GAIN CALCULATIONS FOR THE SYNCHROTRON RADIATION FREE ELECTRON MASER

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Abstract

The gain of the Synchrotron Radiation Free Electron Maser (SRFEM) is studied theoretically and examples for application of SRFEM at compact electron storage rings are considered. The SRFEM scheme that was proposed recently is based on the interaction of the electrons with the field a two-mirror resonator into which the electron ring is partially plunged. It is found that at energies < 100 MeV for typical ring parameters growth rate of several percent is possible for generation of submillimeter waves. When using the parameters of "AURORA" growth rate of about one percent may be obtained under strict requirements on the resonator characteristics.

1 INTRODUCTION

In the recently proposed [1] scheme of a free-electron maser a two-mirror resonator is used to accumulate the synchrotron radiation of a weqk-focused relativistic electron ring. The configuration of such a maser is given in Fig.1.

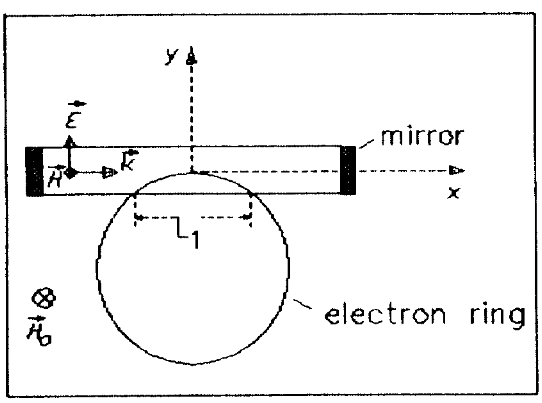

Fig.1

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The electron orbit which is taken to be an exact circle is partially plunged into a resonator composed of two parallel mirrors so that the radiation is reflected back tangentially to the ring. Calculations showed that a self-excitation and corresponding growth of the resonator field are possible under appropriate choice of the parameters: ring radius, electron energy and plunge depth of the electron orbit. This can be used either for generation of submillimeter and millimeter waves or for cooling of the ring.

We first consider a single pass trough the resonator. We average the energy losses of the electron in the combined static and radiation fields over an uniform distribution of electrons at the resonator entrance. Thus obtained one-turn efficiency (mean energy losses) is positive within some range of parameters that we refer to as "generation zone". Stimulated radiation exists during the first turn if the parameters are matched to be inside the generation zone.

Coming now to the problem of many-turns interaction, we first prove that the electron distributions at every successive input can be taken uniform due to the phase mixing during the motion out of the resonator [1]. Each successive turn then leads only to a small shift of the averaged parameters, sothe result obtained for the one-turn case can be simply used to estimate the many-turns energy losses. If for simplicity one takes the d.c. magnetic field to be time independent and the ring center to be fixed then the ring radius decreases continuously.

Finally the gain is estimated and two examples are given for cooling: first - a small ring used for collective acceleration studies [3] and second - the compact synchrotron light source "AURORA" [4].

2 SINGLE PARTICLE ENERGY-LOSSES AND ONE-TURN EFFICIENCY

At the resonator entrance all electrons are assumed to be at radius R_1 (Fig.1) and to have energy $E_1=\gamma_1 mc^2$; the cyclotron angular frequency is then $\omega_{H1}=eH_0/(mc\gamma_1)$. Here m,e are the electronic rest mass and charge; H_0 is the static magnetic field; index 1 means first passage trough the resonator. All of the formulae assume an ideal electron ring (there are no radial and energy distributions) and an ideal resonator field geometry (Fig.1). The resonator field is described as a linearly polarized standing wave and can be presented as a sum of synchronous and oncoming waves:

$$A_y=A\sin(\omega t)\sin(kx) = (A/2)(\cos(\omega t-kx) - \cos(\omega t+kx)), \quad (1)$$

where A_y is the vector potential; $k=\omega/c$.

When considering the single electron dynamics we use that the projection P_y of the canonical momentum on the y-axis conserves. From $P_y=0$ it follows that

$$p_y = (e/c)(A_y + H_0 x), \quad (2)$$

where $\vec{p}$ is the electron momentum.

