ION BEAM ACCELERATION AND FOCSING IN THE POLYHARMONICAL DRIFT TUBE SYSTEMS

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Abstract

The polyharmonical theory of ion beam accelerating field focusing useful for any drift tube system is proposed. The method developed permits to distinguish an electric force time harmonics acting on the ions, and to analyse a motion stability; field space periodicity is not required there. The approximation degree can be choosen in each case by taking into account a desired number of harmonics. High phase acceptance structure parametres are presented as the theory application.

Introduction

The radio-frequency (RF) ion linear accelerators are under interest now as the convenient instrument in many technological and scientific applications. The drift tube accelerating systems are most wide-spread for ion linacs. In the energy range up to tens MeV, ion beam focusing may be effectively accompliched by means of accelerating field itself without any supplementary focusing arrangements.

The electric field space harmonic distinguishing is the wellknown description method for RF accelerating systems based on the autophasing principle[1] (simple harmonic motion). But in the accerating field focused (AFF) systems, the focusing period consists generally of several accelerating gaps. The velocity gain on the focusing period may be essential and structure space periodicity becomes not quite correct in this case. Moreover, the periodical velocity component troubles the AFF systems analysis becaus of tube and gap longitudinal dimentions disaccording with its transit times even at the acceleration absence; the tube and gap dimentions ceases to be the clear characteristics of field action on the particles.

Polyharmonical Analysis Method

The direct analysis of electric forces may be simplified if the gap and tube transit times of so-called syncronous particle Δt_g, Δt_T are accepted as an initial data instead of longitudinal dimentions [2, 3]. When a focusing period structure is determined in phase terms, the inverse conversion to length terms presents no hardness.

The independently phased accelerating gaps sequence will be considered below. Suppose, that the electric field is changed in accelerating gap as

$$E_i(t)=\sum_{K=1}^{M} E_{Ki}\cos(K\omega t+\vartheta_{Ki}),$$

where i is the gap number; ω is the main angular frequency; E_{Ki}, ϑ_{Ki} are the electric oscillation amplitude and initial phase.

Consider, that the electric field acts on the synchronous particle periodically and the focusing period consists of N gaps. If the longitudinal electric field component distribution is approximated by "step function" (E_i in i-th gap and zero in any tube), then the electric field, which acts on the synchronous particle E_S may be presented as follows:

$$E_S=\sum_{n=-\infty}^{\infty} E_n\cos\left(\frac{n}{p}\omega t+\theta_n\right),\ n=0,\pm1,\pm2,\dots$$

$$E_n=\sqrt{a_n^2+b_n^2},\quad tg\,\theta_n=-\frac{b_n}{a_n},$$

$$a_n=\frac{1}{\pi p}\sum_{i=1}^{N}\sum_{K=1}^{M}E_{Ki}\left(\frac{p}{n+Kp}\right)\sin\left[\left(\frac{n+Kp}{p}\right)\frac{\Delta\varphi_{gi}}{2}\right] \times\cos\left[\left(\frac{n+Kp}{p}\right)u_{ci}+\vartheta_{Ki}\right],$$

$$b_n=\frac{1}{\pi p}\sum_{i=1}^{N}\sum_{K=1}^{M}E_{Ki}\left(\frac{p}{n+Kp}\right)\sin\left[\left(\frac{n+Kp}{p}\right)\frac{\Delta\varphi_{gi}}{2}\right] \times\sin\left[\left(\frac{n+Kp}{p}\right)u_{ci}+\vartheta_{Ki}\right],\tag{1}$$

$$u_{ci}=\Delta\varphi_0+\sum_{j=1}^{i-1}\left(\Delta\varphi_{gj}+\Delta\varphi_{Tj}\right)+\frac{\Delta\varphi_{gi}}{2},$$

$$p=\frac{1}{2\pi}\sum_{i=1}^{N}\left(\Delta\varphi_{gi}+\Delta\varphi_{Ti}\right),$$

where u_{ci} is the gap centre phase coordinate; p is normalized phase extent of focusing period. The gap and tube phase extents are determined by its transit times: $\Delta\varphi_{g,T}=\omega\Delta t_{g,T}$. As we can see, at n=0 the synchronous particle is influenced by the constant field which may be thought as the field associated with the synchronous space harmonic; that's why we can accept $E_0=E_S$, $\theta_0=\varphi_S$. Integrating the non-relativistic equation of syncronous particle motion at small velocity increment supposition produces

$$Z_S=V_S t-\sum_{\substack{m=-\infty\\ m\neq0}}^{\infty}\frac{qE_n}{m_0\omega^2}\left(\frac{p}{n}\right)^2\cos\left(\frac{n}{p}\omega t+\theta_n\right),$$

where q, m_0, V_S, Z_S are charge, mass, average velocity and longitudinal coordinate of synchronous particle. Particle forced oscilliation amplittude will be concerned small.

