Alternative Transitionless Racetrack Lattices for Low-Energy Synchrotron

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Abstract

Design of lattices for low-energy high intensity synchrotrons is discussed. The main problem for design is to satisfy simultaneously requirements of achieving high enough transition energy γ_t and sufficiently large dynamic aperture in presence of chromaticity correction. The lack of space in small rings often puts meeting these requirements no so obvious. A racetrack missing magnet lattice with modulated beta function having four superperiods in each arc, with 3 or 4 FODO cells in each superperiod, seems to be most successful for low-energy synchrotrons.

1 INTRODUCTION

The typical low-energy synchrotrons (LES) are called to accelerate $0.6 \div 1$ GeV proton beam to $3 \div 10$ GeV. The top energy of LES is determined by either it's purpose or/and by the cost's minimization requirement. Recently proposed kaon factories [1] — high intensity fast cycling synchrotrons — all involve intermediate booster synchrotrons with energies $3 \div 10$ GeV with the aim to raise the injection energy of main synchrotrons. The injector complex of SSC is also includes the low-energy booster (LEB).

Lattice design for any LES runs into the same requirements, which are directed toward minimizing beam loss, controlling space change detuning and eliminating source of beam instabilities. Since transition crossing is an important factor limiting beam current, the one requirement is that lattice should be designed in such a way that transition crossing has never to be happen. In general, it can be met for LES by putting the transition energy γ_t far enough from top energy.

Another possible source of beam loss is a coupling between the radial and longitudinal phase motion reducing the usable space in the tune diagram. The main source of coupling are the dispersion in rf cavities. Therefore, one should make straight sections filled with rf cavities free of dispersion or at least keep dispersion low.

In order to achieve high current, beam emittances just after injection have to be chosen such that space-charge tune shift ΔQ does not exceed $0.2 \div 0.25$. This condition implies that emittances must be considerably large for high intensity synchrotrons. Consequently, they have rather a large aperture and phase-space distortion due to nonlinearities, especially at large betatron amplitudes, may give the betatron instabilities. To meet design performance, lattice, therefore, must have sufficiently large dynamic aperture at least for full chromaticity correction.

Thus we summarize the common requirements that have to be met in design of low-energy synchrotrons, especially those that accelerate a high current proton beam. A lattice

- must have high enough transition energy: $\gamma_t > 10 \div 20$;
- should be suitable for accelerating intensive proton beams and non-intensive polarized beams up to $3 \div 10$ GeV;
- must have sufficiently large dynamic aperture at least for full chromaticity-corrected ring;
- must have adequate space for beam transfer, correction, monitoring, *etc.* hardware.

This report along with a company paper [2] is a result of the study we have done in a search of improved lattices for the Booster of the Moscow Kaon Factory [3] and for the TRIUMF KAON booster [4].

2 CHOICE OF RING STRUCTURE

2.1 *Circular or racetrack shape?*

Design of an accelerator has to meet the requirement of providing lattice with free spaces for beam transfer hardware and rf cavities. This can be met in a lattice having a circular shape of the ring with relatively large number of superperiods ($S \sim 6 \div 8$), each of them can be designed as a simple array of cells. A modulation of the bending radius ρ by missing magnet cells, in which hardware is located, leads to changes in γ_t. Hence the requirement of avoiding transition crossing can be met in a straightforward way.

Providing only two long enough straight sections, *i.e.* a racetrack lattice, is often more preferable as, in general, leads to the shorter circumference. Besides, in a racetrack lattice the dispersion suppression in the straight sections where rf cavities are located can be easily done either by using dispersion suppressors in regular arcs or by setting horizontal tune q_x of each superperiodic arc to an integer number (first order achromat) [5]. Besides the transfer system looks more simple for design. However, a racetrack lattice being two-fold superperiodic could be more sensitive to structural betatron resonances than a circular lattice having higher superperiodicity. This only drawback of racetrack lattices could result in the problem of obtaining large enough dynamic aperture.

2.2 How to avoid the transition crossing?

It is well known, in order to prevent the transition crossing the structure of arcs has to be either regular with a high betatron tune in horizontal plane $\gamma_t \approx Q_x$ or superperiodic [6,2].

