SPACE CHARGE EFFECT OF BUNCHED BEAM WITH DIFFERENT DENSITY DISTRIBUTIONS IN LINAC

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ABSTRACT

The nonlinear space charge effect due to the nonuniform particle density distribution in bunched beam of a linac is discussed in this paper. The formulae of nonlinear space charge effect are derived for the bunched beam with different density distributions, such as Kapchinskij–Vladimirskij, waterbag, parabolic and Gauss distribution in both of the space charge disk model and space charge cylinder model in the waveguide of a linac.

INTRODUCTION

In high–current beam for Free Electron Laser (FEL) and linear accelerator for high–energy physics and other applications, the space charge force is no longer small compared with the externally applied focusing forces. And the space charge effect is assumed to be one of the fundamental factors governing the beam dynamics. With regard to the current in surrounding structures, many articles have been published. [1–4] However, most of them studied the beam bunch with uniform density distribution except the Ref.[4], in which the general formulae for calculating the space charge effect in a waveguide have been derived.

Furthermore, the nonlinear effect of the space charge is one of the important reasons that induces the emittance growth because of the conversion of field energy to kinetic energy. [5–7] Theoretical study and numerical simulation show that the nonuniform particle distributions have more electrostatic field energy per unit length than that of the equivalent uniform beam with the same current I, RMS radius, and RMS emittance, So, people surmise that this additional field energy is converted into particle kinetic and potential energy (and hence emittance growth) as the distribution tends to become more homogeneous. This concept has been already accepted by some further studies. [8,9] However, it should be pointed it out that above results concerning the calculation of the space charge field energy is based on the continue beam in free space. Therefore, it is necessary to derive the formulae of nonlinear space charge effect for the bunched beam with some common space charge distributions such as Kapchinskij–Vladimirskij (K–V), waterbag (WB), parabolic (PA), and Gauss (GA) distributions in both of the space charge disk model and space charge cylinder model in the waveguide of a linac.

GENERAL FORMULAE

For the convenience of understanding and aplication, here we review the main points of the Ref.[4] in which the general formulae for calculating the space charge effect in a waveguide are obtained.

Assuming the space charge bunch model is central symmetry, we have the potential induced by the space charge with uniform density distribution ρ in a cylindrical coordinate system as follows:

$$\varphi_0 = \rho f_0(r,z;b,L/2), \tag{1}$$

where b and L / 2 are the edges of the model in r and z directions, respectively, f_0 is the potential induced by the unit space charge density and the subscript 0 stands for the uniform density distribution.

Using the Eq.(1), we get the potential induced by the same space charge bunch model, but with nonuniform charge density distribution $\rho(r)$ as

$$\varphi = \int_0^b \rho(\xi)\frac{\partial f_0(r,z;\ \xi,L/2)}{\partial \xi}\,d\xi, \tag{2}$$

Analogously, for the same space charge bunch model with the nonuniform charge distribution $\rho(z)$ one finds the potential as:

$$\varphi = \int_0^{L/2} \rho(\zeta)\frac{\partial f_0(r,z;\ b,\zeta)}{\partial \zeta}\,d\zeta, \tag{3}$$

And ,hence, for the same space charge bunch model with assuming nonuniform charge density distribution $\rho(r,z) = \rho(r)\rho(z)$ we get the potential as follows:

$$\varphi = \int_0^{L/2}\int_0^b \rho(\xi,\zeta)\frac{\partial^2 f_0(r,z;\ \xi,\zeta)}{\partial\zeta\partial\xi}\,d\xi d\zeta. \tag{4}$$

Therefore, according to the Eqs.(2)—(4), the potential induced by the space charge bunch with nonuniform charge distribution can be obtained if the potential induced by the same space charge bunch model but with uniform charge density distribution is known as the Eq.(1).

BUNCHED BEAM WITH DIFFERENT DENSITY DISTRIBUTIONS

Now we apply the Eqs.(2)−−(4) to some common space charge model with different charge density distributions in a linac.

1. Disk model of space charge

We have the potential of uniform disk space charge as [1]

$$\varphi_0 = \frac{\rho b}{\varepsilon_0 a^2} \sum_{l=1}^{\infty} \frac{J_1(k_l b) J_0(k_l r)}{k_l^2 J_1^2(k_l a)} e^{-k_l |z|}, \tag{5}$$

where a is the waveguide radius and b is the disk radius, $J_l(k_l x)$ is Bessel function, and k_l satisfies the equation: $J_0(k_l a) = 0$. Now, we discuss nonuniform space charge density distribution in the disk as follows:

1) K−V distribution

According to the Ref.[5], the charge density in real space with the K−V distribution in phase space is homogeneous, i.e.

$$\rho = \rho_0 = \frac{q}{\pi b^2}, \tag{6}$$

where q is the tatol charge in the disk. Then, the potential induced by the K−V distribution is just as the expression (5), or can be rewritten as:

$$\varphi_0 = \frac{q}{\varepsilon_0 \pi b a^2} \sum_{l=1}^{\infty} \frac{J_1(k_l b) J_0(k_l r)}{k_l^2 J_1^2(k_l a)} e^{-k_l |z|}. \tag{7}$$

2) WB distribution

The charge density in real space with a WB distribution in phase space is parabolic: [5]

$$\rho = \rho_{WB}\left(1 - \frac{r^2}{b^2}\right), \tag{8}$$

where ρ_{WB} can be expressed with the total charge q as $\rho_{WB} = \frac{2q}{\pi b^2}$.

