BEAM ν-SPREAD DUE TO FIELD ERRORS IN RHIC*

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Abstract

The random field error multipoles can produce a significant ν-spread in the beam. Tracking studies, for particles with emittances and $\Delta p/p$ which are within the beam, have shown that a major part of the ν-spread comes from the statistically significant average value of b_3 and b_4 in the main dipoles in the arcs. Because of systematic errors in the construction of the dipoles, this average value may be larger than that which one would expect from a purely random distribution of errors. A correction system for the average b_3 and b_4 in the dipoles appears important. Results will be given for the ν-spread found for the uncorrected multipoles, and for the ν-spread after correction of the average b_3 and b_4.

1. Introduction

Previous work[1] indicated that the random field error multipoles, b_k, a_k can produce appreciable ν-shifts. This study finds the ν-spread in particles that occupy 95% of the beam due to ν-shifts produced by the random field errors. These ν-shifts depend on the transverse emittances, ϵ_x and ϵ_y, and on the momentum $\Delta p/p$ of the particle, and on the range of emittance and $\Delta p/p$ present in the beam that causes the ν-spread in the beam. The performance of RHIC depends on keeping the ν-values of the particles within a range that is free of non-linear resonances, up to and including tenth order resonances. For RHIC, the available range of ν-values free of resonances is $\Delta\nu = 33 \times 10^{-3}$. Thus the ν-spread due to random field errors has to be kept much smaller than $\Delta\nu = 33 \times 10^{-3}$.

The following study finds an appreciable probability for ν-spreads due to random field errors of the order of $\Delta\nu = 21 \times 10^{-3}$ for the worst case. It was also found that a major part of this ν-spread could be corrected by correcting[2] the average value of b_3 and b_4 in all the dipoles. By correcting the average value of b_3 and b_4, the ν-spread due to random field errors can be reduced below $\Delta\nu = 7 \times 10^{-3}$.

2. Results for the ν-Spread in RHIC

The ν-shift of a particle in the RHIC beam is found through tracking. For a given emittance ϵ_x, ϵ_y and a given momentum $\Delta p/p$, the particle is tracked for 400 turns in the presence of the expected random field errors. These field errors are listed in the RHIC Conceptual Design Report.[3] Because the ν-shift is a non-linear effect that depends on ϵ_x, ϵ_y, the ν-shift is found by Fourier analyzing the particle motion over 400 turns. Because of the presence of the random a_k, the x and y motion are coupled, and more than one ν-value may sometimes be seen in the x or y motion.

In order to find the ν-spread in the beam, one should in principal find the ν-shift for all particles in the beam, covering the range of ϵ_x, ϵ_y and $\Delta p/p$ in the beam. The largest ν-spread is expected for a heavy ion like Au at the lowest colliding energy of interest which is $\gamma = 30$. The heavy ions experience the largest growth due to intrabeam scattering and have the largest emittance ϵ_x, ϵ_y. After 10 hours at $\gamma = 30$, the beam parameters for Au will grow to $\epsilon_t = \epsilon_x + \epsilon_y = 1.92\ \pi$mm-mrad, $\Delta p/p = \pm 0.005$ and $\sigma_x = 3.1$ mm. ϵ_t is the total emittance that contains 95% of the beam, σ_x is the rms horizontal beam size, and ϵ_t is given by[4] $\epsilon_t = 10\sigma_x^2/\beta_x$.

The ν-shift was explored previously[1] as a function of ϵ_x, ϵ_y and $\Delta p/p$, and it was found that the largest ν-shift for a given ϵ_t and $\Delta p/p$ occurred when $\epsilon_x = \epsilon_t$, $\epsilon_y = 0$. It was assumed in the following studies that the largest ν-shift will occur when $\epsilon_x = \epsilon_t = 1.92$ $\epsilon_y = 0$, $\Delta p/p = \pm 0.005$ which corresponds to the initial coordinates $x_0 = 9.8$ mm, $y_0 = 0$, $x_0' = y_0' = 0$.

Table 1 lists the results for the ν-spread in a beam of Au ions after 10 hours at $\gamma = 30$ for a lattice with $\beta^* = 6$ in all insertions. Twenty different distributions of random field errors, b_k, a_k, $k =$ 2 to 10, were tracked. The ν-spread, $\Delta\nu$, listed includes only the ν-spread due to random errors. It does not include the ν-spread due to the chromaticity sextupole.