The longitudinal and transverse electric field components E_z, E_r may be defined near the accelerator axis in terms of $E_s(t)$ and $Z_s(t)$:

$$E_z(z,t)=\sum_{K=1}^{M}\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty} E_n \cos\left[\left(\frac{n+Kp}{p}\right)\left(\frac{\omega}{V_s}z+\delta\varphi_c\right)-K\omega t+\theta_n\right],$$

$$E_r(r,z,t)=\frac{r\omega}{2V_s}(1-\delta\varphi_s)\sum_{K=1}^{M}\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty} E_n \frac{n+Kp}{p} \times\sin\left[\left(\frac{n+Kp}{p}\right)\left(\frac{\omega}{V_s}z+\delta\varphi_c\right)-K\omega t+\theta_n\right],$$

$$\delta\varphi_c=\frac{\omega}{V_s}\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty}\frac{qE_n}{m_0\omega^2}\left(\frac{p}{n}\right)^2\cos\left(\frac{n}{p}\frac{\omega}{V_s}z+\theta_n\right),$$

$$\delta\varphi_s=\frac{\omega}{V_s}\sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty}\frac{qE_n}{m_0\omega^2}\left(\frac{p}{n}\right)^2\sin\left(\frac{n}{p}\frac{\omega}{V_s}z+\theta_n\right).$$

The transverse and longitudinal motion equations of any general particle are:

$$m_0\frac{d^2(\Delta z)}{dt^2}=\left[E_z(z_s+\Delta z,t)-E_z(z_s,t)\right], \qquad (2)$$

$$m_0\frac{d^2 r}{dt^2}=eE_r(r,z_s+\Delta z,t), \qquad (3)$$

where Δz is the particle longitudinal displacement from the synchronous one; r is the transverse coordinate.

When considering Δz as an independent slow changed parameter, then eq. (2, 3) may be transformed for small Δz, r as follows:

$$\frac{d^2\psi}{dt^2}+\frac{1}{2\pi\beta_s}\left[A_s\sin\varphi_s-Q+\sum_{K=1}^{M}\sum_{m=1}^{\infty}R_{km}\sin\left(\frac{m}{p}\varphi+\varphi_{km}\right)\right]\psi=0, \qquad (4)$$

$$\frac{d^2 r}{dt^2}-\frac{1}{4\pi\beta_s}\left[A_s\sin(\varphi_s+\psi)-Q+\sum_{K=1}^{M}\sum_{m=1}^{\infty}R_{km}\sin\left(\frac{m}{p}\varphi+\varphi_{km}\right)\right]r=0, \qquad (5)$$

where

$$Q=\frac{1}{4\pi\beta_s}\sum_{K=1}^{M}\sum_{m=1}^{\infty}\left[A_m^2\left(\frac{Kp+m}{m}\right)-A_{-m}^2\left(\frac{Kp-m}{m}\right)+2A_mA_{-m}\cos(\theta_m+\theta_{-m}),\right.$$

$$R_{km}=\left[A_m^2\left(\frac{Kp+m}{p}\right)^2+A_{-m}^2\left(\frac{Kp-m}{p}\right)^2-2A_mA_{-m}\frac{(Kp+m)(Kp-m)}{p^2}\cos(\theta_m+\theta_m)\right]^{\frac{1}{2}},$$

$$tg\varphi_{km}=\frac{A_m(Kp+m)\sin\theta_m+A_{-m}(Kp-m)\sin\theta_{-m}}{A_m(Kp+m)\cos\theta_m-A_{-m}(Kp-m)\cos\theta_{-m}};$$

$$A_{\pm m}=\frac{eE_{\pm m}\lambda}{m_0c^2}, \qquad m=1,2,\ldots$$

$\psi=\omega\Delta z/V_s$ is particle small phase deviation there; $\varphi=\omega t$ is current phase; β_s is normalized average synchronous velocity; c is lihgt velocity. Each value m in eq. (4, 5) corresponds with two harmonics, the numbers of which are m and $-m$.

To simplify the analysis procedure, consider the monofrequency systems, $K=1$. Suppose, that the fundamental contribution to beam focusing is made by two harmonics of electric forces under m, $-m$ numbers which influence upon the syncronous particle with the same period that the system main focusing period p. Taking into account only these two harmonics, we can transform eq. (4, 5) to the Matieau's equations:

$$\frac{d^2\psi}{du^2}+(\delta_\varphi+\mathcal{E}_\varphi\sin u)\psi=0, \qquad (6)$$

$$\frac{d^2 r}{du^2}+(\delta_r+\mathcal{E}_r\sin u)\psi=0, \qquad (7)$$

where

$$\delta_\varphi=\frac{p^2}{2\pi\beta_s}(A_s\sin\varphi_s-Q),$$

$$\delta_r=-\frac{p^2}{4\pi\beta_s}\left[A_s\sin(\varphi_s+\psi)\right]-Q],$$

$$\mathcal{E}_\varphi=\sqrt{\mathcal{E}_1^2+\mathcal{E}_{-1}^2-2\mathcal{E}_1\mathcal{E}_{-1}\cos(\theta_1+\theta_{-1})}, \quad \mathcal{E}_r=-\frac{1}{2}\mathcal{E}_\varphi,$$

$$u=p\omega t+\varphi_1, \quad \mathcal{E}_{\pm1}=\frac{p(p\pm1)A_{\pm1}}{2\pi\beta_s}.$$

Q, φ_1, $A_{\pm1}$ and $\theta_{\pm1}$ are defined by formulas given above at $K=1$, $m=1$, $n=\pm1$.