Acoording to the Eq.(2), one yields the potential induced by the WB distribution as follows :

$$\begin{aligned}\varphi &= 2\rho_{WB} \sum_{l=1}^{\infty} \frac{J_2(k_l b) J_0(k_l r)}{\varepsilon_0 a^2 k_l^2 J_1^2(k_l a)} e^{-k_l |z|} \\ &= \frac{4qa}{\varepsilon_0 \pi b^2} \sum_{l=1}^{\infty} \frac{J_2(k_l b) J_0(k_l r)}{(k_l a)^3 J_1^2(k_l a)} e^{-k_l |z|}.\end{aligned} \tag{9}$$

The disk charge with the WB distribution becomes to a point charge when the disk radius $b \to 0$, and hence, the Eq.(9) becomes to

$$\varphi = \sum_{l=1}^{\infty} \frac{q J_0(k_l r)}{2\pi\varepsilon_0 a \quad (k_l a) J_1^2(k_l a)} e^{-k_l |z|}, \tag{10}$$

which agrees with the potential of a point charge in the Ref.[1].

3) PA distribution

The charge density in real space with a PA distribution in phase space has the following form : [5]

$$\rho = \rho_{PA}\left(1 - \frac{r^2}{b^2}\right)^2, \tag{11}$$

where ρ_{PA} can be expressed with the total charge q as $\rho_{PA} = \frac{3q}{\pi b^2}$.

According to the Eq.(2), we obtain the potential induced by the PA distribution as follows :

$$\begin{aligned}\varphi &= 8\rho_{PA} \sum_{l=1}^{\infty} \frac{J_3(k_l b) J_0(k_l r)}{\varepsilon_0 b a^2 k_l^2 J_1^2(k_l a)} e^{-k_l |z|} \\ &= \frac{24qa^2}{\varepsilon_0 \pi b^3} \sum_{l=1}^{\infty} \frac{J_3(k_l b) J_0(k_l r)}{(k_l a)^4 J_1^2(k_l a)} e^{-k_l |z|}.\end{aligned} \tag{12}$$

The disk charge with the PA distribution becomes to a point charge when the disk radius $b \to 0$, and hence, the Eq.(12) also becomes to the Eq.(10) of the potential for a point charge.

4) GA distribution

The charge density in real space with a GA distribution in phase space can be expressed as follows: [5]

$$\rho = \rho_{GA} e^{-\frac{r^2}{2\alpha^2}}, \tag{13}$$

where $\alpha^2 = \langle x^2 \rangle$, and ρ_{GA} can be expressed with the total charge q as $\rho_{GA} = \frac{q}{2\pi\alpha^2}$.

According to the Eq.(2), we get the potential induced by the GA distribution as follows:

$$\begin{aligned}\varphi &= \rho_{GA} \sum_{l=1}^{\infty} \frac{\alpha^2 J_0(k_l r)}{\varepsilon_0 a^2 k_l J_1^2(k_l a)} e^{-\frac{k_l^2 \alpha^2}{2}} e^{-k_l |z|} \\ &= \frac{q}{2\pi\varepsilon_0 a} \sum_{l=1}^{\infty} \frac{J_0(k_l r)}{(k_l a) J_1^2(k_l a)} e^{-\frac{k_l^2 \alpha^2}{2}} e^{-k_l |z|}.\end{aligned} \tag{14}$$

The disk charge with the GA distribution becomes to a point charge when the disk radius $b \to 0$, (and hence $\alpha \to 0$ too), and, then we have the expected point charge potential from the Eq.(14).

It should be pointed out that, we expanded the upper limit b of the integral into ∞ in the above procedure. This is reasonable due to the character of Gauss distribution of the beam.

2. Cylinder model of space charge

We have the potential of this cylinder space charge as [1]

$$\left.\begin{aligned}\varphi_{1,2}&=\frac{2\rho b}{\varepsilon_0 a^2}\sum_{l=1}^{\infty}\frac{J_1(k_l b)J_0(k_l r)}{k_l^2 J_1^2(k_l a)}sh\frac{k_l L}{2}e^{-k_l|z|}, \quad (|z|>\frac{L}{2}),\\ \varphi_3&=\frac{2\rho b}{\varepsilon_0 a^2}\sum_{l=1}^{\infty}\frac{J_1(k_l b)J_0(k_l r)}{k_l^2 J_1^2(k_l a)}(1-e^{-\frac{k_l L}{2}}chk_l z), \quad (|z|<\frac{L}{2}).\end{aligned}\right\} \quad (15)$$

where b and L are the radius and length of the cylinder, respectively.