Table 1: $\Delta\nu$ spread due to random b_k, a_k.

Field Error Distribution Number	$\Delta\nu/10^{-3}$
1	3
2	2
3	4
4	4
5	10
6	2
7	4
8	3
9	2
10	2
11	0
12	7
13	1
14	7
15	0
16	1
17	4
18	9
19	2
20	14

The largest ν-spread found was $\Delta\nu = 14 \times 10^{-3}$ which is to be compared with the available ν-range free of resonances of $\Delta\nu = 33 \times 10^{-3}$. Of the 20 error distributions, 5 error distributions have ν-spreads larger than 7×10^{-3}.

The ν-spread in the beam due to random field errors is likely to be larger than the above results because of the possible presence of a large average b_3 or large average b_4 around the ring. This is discussed in the next section.

3. Effect of an Enhanced Average b_3 and b_4

The random b_k, a_k used in the tracking runs to find the results given in Table 1, were generated using a random Gaussian distribution generator for given rms values $b_{k,rms}$ and $a_{k,rms}$. In each error distribution, the expected average b_k, a_k value in the 144 dipoles is given by

$$b_{k,av} = \frac{1}{\sqrt{144}}\, b_{k,rms} = 0.085\ b_{k,rms}\ . \tag{3.1}$$

It appears likely from previous results of magnet measurements[5] that $b_{k,av}$ will be larger than the above result, and a possible

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estimate for the lower b_k, a_k is

$$b_{k,av} \simeq \frac{1}{3}\, b_{k,rms}\,. \qquad (3.2)$$

It will be seen below that this larger $b_{k,av}$ will increase the largest ν-spread found from $\Delta\nu = 14 \times 10^{-3}$ to $\Delta\nu = 21 \times 10^{-3}$.

To find the effect of the larger $b_{k,av}$ as given by Eq. 3.2, it is interesting first to find how much $b_{k,av}$ was present in the tracking study to compute the ν-spread when the b_k are generated using a random Gaussian distribution. This is shown in Table 2 for the two worst field errors for the lower multipoles.

Table 2: $b_{k,av}$ generated by a random gaussian distribution.

$b_{k,av}/b_{k,rms}$	Field Error 20	Field Error 5
b_2	-0.049	0.099
b_3	0.164	0.149
b_4	-0.053	-0.088

It is also interesting to see the average b_k present in the tracking study for all the multipoles present. This is shown in Table 3 for field error 20.

Table 3: The average b_k in the dipoles for $k = 1$ to 10 for field error 20.

k	$b_{k,av}/b_{k,rms}$	$a_{k,av}/a_{k,rms}$
1	0.0000	0.0000
2	-0.0482	-0.1170
3	0.1640	-0.0844
4	-0.0530	-0.0636
5	0.1436	0.0039
6	-0.0186	-0.0065
7	-0.1005	0.1809
8	-0.0601	0.0467
9	0.0820	-0.0874
10	-0.0044	0.0089

The results shown in Table 2 are in good agreement with the prediction for $b_{k,av}$ given by Eq. 3.1. It shows that $b_{k,av}$ present using a random Gaussian distribution is about one half the amount expected as given by $b_{k,av} \simeq (1/3)\, b_{k,rms}$. To simulate the effect of the larger $b_{k,av}$, the ν-spread was computed when the $b_{k,av}$ for b_3, b_4 was increased in each dipole by a factor of 2 by adding a constant amount to these multipoles in each dipole. The ν-spread $\Delta\nu$ was computed in this way for the 2 worst distributions and the results are shown in Table 4.

Table 4: $\Delta\nu$ spread due to random b_k, a_k when $b_{k,av}$ is doubled for b_3 and b_4.

Field Error	$\Delta\nu/10^{-3}$
20	21
5	16

4. Effects of Correcting $b_{3,av}$ and $b_{4,av}$

It was found that a major part of the ν-spread could be corrected by correcting the average b_3 and the average b_4 in the dipoles.