The stable solution region of Matieau's equation may be appropximated as $-\mathcal{E}^2/2<\delta<1/4-\mathcal{E}/2$ [4]. To provide both transverse stability of accelerating ion bunches and longitudinal one, this condition must be satisfied for eq. (6, 7) simultaneously. The "focusing coefficient" $\mathcal{E}$ defines the accelerating channel acceptance and beam current capability. A correlation between the transverse acceptance and longitudinal one is associated with the coefficient δ. Periodical particle velocity component leads to so-called "static" focusing effect on the analogy of axial symmetric electrostatic linzes, where the focusing effect is caused partly by particle velocity oscillation. As the practical calculations confirms, the "static focusing coefficient" is usually small and it may be neglected in focusing period structure preliminary choice. Thus

$$\delta_\varphi=\frac{p^2}{2\pi\beta_s}A_s\sin\varphi_s,\quad \delta_r=-\frac{p^2}{4\pi\beta_s}A_s\sin(\varphi_s+\psi).$$

Theory Practical Application

The method developed permits to analyse any focusing period structures in the various drift tube accelerating systems. Consider some of this systems, used in practice.

1. Single multigap resonator. Fourier coefficients (1) may be presented in following form:

$$a_m=\frac{1}{\pi(p+m)}\sum_{i=1}^{N}E_i\sin\left[\left(\frac{p+m}{p}\right)\frac{\Delta\varphi_{gi}}{2}\right]\cos u_i,$$
$$b_m=\frac{1}{\pi(p+m)}\sum_{i=1}^{N}E_i\sin\left[\left(\frac{p+m}{p}\right)\frac{\Delta\varphi_{gi}}{2}\right]\cos u_i, \tag{8}$$

where

$$u_i=\frac{p+m}{p}\left(\pi\sum_{j=1}^{i-1}k_j+\varphi_{ci}\right)+\pi(i-1).$$

k_j is j-th tube multiple coefficient there; φ_{ci} is the electric field oscillation phase (in cos coordinate) at which syncronous particle cross the gap electric center. $\pi(i-1)$ occurs for π-mode resonators only in last expression but it is absent for 2π-mode (Alvaretz) resonators.

2. The independently phased monogap resonator sequence. Keeping the form (8), we have

$$u_i=\frac{p+m}{m}\left[\Delta\varphi_H+\sum_{j=1}^{i-1}(\Delta\varphi_{gj}+\Delta\varphi_{Tj})+\frac{\Delta\varphi_{gi}}{2}\right]+\vartheta_i, \tag{9}$$

where ϑ_i is the oscillation initial phase in i-th gap, or

$$u_i=\frac{p}{m}\left[\Delta\varphi_H+\sum_{j=1}^{i-1}(\Delta\varphi_{gi}+\Delta\varphi_{Ti})+\frac{\Delta\varphi_{gi}}{2}\right]+\varphi_{ci}, \tag{10}$$

where φ_{ci} is the gap center crossing phase determined above.

3. The independently phased two-gap resonator sequence useful for heavy ions may be described also by formulas (8-10).

In focusing period structure preliminary choosing and in accelerator channel parameters estimating, it is advisable to limit the harmonics number by value of three: one synchronous (or accelerating) harmonic and two non-synchronous (focusing) ones. The Matieau's equations (6,7) may be considered in this case.

Calculation Results

There are accelerating field focusing varieties, characterized in general by one focusing harmonic ($m=1$ or $m=-1$). But if the influence of both focusing harmonics is more or less equivalent, then the focusing variety is most interesting. Thus, so-called "hybrid" focusing which combines the center gap phase alternating [5,6] with the gap transit time periodical changing (at large gap phase extent, more then π) [7] is proposed. Using this focusing variety, the following proton accelerator parameters are received by means of theory application and computer beam dynamics simulation [8]. The longitudinal acceptance phase extent is as large as 250° at 8% initial energy spread, injection voltage is less then 30kV. The normalized transverse acceptance ~50mrad*mm is usual for alternating phase focusing. The accelerator length equals 1,2m; the final energy 1,3MeV is gained at the electric field strength in gap centers about (3-5)MV/m.

Conclusion

The polyharmonical method developed may be useful for all drift tube systems with any gap number per focusing period. We can choose the approximation degree in each case by taking into account a desired number of harmonics. Beam dynamics simulation is certainly required in accelerator designing but the theory permits to make the preliminary evaluations and simplify sufficiently the effective focusing period structure choice.

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