By the procedure analogous to that of deducing the above disk model of space charge and taking notice of the relationship between the charge density ρ and the total charge q in the cylinder model of space charge, we obtain the potentials of the space charge cylinder with K–V, WB, PA, GA distributions in the following:

1) K–V distribution

The potential induced by the K–V charge density distribution is just as the expression (15), or can be rewritten as:

$$\left.\begin{aligned}\varphi_{1,2}&=\frac{2qa}{\pi\varepsilon_0 bL}\sum_{l=1}^{\infty}\frac{J_1(k_l b)J_0(k_l r)}{(k_l a)^3 J_1^2(k_l a)}sh\frac{k_l L}{2}e^{-k_l|z|}, \quad (|z|>\frac{L}{2}),\\ \varphi_3&=\frac{2qa}{\pi\varepsilon_0 bL}\sum_{l=1}^{\infty}\frac{J_1(k_l b)J_0(k_l r)}{(k_l a)^3 J_1^2(k_l a)}(1-e^{-\frac{k_l L}{2}}chk_l z), \quad (|z|<\frac{L}{2}).\end{aligned}\right\} \quad (16)$$

2) WB distribution

The potential induced by the WB charge density distribution is

$$\left.\begin{aligned}\varphi_{1,2}&=\frac{8qa^2}{\pi\varepsilon_0 b^2 L}\sum_{l=1}^{\infty}\frac{J_2(k_l b)J_0(k_l r)}{(k_l a)^4 J_1^2(k_l a)}sh\frac{k_l L}{2}e^{-k_l|z|}, \quad (|z|>\frac{L}{2}),\\ \rho_3&=\frac{8qa^2}{\pi\varepsilon_0 b^2 L}\sum_{l=1}^{\infty}\frac{J_2(k_l b)J_0(k_l r)}{(k_l a)^4 J_1^2(k_l a)}(1-e^{-\frac{k_l L}{2}}chk_l z), \quad (|z|<\frac{L}{2}).\end{aligned}\right\} \quad (17)$$

3) PA distribution

The potential induced by the PA charge density distribution is

$$\left.\begin{aligned}\varphi_{1,2}&=\frac{48qa^3}{\pi\varepsilon_0 b^3 L}\sum_{l=1}^{\infty}\frac{J_3(k_l b)J_0(k_l r)}{(k_l a)^5 J_1^2(k_l a)}sh\frac{k_l L}{2}e^{-k_l|z|}, \quad (|z|>\frac{L}{2}),\\ \varphi_3&=\frac{48qa^3}{\pi\varepsilon_0 b^3 L}\sum_{l=1}^{\infty}\frac{J_3(k_l b)J_0(k_l r)}{(k_l a)^5 J_1^2(k_l a)}(1-e^{-\frac{k_l L}{2}}chk_l z), \quad (|z|<\frac{L}{2}).\end{aligned}\right\} \quad (18)$$

4) GA distributon

The potential induced by the GA charge density distribution is

$$\left.\begin{aligned}\varphi_{1,2}&=\frac{q}{\pi\varepsilon_0 L}\sum_{l=1}^{\infty}\frac{J_0(k_l r)}{(k_l a)^2 J_1^2(k_l a)}e^{-\frac{k_l^2 a^2}{2}}sh\frac{k_l L}{2}e^{-k_l|z|}, \quad (|z|>\frac{L}{2}),\\ \varphi_3&=\frac{q}{\pi\varepsilon_0 L}\sum_{l=1}^{\infty}\frac{J_0(k_l r)}{(k_l a)^2 J_1^2(k_l a)}e^{-\frac{k_l^2 a^2}{2}}(1-e^{-\frac{k_l L}{2}}chk_l z), \quad (|z|<\frac{L}{2}).\end{aligned}\right\} \quad (19)$$

Obviously, the cylinder charge becomes to a disk charge when the cylinder length $L\to 0$. And hence the above potential formulae Eqs.(16), (17), (18) and (19) become to the potentials of Eq.(7), Eq.(9), Eq.(12), and Eq.(14) of the disk space charge density with K–V, WB, PA and GA distributions, respectively.

It should be pointed out that all the formulae are derived here in the frame of reference moving with the space charge bunch in the same velocity. However, the motion of the space charge bunch in the longitudinal direction can be relativistic in a linac. Therefore, the formulae should be transformed into that in laboratory system according to the Ref.[3].

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