The correction of the average b_3 and the average b_4 was simulated by first generating a field error distribution, b_k, a_k using a random gaussian distribution. For this field error distribution the average b_3, $b_{3,av}$ and the average b_4, $b_{4,av}$, in the dipoles was then computed. The b_3 and b_4 in the dipoles was then modified by subtracting $b_{3,av}$ from b_3 and $b_{4,av}$ from b_4 in each dipole. This then assures that the modified distribution of b_3 and b_4 in the dipoles has zero average b_3 and zero average b_4. The results[6] of this correction is shown for the 5 worse field errors in Table 5.

Table 5: Computed ν-spread with and without correction of the average b_3 and the average b_4 for the five worse error field distributions.

Error Field	Uncorrected	Corrected $b_{3,av}$, $b_{4,av}$
5	10	4
12	6	2
14	7	7
18	7	5
20	14	7

The largest ν-spread has been reduced to $\Delta\nu = 7 \times 10^{-3}$. This result will still hold if the $b_{k,av}$ are larger of the order of $(1/3)\, b_{k,rms}$.

The importance of the average value of b_k is evident in the case where the random a_k are absent and only random b_k are present. In this case, analytical expressions for the ν-shift are available. For example for the random b_3 one can write for $\Delta\nu_x$

$$\Delta\nu_x = \frac{3}{4\pi\rho}\int ds b_3 \left\{\beta_x\,(X_p\delta)^2 + \frac{1}{4}\beta_x^2\epsilon_x - \frac{1}{2}\beta_x\beta_y\epsilon_y\right\} \qquad (4.1)$$

where $\delta = \Delta p/p$ and X_p is the horizontal dispersion. If one considers the contribution to $\Delta\nu_x$ from the 144 dipoles in the arcs in RHIC, one can see that the contribution is proportional to the average b_3 in these dipoles. For all these dipoles X_p, β_x and β_y give the same contribution to the above integral. Thus the contribution to $\Delta\nu_x$ from all 144 dipoles will vanish if the average b_3 is zero. The residual $\Delta\nu$ found from tracking studies after the average b_3 and b_4 have been made zero are due to effects not included in Eq. (4.1). These include higher order terms,[6] particularly terms proportional to b_2^2, effects due to the presence of the random a_k, effects due to the presence of nearby resonances, and contributions coming from other magnets[6] than the arc dipoles.

The above study indicates that it will be important to correct $b_{3,av}$ and $b_{4,av}$ in the dipoles. The procedure used in the tracking study, where the $b_{3,av}$ and $b_{4,av}$ present were subtracted from the b_3 and b_4 in each dipole, can not be carried out for the actual accelerator. Instead, this procedure has to be approximated by placing b_3 and b_4 correction coils at certain places around the ring. A correction coil can be put at each end of the dipole. A correction coil at the center of the dipole is not practical for RHIC, but can be approximated by putting a correction coil in the insertions. This correction arrangement is described in reference 7. A correction coil at the center of the dipole, or a coil that simulates this, may be required in order that the correction coils should provide a good correction of $b_{3,av}$ and $b_{4,av}$.

5. Conclusions

This study has found that the random b_k, a_k can produce an appreciable ν-spread in the beam. For the worst case, a ν-spread of $\Delta\nu = 21 \times 10^{-3}$ was found. By correcting the average value of b_3 and b_4 in all the dipoles, the ν-spread can be reduced below $\Delta\nu = 7 \times 10^{-3}$.

These results may present an overly pessimistic picture. To keep a proper perspective on this problem, one should keep in mind the following aspects of the problem:

1. A small fraction, about 25%, of accelerators will have random error distributions that cause large ν-spreads.
2. The ν-spread computed above is for the beam dimensions after 10 hours of growth due to intrabeam scattering for the worst case of Au at $\gamma = 30$.

3. Only particles with large x and small y exhibit the large ν-shifts that cause the large ν-spread. This again is a fraction of all the particles.
4. The largest source of ν-spread in the beam is due to the beam-beam interaction, which could produce a ν-spread of $\Delta\nu \simeq 25 \times 10^{-3}$ at the start, which gradually decreases as the beam grows due to intrabeam scattering. The beam-beam ν-spread and the ν-spread due to random errors are not simply additive. The beam-beam ν-shift is smaller at large betatron oscillations where the ν shift due to the random b_k, a_k is largest.

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