = L_c/v_{group}$ Taking in account that $\tau_{space} k = t$, and $1/(\alpha * v_{group}) = 2 * Q/w$ we can get $E_k \sim (1 - e^{-t/\tau})$, where $\tau_{time} = 2Q/w$ is the constant of damping of standing wave in time. The relation between τ_{time}, τ_{space} and the beam pulse length τ_{beam} defines four types of accelerating structure:

1. $t_{beam} \gg \tau_{time} \gg \tau_{space}$ is the INR variant.
2. $t_{beam} \gg \tau_{space} \gg \tau_{time}$ is the waveguide variant.
3. $t_{beam} \gg \tau_{space} \sim \tau_{time}$ is the LAMPF variant.
4. $t_{beam} \sim \tau_{space} \sim \tau_{time}$ is the SSC variant.

As it will be shown below this relation determinates the parameters of beam during the transient. During τ_{space} the cavity can be considered as waveguide where the spectrum of modes with μ variation is continuous. Approximately after one-two runs of the waves the modes with discrete μ predominate. The general solution is found as the expansion in a series of eingen functions for cavity in the following form not forgetting that during first τ_{space} we have to integrate over all μ range:

$$E = \sum_s^N A_s(t) E_s(r) e^{-jwt} \tag{6}$$

$$H = \sum_s^N B_s(t) H_s(r) e^{-jwt} \tag{7}$$

Here E_s is the sum of the direct and reflected waves:

$$E_s = E_s^w + E_{-s}^w = E_s^0 cos(\mu z/d), \tag{8}$$

besides μ has additional requirement $\mu N = \pi s$.

$$E_s = E_s^0 cos(\pi s n/N), \tag{9}$$

where n is number of cell. The same is for eigenfunction of magnetic field H_s. The eigen functions E_s and H_s are connected with each other by the Maxwell gomogeneouse equations:

$$rot E_s = j\mu_0 w_s H_s \tag{10}$$

$$rot H_s = -j\varepsilon_0 w_s E_s \tag{11}$$

Now substituting (6,7,10,11) in Maxwell ungomogeneous equations we get

$$\frac{\partial A_s}{\partial t} - jwA_s + jw_s B_s = -\frac{1}{N_s}\int_V j^e E_s^* dv \tag{12}$$

$$\frac{\partial B_s}{\partial t} - jwB_s + jw_s A_s = -\frac{1}{N_s}\int_V j^m H_s^* dv \tag{13}$$

$$N_s = \varepsilon \int_V E_s E_s^* dv \tag{14}$$

where j^m and j^e are magnetic and electric currents. Eliminating A_s and B_s and taking in account that $w_s = w_{0s}(1 - j/Q_s)$ we can receive the following equations:

$$\frac{\partial A_s}{\partial t} + \frac{j(w_{0s}^2 - w^2) + w_s^2/Q_s}{2w} A_s = \tag{15}$$

$$-\frac{1}{2N_s}\int_V j^e E_s^* dv - \frac{w_{s0}}{2N_s w}\int_V j^m H_s^* dv \tag{16}$$

Solving this differential equation we have:

$$A_s = \frac{Q_s/(N_s w_S)[\int_V j^e E_s dv + \int_V j^m H_s dv]}{1 + j2\Delta_s w Q_s/w_s} [1 - e^{-j\Delta_s wt} e^{-w_s t/(2Q_s)}], \tag{17}$$

where $\Delta_s w = w - w_s$..

The equation with B_s is similar. For simplification the norm N_s is determinated as the stored energy in cavity at the nominal level of accelerating field. Then integral $\int_V j^e E_s dv$ is a power of Cherenkov's beam radiation of $s-th$ mode and other integral $\int_V j^m H_s dv$ is the generator power of $s-th$ mode.

It is obviously that for proton linac the beam has to be injected only after stabilizing of electromagnetic field at nominal level ($A_s = 1$ for fundamental mode). Before beam injection full power from generator is spend on support electromagnetic field and goes to losses(P_{losses}). In moment of beam switching the generator has to increase a power on value approximately equals to the beam power. Here we would like to emphasize two features.

1. First moment. Generator inputs power in some point, but beam radiates power distributively almost uniformly along cavity. So if amplitudes j^e and E_s don't depend on the longitudinal coordinate then integral $\int_V j^e E_s dv$ equals to zero for all modes except fundamental mode of beam. And we can suppose that for fundamental mode $P_{generator} = P_{beam}$. In the same time the generator excites all modes proportionally to ratio $RP = P_{beam}/(cos\varphi_s P_{losess})$ This value is usually about 0.3 (LAMPH,INR,SSC).
2. Second moment. The velocity of beam propagation along cavity equals to phase velocity (synchronization condition). But the velocity of the wave front of generator equals to the group velocity ($v_{group} = K_c \pi v_{phase}/4$). K_c is the couple coefficient of structure (for SSC and LAMPF $K_c = 0.05$ for INR $K_c = 0.5$). Besides the coefficient of space damping for a generator wave (K_c) times as much as the coefficient of space damping for Cherenkov's wave excited by beam.

