# Influence of a Wavelength Shifter on the Performence of the Synchrotron Light Source BESSY II

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## 1 INTRODUCTION

The planned storage ring BESSY II [1] is a synchrotron radiation source with a low emittance lattice that is optimized for the operation of insertion devices. To study especially the linear and nonlinear effects of a superconducting wavelength shifter on the dynamical aperture we developed software tools for fast canonical tracking calculations based on generating functions. For a simple wavelength-shifter model we derived analytical formulas to describe the change of the natural emittance and the beam polarization and compared the results with numerical calculations. In this paper we summarize results of the technical report [2].

## 2 CANONICAL TRACKING CALCULATIONS

A general canonical tracking routine, based on a generating function, for particle motion in arbitray magnetic fields is presented by J.Bahrdt and G.Wüstefeld [3]. However, the applied expansion shows slow convergence in strong fields, where the typical bending radius of the particle trajectory is small. In the case of our WLS the bending radius can be smaller than $0.5m$. Therefore, a different method that is better suited for strong linear focussing terms is applied for WLSs. We have written a computer code that tracks a set of particles through the WLS by an integration method and fits numerically the Taylor-expansion

$$F(q_{x_i}, p_{x_f}, q_{y_i}, p_{y_f}) = \sum_{k+l+m+n=1}^{M} a_{klmn}\, q_{x_i}^{k} p_{x_f}^{l} q_{y_i}^{m} p_{y_f}^{n} + \cdots$$

of the generating function. The expansion is done with respect to the canonical variables. Let $Q_{x_i}\, P_{x_i}\, Q_{x_f}$ $P_{x_f}\, Q_{y_i}\, P_{y_i}\, Q_{y_f}$ and $P_{y_f}$ be the canonical variables of a set of tracked particles and $q_{x_f}\, p_{x_i}\, q_{y_f}$ and $p_{y_i}$ the variables as calculated from the generating function. We define those coefficients to be the best representation of the $a_{klmn}$ such that the variable $\chi^2$, given by

$$\chi^2 = \frac{1}{N}\sum_{\nu=1}^{N}[(q_{x_f,\nu} - Q_{x_f,\nu})^2 + (p_{x_i,\nu} - P_{x_i,\nu})^2 + (q_{y_f,\nu} - Q_{y_f,\nu})^2 + (p_{y_i,\nu} - P_{y_i,\nu})^2],$$

is minimized. The number of the tracked particles is denoted by N. The expansion up to the $4th$ order is normally sufficiently accurate and includes octupole-like terms of the generating function. The coefficents of the generating function are used by a canonical tracking routine that bypasses the internal insertion-device-tracking routine of BETA [5].

Divided by the mean length of the vector $(q_{x_f}, p_{x_i}, q_{y_f}, p_{y_i})$, $\chi$ provides a criterion for the convergence of the expansion. It gives the error between the canonical variables calculated from the generating function and those from the integration routine. For a typical WLS and a beam energy of $1.7\ GeV$, a $4th$ order expansion yields errors of less than $10^{-5}$. A $6th$ order expansion gives values of about $10^{-7}$. Thus, using the generating function, we can track a particle with an acceptable precision in a single step through a WLS. A high precision tracking is in fact required because the tracking calculations deal with up to $10^4$ turns, and errors might accumulate. If we use only the $2nd$ order terms of the generating function, i.e. a linear mapping of the canonical variables, the error reaches values of the order $10^{-3}$. Thus the wavelength shifters considered behave within this limits as linear devices. The nonlinearities increase if, for example, we look at the BESSY I energy of $0.8\ GeV$ where they reach values of about 1-1.5 percent.