Since $cp_y \ll E$ we can write

$$p_x \approx (E/c)\left(1-(mc^2+c^2p_y^2)/(2E^2)\right). \quad (3)$$

Equations describing the energy E and the time t are:

$$dE/dx=(e/c)(p_y/p_x)\partial A_y/\partial t, \quad dt/dx=E/(c^2p_x). \quad (4)$$

After substituting (1),(2), (3) into (4) we obtain for the electron energy losses W and phase φ with respect to the synchronous wave

$$\frac{dW}{dx} = \frac{1}{1-W}\left(\alpha\rho\tilde{x}\sin(\varphi)+(1/4)\alpha^2\sin(2\varphi)\right) \quad (5)$$

$$\frac{d\varphi}{dx} = \frac{1}{(1-W)^2}\left(1+\rho^2\tilde{x}^2+\alpha\rho\tilde{x}\cos(\varphi) + (1/8)\alpha^2(2+\cos(2\varphi))\right). \quad (6)$$

$$W(\tilde{x}=-\tilde{L}_1/2)=0 \ ; \quad \varphi(\tilde{x}=-\tilde{L}_1/2)=\varphi_1 .$$

Here: $W \equiv 1-E/E_1$ $\quad \rho\equiv (2eH_0\gamma_1^2)/(mc\omega)$

$\varphi \equiv \omega t-kx$ $\quad \tilde{x}\equiv kx/(2\gamma_1^2)$

$\alpha\equiv eA/(mc^2)$ $\quad \tilde{L}_1\equiv kL_1/(2\gamma_1^2)$;

The one-turn efficiency is

$$\eta_1(\alpha,\rho,\tilde{L}_1)= \langle W\rangle_{\varphi_1}, \text{ where } \langle\rangle_\varphi = \frac{1}{2\pi}\int_0^{2\pi} d\varphi .$$

The Eqs. /5/,/6/ were solved by perturbation method for $\alpha\rho \ll 1$ and numerically for arbitrary α,ρ. For a fixed optical-wave amplitude, η_1 is positive inside some region of the $(\rho,\tilde{L}_1)$-plane (generation zone). Fig.2 shows $\eta_1(0.1,\rho,\tilde{L}_1)$ inside the generation zone. The border of the zone is described by the condition of constant cinematic phase shift (phase shift caused by the curvature of the trajectory; the corresponding term in /6/ is $1+\rho^2\tilde{x}^2$):

$$\tilde{L}_1 + 1/12\ \rho^2\ \tilde{L}_1^3 = 4.9,$$

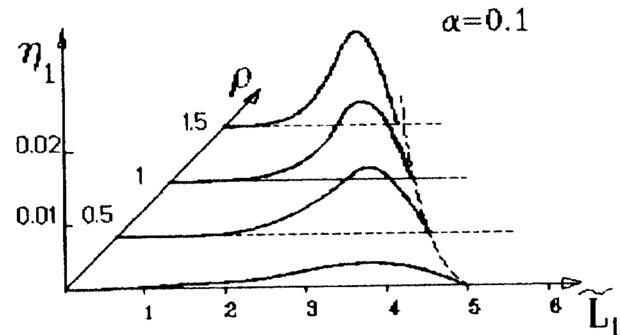

Fig. 2

The parameters $\rho,\tilde{L}_1$ should be chosen on the left and close to the maximum of η_1 (Fig.2). If so, the slope of the curve $\eta_1=\eta_1(\tilde{L}_1)$ ensures stability when dealing with a real ring. Optimal values of the parameters correspond to high efficiencies and high optical field amplitudes. Thus the absolute maximum of η_1 is ~30% for $\alpha=2$, $\rho=12$, $\tilde{L}_1=0.6$.

2 MANY-TURNS ENERGY LOSSES

Here we quote briefly the results in [1]. We assume point-like energy losses at each pass trough the resonator. Consider at the i-th turn (i=1,2,3,...) an electron having energy γ_i and moving on radius R_i (corresponding interaction length L_i) which is entering into the resonator in a phase φ_i. At the next input

$$\varphi_{i+1}=\varphi_i+2\pi s\gamma_{i+1}/\gamma_i ; \quad \gamma_{i+1}= (1-W_i)\gamma_i .$$

Here $s=\omega/\omega_{H1}$; $\tilde{L}_i=kL_i/(2\gamma_i^2)$; $W_i=W(\tilde{L}_i,\varphi_i)$ is the small-signal solution of (5), (6).