3 THE STEADY REGIME

In general case the excitation of any mode can be estimated by following expression:

$$A_s = \frac{RP(1 - j2\Delta_s wQ_s/w_s)}{1 + 4(\Delta_s wQ_s/w_s)^2}[1 - e^{-j\Delta_s wt} e^{-w_s t/2Q_s}] \tag{18}$$

In the steady regime when $t \gg \tau_{time}$ and $2\Delta_s wQ_s/w_s \gg 1$, then

$$A_s \simeq \frac{RP}{2\Delta_s wQ_s/w_s} \qquad (19)$$

Let's calculate the sum of two neighbor modes nearest to fundamental mode $S = N/2 \pm 1$ which are the slope of field along a cavity with the dispersion relation $w_s = w_{N/2}(1 - K_c/2cos(\pi s/N))$

$$A_{-s+s} \simeq \frac{2RP}{(2\Delta_s wQ_s/w_s)^2} \qquad (20)$$

The last demonstrates the main advantage of structure with $\frac{\pi}{2}$ fundamental mode.
Substituting $\Delta_s w/w = \frac{\pi}{2}\frac{K_c}{N}$ in (20), which is satisfied for nearest modes, get:

$$A_{-s+s} \simeq RP(\frac{\sqrt{2}N}{\pi K_c Q_s})^2 \qquad (21)$$

The slope amplitude equals $2 * A_{-s+s}$As long as the expression between the brackets much less unit the structure with fundamental $\pi/2$ mode saves its advantages and has enough small distortion along cavity. For instance, this slope of field for INR equals $2 * 10^{-4}$, for LAMPF $1 * 10^{-2}$ and SSC $6 * 10^{-2}$.

4 THE TRANSIENT

During the transient $t \sim \tau_{time}$ (for INR,LAMPF,SSC $\tau \simeq 3 \div 5\mu sec$) we have to take in account all expression for A_s.
Eliminating the real part of A_{-s+s} in (18) let 's write the expression for two symmetrical modes:

$$ReA_{-s+s} = RP\frac{sin\Delta_s wt}{\Delta_s wQ_s/w}e^{-w_s t/2Q_s} = \frac{2N}{\pi K_c Q_s}RPsin(\Delta_s wt)e^{-w_s t/2Q_s} \qquad (22)$$

Now calculate the distortion of field along cavity for all modes taking in account its distribution (8)

$$\frac{\delta E}{E}(z,t) = RPe^{-t/\tau}\sum_{s=0}^{N/2}\frac{sin(\Delta_s wt)}{\Delta_s w/w_s Q_s}cos\frac{2\pi sz}{T} \qquad (23)$$

where $T = 2L_c$ and $\tau = 2Q_s/w_s$ is taking approximatly the same for all modes.
Now we can do the convolution of this series using $t = z/v_{group}$ and the linearizated dispersion relation:

$$\frac{\delta E}{E}(z,t) = e^{-t/\tau}\frac{N}{K_c Q}RP\Re(z - v_{group}t) \qquad (24)$$

where $\Re = 1$, if $z \leq v_{group}t$ and 0 if $z \geq v_{group}t$ During τ_{space} we solve this problem through waveguide eingen function $E = \int A_s(z)E_s e^{jh_s z}ds$ and just overwrite the final result:

$$\frac{\delta E}{E}(z) = e^{-t/\tau}wL_{cav}\Re/(2Qv_{group}) \qquad (25)$$

5 CONCLUSION

It is obviously that if we will excite cavity in middle of cavity then $\delta E/E$ will be approximated by series with the less number of modes, but the distortion will remain almost the same. After injection of beam and the simultaneous input of power in cavity three types of wave run alone cavity: wave of generator fundamental mode, Cherencov's wave of beam and additional wave of nonfundamental modes with amplitude proportional to τ_{space}/τ_{time}. The nonfundamental wave being damped proportionally to $e^{-t/\tau_{time}}$ propagates with group velocity. After reflection at end of cavity the wave changes sign every times. Since the beam velocity is much more than the group velocity of the nonfundamental modes wave the beam "sees" significant distortion of field during transient. This effect will give additional momentum spread of beam largely to side of high energy. One may suppose that the beam's excitation of the waves with phase velocity only smaller than the beam velocity will lead to changing of the advanced phase per cell and hence to changing of the structure phase velocity. To decrease this effect it should use the beam pulse with $\tau_{front} \simeq 1/\Delta w_s$. However the beam intensity modulation can lead to the intensity modulation of current in the synchrotron if the injection time is comparable with the beam pulse length in linac. Other way is to use regime with more longer pulse of beam and less loading. Nevertheless in any case the structure with high coupling coefficient improves quality of beam.

6 REFERENCES

[1] Yu.V.Senichev *et al.*,"Problems of the Beam Loss in Intence Ion Linear Accelerators",Proc. of the 1988 Linear Accelerator Conference., CEBAF-Report-89-001, pp.669-671.

[2] Yu.V.Senichev and E.N.Shaposhnikova,"The Effect of Perturbations on the Longitudinal Motion of Particles in a Stepped-Phase-Velocity Linear Accelerator", Particle Accelerator , 1988, Vol. 22,pp.

[3] Yu.V.Senichev and E.N.Shaposhnikova,"The Quasi-Synchronous Motion in a Stepped Phase Velocity Linac", Journal of Technical Physics, V.57, no.6,1987.

[4] K.R.Crandall,"The Δ-T Tune up Procedure for the LAMPF 805-MHz Linac", LAMPF Report, LA-6374-MS ,1976.

[5] Yu.V.Senichev *et al.*,New Features of The ΔT- Procedure for an Intensive Ion Linac",Proc. of the 1988 Linear Accelerator Conference, CEBAF-Report-89-001, pp.666-668.

[6] Yu.V.Senichev and Shaposhnikova,"Problems of Tuning of Ion Linac Consisting of Sections with Constant Phase Velocity," Proc. of the XIII International Conference on High Energy Accelerators, Novosibirsk, V.1,pp.244-249,1987.

[7] E.A. Knapp *et al.*,, "Standing Wave High Energy Linear Accelerator Structures", Rev. Sci. Instrum., Vol.39, No.7,p.979,1968.

[8] D.E.Nagle *et al.*,, "Coupled Resonator Model for Standing Wave Accelerator Tanks", Rev. Sci. Instr. 38, No.11,p.1583-1587,1967.