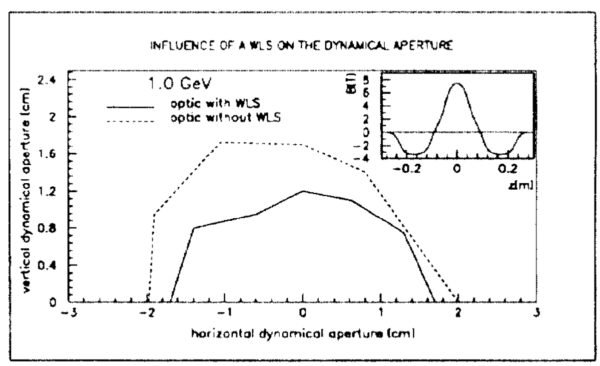


Fig. 1: Dynamical aperture of the BESSY II optics including a WLS proposed by Kulipanov et al.[6]; magnets errors are taken into account; 1000 stable turns are demanded. The field of the WLS is shown in the upper right corner.

The WLS introduces a vertical focusing that must be compensated by an appropriate setting of the quadrupoles adjacent to the WLS. This leads to a strong breaking of the symmetry and causes a reduction of the dynamic aperture. Furthermore, the detuning of the optics disturbs the sextupole compensation scheme and enhances, together with the nonlineare field of the WLS, the nonlinearities of the optics. The resulting dynamical aperture is still acceptable

(Fig. 1). The tracking calculations were performed using BETA with our canonical tracking routine implemented. The field is taken from a Maxwellian conformal two dimensional approximation of a device that is proposed by Kulipanov et al. [6]. The beam energy of $1\,GeV$ was chosen to get an upper limit for the reduction of the aperture, because BESSY II is planned to run also at energies lower than the nominal $1.7\,GeV$.

## 3 EMITTANCE CHANGE BY A WLS

The operation of strong WLSs in a low emittance lattice requires a careful adjustment of both the storage ring and WLS parameters. The transversal natural emittance $\epsilon_n$ for a storage ring is calculated by evaluating the radiation integrals [4] along the ring circumference. We are explicitly interested in the integrals $I_2$ and $I_5$ (neglecting $I_4 << I_2$), which are given by:

$$\begin{aligned} I_2 &= \oint ds\,/\,\rho^2 \quad , \quad I_5 = \oint H\,/\,|\rho^3|\;ds \qquad \text{with} \\ H &= \{\eta^2 + (\beta\eta' - \beta'\eta\,/\,2)^2\}\,/\,\beta \end{aligned}$$

The natural emittance is then calculated by:

$$\epsilon_n \;=\; C_q\,\gamma^2 \sum I_{5i} / \sum I_{2i}$$

with the relativistic parameter $\gamma$ and the quantum constant $C_q = 3.832 \cdot 10^{-13} m$. The summation is done over all devices $i$ in the ring. We can rearrange the summation to write it in the form of a weighted mean:

$$\sum I_{5i} / \sum I_{2i} \;=\; \sum I_{2i} \left(\frac{I_5}{I_2}\right)_i / \sum I_{2i}$$

which shows how the ratio $(I_5/I_2)_i$ of a given device contributes to the size of the natural emittance. The weight of this contribution is $I_{2i}$, which is proportional to the radiated power of that device. If $(I_5/I_2)_i$ of a device is smaller than the average value, the overall ring emittance decreases. In this case we can add several devices of this type at places with comparable beta and dispersion functions without increasing the ring emittance. A value $(I_5/I_2)_i$ larger than the average would not increase the natural emittance much if $I_{2i}$ is small compared to $\sum I_{2i}$. A WLS gives a relative strong contribution to $I_2$ of the ring - about 10% for the BESSY II optics.

The dependence of the low emittance optics on a strong WLS field was studied analytically by a simple WLS model and also by explicit numerical integration using realistic WLSs. Only the analytical results are discussed here. The WLS model (Fig. 2) is composed of a strong central pole and two end poles. The shape of the field is given by *COS*-functions, with a period length $\lambda_0$ and a minimal bending radius $\rho_0$ for the main pole. The maximal strength of the end pole is reduced by a factor $n$ which yields a bending radius of $-\rho_0 n$. The length is adjusted to cancel the overall dipole moment of the WLS. Three parameters are sufficent to describe this WLS model.