Regular lattice
Obviously, a lattice with a high betatron tune has to have either a high number of cells or a large beta function. Therefore, such kind of lattice can be used only for design LES with higher energy (10 GeV) [7]

Superperiodic lattice In this method a superperiodicity S are introduced in arc's structure that can be done either by modulation of the beta function, or by modulation of the bending radius ρ or both simultaneously [6]. It can be easily shown that to achieve high γ_t and in order to avoid high peaks of the lattice functions, the tune of each superperiod $\nu_x = q_x/S$ should be near 1. Therefore, the optimal value of q_x is $S - 1$ and,

$$\nu_x = 1 - 1/S. \tag{1}$$

2.3 β-modulation scheme

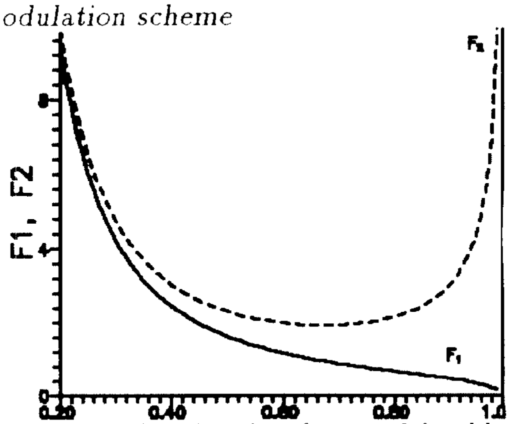

Figure 1: Peak lattice functions in a beta-modulated lattice.

The modulation of the beta function is created by introducing perturbation in quadrupole strengths of regular cells [6,5]. In Fig. 1 the variation of the parameters $\mathcal{F}_1, \mathcal{F}_2$, computed by using an analytic approach ref. [8], with betatron tune are shown for the case of $1/\gamma_t = 0$, where $\mathcal{F}_1, \mathcal{F}_2$ are characteristics of peak beta function and dispersion, respectively. One can see that in order to avoid high peak beta function ν_x has to lie in the range $0.8 \div 0.88$ and hence, the optimal number of superperiods per arc must be rather high ($S = 5 \div 8$). But LES may have, in general, no more than 4 superperiods. Thus, to obtain high γ_t a strong modulation of the beta function is required that leads to reduction of the dynamic aperture. In order to improve the dynamic aperture, one should try to keep the β-function low in the place of sextupoles by setting ν_x closer to 1 and, therefore, rejecting arcs being a first order achromat and hence zero dispersion in straight sections. However, the problem may be reduced to design a structure of the superperiod in which η is zero at it's ends or at least has a low value. An example of design of such lattice can be found in ref. [4].

2.4 Missing magnet structure

In this method γ_t is raised by eliminating some dipoles from the lattice — "missing magnets", β-function being nearly undisturbed and hence has a lower peak value. The method can be used in a straightforward way in the design of circular rings. But the application in racetrack designs is more difficult. Really, in order to have long enough straight section, the acceptable ratio of number N of cells in each superperiod to a number N_e of it's empty cells must be $N_e/N < 0.25 \div 0.33$. Using analytic formulas for lattice parameters from ref. [9] we can compute the variation of $\gamma_t^{-2}/\gamma_{t0}^{-2}$ and peak of η/η_0 with betatron tune in a superperiod having N_e empty cells, placed at it ends, where γ_{t0}, η_0 are transition energy and dispersion of unperturbed lattice having all cells filled with dipole magnets. The results are shown on Fig. 2.

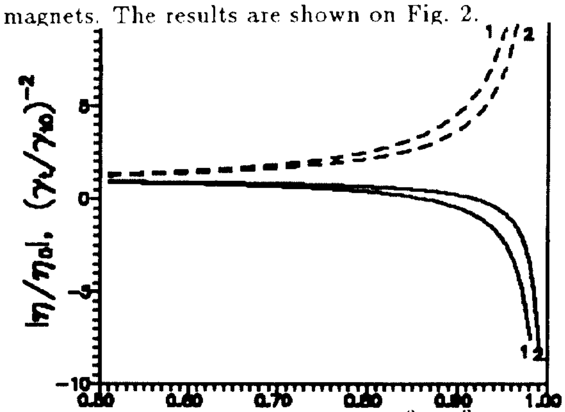

Figure 2: η/η_0 (dashed lines) and $\gamma_t^{-2}/\gamma_{t0}^{-2}$ versus betatron tune ν_x of a superperiod in a missing magnet lattice. 1: $N_e/N = 1/3$, 2: $N_e/N = 1/4$.

One can see, that for $N_e/N = 0.25 \div 0.33$ the horizontal tune of each superperiod has to be close to 1 ($\nu_x \approx 0.9$) in order to raise γ_t significantly.