The many-turn efficiency (mean energy losses after N turns) is [1]

$$\eta_N \equiv \langle 1-\gamma_N/\gamma_1 \rangle_{\varphi_1} = 1-\langle \prod_{i=1}^{N-1} (1-W_i) \rangle_{\varphi_1} \approx \sum_{i=1}^{N-1} \langle W_i \rangle_{\varphi_1}$$

where

$$\langle W_i \rangle_{\varphi_1} = \alpha \rho m \langle \sin(\varphi_i+\delta) \rangle_{\varphi_1} + \eta_1(\alpha,\rho,\tilde{L}_i), \quad (7)$$

m and δ being roughly constant on each turn. As it was pointed above we neglect the first term which corresponds to bunching in the optical-field-free part of the trajectory. An estimation of the oscillating integral carried out for i=2 by the method of stationary phase shows that the first term is about $\alpha s^{1/2}$ times smaller; (s>>1). We have then

$$\eta_N = \sum_{i=1}^{N-1} \eta_1(\alpha,\rho,\tilde{L}_i) \approx N \max_{\tilde{L}_1} \eta_1(\alpha,\rho,\tilde{L}_1) \quad (8)$$

So if one takes the starting parameters near the maximum of $\eta_1(\rho,\tilde{L}_1)$ then during the first several turns η_N increases almost linearly.

3 CALCULATION OF THE SMALL-SIGNAL GAIN

A part of the stored energy can be extracted trough a small hole in one of the mirrors. The reflection coefficients of the mirrors are then $r_1<1$ and $r_2 \approx 1$.

We denote: S - area of one of the mirror plates; L_{res}- distance between the mirrors; N_e-total number of electrons in the ring; $T= 2\pi R_1/c$; $\varepsilon_v = k^2A^2/(4\pi)$- energy/volume of the optical field; $\varepsilon_{res}= S L_{res}\varepsilon_v$- total energy stored into the resonator.

Since a linear increase of η_N as in (8) is assumed the power filling the resonator is

$$P_{in}= N_e mc^2 \gamma_1 \eta_1(\alpha,\rho,\tilde{L}_1)/T .$$

The radiated power is $P_{rad}=S \varepsilon_v c(1-r_1)$.

The growth rate of the optical field is

$$\Gamma = (P_{in}-P_{rad})/\varepsilon_{res}.$$

The round-trip gain is then

$$G= \Gamma T = N_e mc^2 \gamma_1 \eta_1(\alpha,\rho,\tilde{L}_1)/\varepsilon_{res} - \omega T/Q \quad (9)$$

In the cooling regime (Q= ∞) from (9) we get

$$G_c = N mc^2 \gamma_1 \eta_1/ \varepsilon_{res}= = 0.9\times10^{-13} N_e \gamma_1 \lambda^2 \eta_1(\alpha,\rho,\tilde{L}_1)/(\alpha^2 L_{res}S) \quad (10)$$

Here $\lambda=\pi R_1 \rho/\gamma_1^3$ is the radiated wavelength.

4 EXAMPLES

4.1 First example (low energy ring)

Ring radius R_1=10cm; energy γ_1=10; number of electrons $N_e=10^{12}$; L_1=2.8cm;$L_{res}=3L_1$ (the ring is plunged in a depth δ=0.1cm); reflecting area of the mirror $S\approx 0.5\delta^2$= =0.005cm²; λ=0.046cm. The corresponding point in the $\rho,\tilde{L}_1$-plane is ρ=1.5, $\tilde{L}_1$=1.9 and the one-turn efficiency is $\eta_1 \approx \alpha^2$ for α=0.1 (6MV/cm). From (10) we obtain cooling round-trip gain $G_c \approx 4\%$.

4.2 Second example ("AURORA" case)

Ring radius R_1=50cm; energy γ_1=630; number of electrons $N_e=2\times10^{10}$; L_1=6.3cm; $L_{res} \approx L_1$ (the ring is plunged in a depth δ=0.1cm); reflecting area of the mirror $S\approx 0.5\delta^2$= =0.005cm²; λ=0.012cm. The corresponding point in the $\rho,\tilde{L}_1$-plane is $\rho=2\times10^4$, $\tilde{L}_1=4\times10^{-3}$ and the ratio η_1/α^2 is about 1.5 for $\alpha=10^{-4}$ (30kV/cm). From (10) we obtain cooling round-trip gain $G_c \approx 1\%$.

High electron energies (γ>100) require high working values of ρ (>10^2), and hence, very low beam transversal dimensions (the generation zone becomes narrower - Fig.2).

5 REFERENCES

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