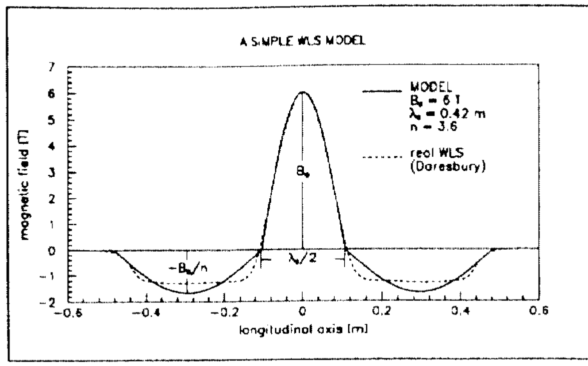


Fig. 2: Magnetic field of the simple WLS model with parameters $B_0$, $\lambda_0$ and $n$

For a gradient free WLS, the horizontal beta function, symmetrically centered in the WLS, is approximated by a drift transformation:

$$\beta \;=\; \beta_0 + s^2/\beta_0$$

where $s$ is the longitudinal coordinate. The internal dispersion $\eta_i$ of the WLS is easily calculated if the horizontal slope $x'^2 << 1$ of the particle trajectory is negligible. We obtain :

$$x'' = -1/\rho(p) = -1/\rho(p_0)(1 - \Delta p/p_0) \equiv x_0'' + \eta_i'' \Delta p/p_0.$$

Integrating twice we get $\eta_i = -x_0 + \eta_e$, where $\eta_e$ is an external, arbitrary dispersion offset and $x_0$ the closed orbit bump.

These relations for the optical functions are used to calculate $I_2$ and $I_5/I_2$ for the simple WLS model. We obtain:

$$\begin{aligned} I_2 &= \lambda_0\,(n+1)\,/\,(4\rho_0^2 n)\,; \\ I_5/I_2 &= (A+D)/\beta_0 + B\beta_0. \end{aligned}$$

With the abbreviation $\theta = \lambda_0/\rho_0$ for the WLS parameters and $\tilde{\eta_e} = \eta_e/(\theta^2\rho_0)$ for the external dispersion offset the coefficients are:

$$\begin{aligned} A &= 0.0003360\,\theta^3\,\lambda_0\,(n^4 + 2.813n^3 + 2.471n^2 \\ &\quad +1.497n + 1.2)/(n^2+n) \\ B &= 0.004300\,(\theta^3/\lambda_0)\,(n^2+1.5)/(n^2+n) \\ D &= 0.03377\,\theta^3\,\lambda_0(n^3 + 25.13n^2\tilde{\eta_e} + 1.406n^2 \\ &\quad +0.6484n + 25.13\tilde{\eta_e} + 1)\tilde{\eta_e}/(n^2+n). \end{aligned}$$

Only the coefficient $D$ dependends on the external dispersion. Using these results we are able to derive a minimization procedure for $I_5/I_2$. We see that for small $\beta_0$ the result becomes sensitive to the dispersion-dependent term $D$. Actually, $D$ is a second order function in $\eta_e$, and $I_5/I_2$ can be minimized by adjusting the external dispersion $\eta_e$. The best value $D_{op}$ is obtained when the external dispersion just cancels the internal dispersion of the WLS around the center of the strong field. This requires only some few centimeters for $\eta_e$. The expression for $I_5/I_2$ can be further minimized with respect to $\beta_0$. We obtain:

$$(I_5/I_2)_{min} \;=\; 2\sqrt{(A + D_{op})\,B}\,.$$

Performing some manipulations we can decompose the right hand side into a form factor $F_W$ which is dependent only on the parameter n:

$$F_W = 0.001065(1 + 0.2369/n + 0.3691/n^2 + \ldots)$$

and the field strength dependent factor $\theta$. The minimum value of $I_5/I_2$ is simply given by

$$(I_5/I_2)_{min} = F_W \theta^3.$$

This result shows the strong dependence of $(I_5/I_2)_{min}$ on the WLS parameter $\theta$. This minimum is only obtained for the optimized beta function

$$\beta_{op} = \sqrt{(A + D_{op}) / B}.$$

The optimized beta function $\beta_{op}$ can be rather small - about $7\,cm$ for $\lambda_0 = 0.4m$. We define a form factor for the beta function and write:

$$\begin{aligned} \beta_{op} &= F_B \lambda_0 \quad \text{with} \\ F_B &= 0.1238(1 + 1.237/n - 0.9000/n^2 + \ldots). \end{aligned}$$