Obviously, this cannot be achieved in a racetrack lattice if the arcs are to be first order achromats, where $q_x = S - 1$. However, it can be done by using the same idea described in the previous section, *i.e.* design a lattice with non-zero (but perhaps low) dispersion in the straight sections. The lattice of this type was investigated for the MKF Booster [3]. We have found, unfortunately, that lattice had a large dispersion and unsufficient dynamic aperture.

Another possibility is a lattice with different straight sections and dispersion cancellation in only one of them [3,2]. This version have to be rejected because of the superperiodicity of 1 for the whole ring.

2.5 Missing magnet lattice with modulated β-function

The basic ideas are the following:

- In order to achieve sufficiently large d.a. the beta function must be low in sextupole's location that can be met in a missing magnet lattice. Then, in addition to variation of ρ function, a small modulation of the beta function γ_t can increase γ_t significantly.

- Since the acceptable total number of cells in arcs is 24 for 3 GeV ring or 32 for higher energy LES (7-10 GeV), there are two possibilities in a choosing of the arc superperiodicity S and the number of cells per superperiod N:

  1. three-fold superperiodic arc with $N = 4$ cells in each superperiod (3 GeV LES);
  2. four-fold superperiodic arc with $N = 3$, or $N = 4$ (for higher energy rings);

Three-fold superperiodic arcs. Possible lattice for LES with top energy up to 3–4 GeV may have a three-fold superperiodic arcs. Each superperiod contains 4 FODO cells with the central cell being empty. In this cell the chromaticity-correcting sextupoles are located. However, the arc having only three superperiods is not a second-order achromat and there is a large nonlinear distortion of the betatron motion. As a consequence, the dynamic aperture is relatively small, $\approx 500\pi$ mm mrad. The only way to improve the d.a. in this structure is to find a superperiod with sextupoles introduced in pairs separated by a phase advance of π. Then the geometric aberrations introduced by the sextupoles are compensated over the superperiod. This method was tested in the following lattice.

Example 1. $S = 3$, $N = 4$. Empty half-cells are located at the ends of the superperiod. Since the phase advance in each cell is chosen to be close to 90° in both planes, three pairs of sextupoles separated by an angle π in phase advance are created by placing two additional sextupoles in the superperiod. The d.a. is quite large ($\approx 1300\pi$ mm mrad) and the betatron motion looks very linear for high initial amplitudes. The disadvantage of the lattice lies in high peak-values for the β-functions and the dispersion, and hence large magnet aperture.

Four-fold superperiodic arcs. The lattice with $S = 4$ has the following advantages:

- The tune of each superperiod can be set closer to 1 ($\nu_x = 0.75$, $q_x = 3$) compared to $S = 3$ where $q_x = 2$ and, thus, $\nu_x = 0.667$. And, obviously, to obtain $\gamma_t > 10$ less modulation of the β-function is needed;

- The arcs may form second-order achromats with no low-order geometric aberrations introduced by the chromaticity-correcting sextupoles.

Originally this scheme of arc was used for the design of the 7.5 GeV MKF Booster [3]. The lattice has arc's superperiodicity of 4 and 4 FODO cells in each superperiod with one empty cell being central ($N_e/N = 1/4$). The high $\gamma_t \approx 20$ is achieved by using small modulation of the beta function. We have found that the lattice has sufficiently large d.a. and satisfies other design performances. For lower energy synchrotrons (3 GeV), the preservation of the 4-cell's structure of superperiods is not longer possible due to free-space problem. Therefore 3-cell's structure should be considered.

Example 2. The largest dynamic aperture was achieved in a lattice having two additional sextupoles per superperiod, the empty half-cells being at the ends of the superperiod and phase advance in each cell being close to 90°. The drawback of the lattice lies in appearing strong $Q_x - Q_y$ difference resonances and larger dispersion [4].

A version of transitionless racetrack lattice having 4-fold superperiodic arcs, with 3 FODO cells and empty central cell in each superperiod was recently proposed as an alternative lattice for TRIUMF KAON booster [10]. We have found that the dispersion has lower peaks and d.a. is somewhat larger ($\approx 1500\pi$ mm mrad), though a stronger modulation of the β-function is required to achieve $\gamma_t > 10$, if a superperiod starts with defocusing lens (DOFO central cell) in comparison with one starting with F-quadrupole.

3 CONCLUSION

Different racetrack lattices having a high γ_t and large dynamic aperture are discussed. The lattices with a "missing magnet" structure of the superperiod and with modulated β-function appear to be most successful for low-energy synchrotrons.

4 ACKNOWLEDGMENTS

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