The last results are derived in analogy to minimum emittance considerations for dipoles, where different form factors for beta and $I_5/I_2$ are given [4], dependent on the lattice parameters.

If the beta function is not optimized we have:

$$I_5/I_2 = (I_5/I_2)_{min} \left[ \frac{\beta_0}{\beta_{op}} + \frac{\beta_{op}}{\beta_0} \right] / 2$$

For a large beta function $\beta_0$ and a small dispersion $\eta_e$ the second term in the bracket can be ignored. The overall result for $I_5/I_2$ is approximated by

$$I_5/I_2 = \beta_0 B.$$

It is independent of the external dispersion, but dependent on $\eta'$.

A numerical example for a WLS in the BESSY II optics at $1.7\,GeV$ is given in Fig. 3. Depending on the beta function $\beta_0$ in the WLS center the value $I_5/I_2$ is calculated for $\eta_e = 0$ and for the optimal $\eta_e = -1.8cm$. The value $I_5/I_2 = 0.00147$ of the ring dipoles is also given. At the intersection of this straight line and the corresponding WLS curves, the WLS does not change the overall emittance. This point is reached for a beta function of about $1.7m$, a value which can be realized in an appropriate lattice design. Smaller beta functions lead to a still better emittance. If care is taken with respect to these considerations several WLSs can be included without increasing the low emittance of the machine.

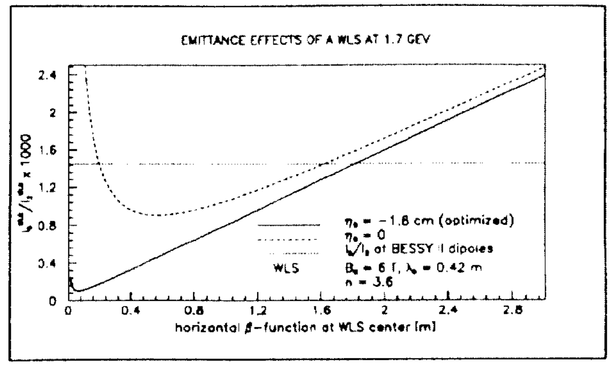


Fig. 3: Ratio $I_5^{WLS}/I_2^{WLS}$ of the WLS which contributes to the total emittance according to its weight $I_2^{WLS}$

## 4 BEAM POLARIZATION

In dedicated runs BESSY II will be operated at low beam energies of about $1\,GeV$. The WLS is planned to reduce the beam polarization time in these runs. An effective reduction of the polarization time can be achieved if $\int |B|^3 ds$ of the WLS field is large. At the same time a high degree of polarization can be retained if $\int B^3 ds$ of the WLS field approaches $\int |B|^3 ds$. This implies weak end poles for the WLS. However, a WLS optimized for the reduction of the polarization time at $1\,GeV$ may cause a significant increase of the emittance. Thus the design of the WLS for BESSY II must be a compromise with respect to the beam polarization and the emittance. Making use of the simple WLS model the best compromise can be found by a systematic variation of the model parameters.

## 5 CONCLUSION

Canonical tracking calculations show that the reduction of the dynamical aperture of BESSY II due to a WLS is mainly caused by linear effects. The resulting aperture was found to be acceptable. The natural emittance of the machine might be significantly affected by the WLS. However, an appropriate optics setting allows the low emittance features of BESSY II to be retained.

## 6 REFERENCES


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