We discuss here the measurement of the coupling impedance of individual accelerator vacuum chamber components by wire method using a synthetic pulse technique. A standard instrumentation, like a Network Analyzer able to work in the microwave region, is used in order to perform measurements in the frequency domain. An off-line Fast-Fourier Transform can be used to treat the data and to identify some signal characteristics. This option allows to generate a synthetic pulse in the time domain via FFT from the original data measured in the frequency domain. Potential errors intrinsic to the method of coaxial wire are pointed out and analyzed. Some preliminary results are presented.

Introduction

An indirect method, known as “coaxial wire method”, for the measurement of the energy loss of a stored beam to a cavity was proposed by Sands and Rees [1]. They argued that the energy lost by a short current pulse on a central wire is the same as that lost by a bunch of particles having equal time shape. Some examples of bench measurements of the energy loss from a stored beam and machine components impedance, performed in time or frequency domain, are reported in Ref. [2-6].

In this paper the measure of the longitudinal beam impedance \( Z(\omega) \) [7] of a reference cavity is presented. \( Z(\omega) \) is related to the scattering matrix transmission coefficient, \( S_{12}(\omega) \), of the same cavity. The use of transmission wave instead of the reflected one reduces the errors on \( Z \), specially in case of distributed impedance \[6\].

The time domain transform of the detected signal (frequency domain) allows the identification of unwanted reflections and the possibility of eliminate them. The technique employed for this data treatment can be seen like the time-domain analogy of a band-pass filter. This filtering is known as "gating": When the 'gate' is applied, the signal outside the chosen range is eliminated leaving intact the reflections of interest.

As to be mentioned that this method must be applied carefully in order to "cut" the \( t \)-domain signal without producing big errors or meaningless results in the corresponding \( f \)-domain after the inverse FFT.

Impedance calculation

A current pulse \( i_1(t) \) is fed into a cylindrical coaxial structure (reference pipe) with characteristic impedance \( Z_L \) (Fig.1a). The energy contained in the pulse is given by

\[
U_1 = \int_{-\infty}^{+\infty} Z_L i_1^2(t) \, dt \quad (1)
\]

The same pulse is fed into the structure with the testing object that replaces a part of the coaxial line (Fig. 1b). Assuming that the pulse is only slightly modified by the testing object, i.e. \( i_2(t) = i_1(t) + \Delta i(t) \), the energy contained in the pulse so perturbed can be written as:

\[
U_2 = \int_{-\infty}^{+\infty} Z_L i_2^2(t) \, dt = \int_{-\infty}^{+\infty} Z_L i_1^2(t) \, dt + \int_{-\infty}^{+\infty} 2 Z_L \Delta i(t) \, dt \quad (2)
\]

where it is assumed (small perturbations)

\[
|\Delta i(t)| \ll |i_1(t)| \quad (2a)
\]

Comparing the energy lost by the pulse, \( U = U_1 - U_2 \), with the expression of the energy loss of a bunch of particles

\[
U = q \int_{-\infty}^{+\infty} W_b i(t) \, dt \quad (3)
\]

it can be derived that the signal difference \( \Delta i(t) \) times \(-2 Z_L/Q\) is equal to the wake potential \( W_b(t) \) of a bunch of particles with the same shape

\[
W_b(t) = -2 Z_L \Delta i(t) \quad (4)
\]

Transforming eq. (4) into the frequency domain yields

\[
Z(\omega) \Delta i(\omega) = -2 Z_L \{1(\omega) - i_2(\omega)} \quad (5)
\]

The eq. (5) suggests to perform the measurement directly in frequency domain (Fig. 1) measuring the scattering parameters \( S_{12}(\omega) \).

According to eq. (5) the longitudinal coupling impedance is given by:

\[
Z(\omega) = \frac{2 Z_L S_{12} - S_{12}^{\text{ref}}}{S_{12}^{\text{ref}}} \quad (6)
\]

where \( S_{12} \) and \( S_{12}^{\text{ref}} \) stand for the structure with the cavity and for the reference pipe respectively.

It has to be pointed out that the use of the small perturbation approximation (eq. 2a) is better satisfied the thinner is the central wire (\( Z_L \) increases). But to keep \( \Delta i(t) \) at a measurable value the wire cannot be reduced too much. A good compromise needs[1,6].

Fig.1 - A schematic diagram of the used experimental set-up : a) reference pipe, b) test cavity structure.
Measurements and discussion of results.

We use a 69 mm copper tube for the beam pipe and a calculable pill-box test cavity (250 mm, diameter; 377.6 mm long). A 0.9 mm copper wire is stretched in the pipe to simulate the bunch and it is connected at the far ends with suitable coax-to-air line transition.

The reference pipe is long 477.6 mm, corresponding to the cavity length plus two short arms. The theoretical value of the tube characteristic impedance, under the assumption of TEM (Transverse Electromagnetic) propagation, is

\[ Z_L = 60 \log \left( \frac{b}{a} \right) = 257.12 \, \Omega, \]

where \( b \) and \( a \) are the pipe and the wire radii respectively.

The cable-pipe transitions are designed in order to match the cable (generally with an impedance of 50 \( \Omega \)) to the pipe without reflections. We mention that these transitions are a critical part of the measurement bench.

As reference we compute the transmission coefficient of the cavity when it is considered as a line with a diameter variation (see Fig. 1):

\[
Z_1 = \frac{Z_2 + jZ_L \tan(\beta_1)}{Z_L + jZ_2 \tan(\beta_1)}; \quad Z_3 = 50 \, \Omega
\]

\[ Z_2 = Z_C \frac{Z_3 + jZ_C \tan(\beta_2)}{Z_C + jZ_3 \tan(\beta_2)} \]

\[ S_{12} = \frac{2Z_L}{Z_1 + Z_L} \]

where \( Z_C \) is the characteristic impedance of the cavity, \( Z_1, Z_2 \) and \( Z_3 \) are respectively the input impedances for the portions \( I_1, I_2 \) and \( I_3 \).

In equs. (7) and (8) a perfect matching pipe-line is assumed and the reactance due to the step is neglected, in first approximation. In Fig. 2 it is reported the amplitude of \( Z_1(\omega) \) computed according to equs. (7) and (8) in the frequency range 0.1 to 2.4 GHz.

![Fig. 2 - Amplitude of cavity impedance, \( Z(\omega) \).](image)

The layout of the set-up used for the measurement is reported in Fig. 3. For the data acquisition and for the control of the Sweep oscillator we've used an HP 9000/300 computer connected to the measurement system (Network Analyzer (N.A.), Sweep oscillator, etc.) through an HP-IP bus. The signals (Amplitude and Phase) from the N.A. are sent to an A/D convert module inside the computer and registered for the analysis as Complex number.

The measured scattering parameter \( S_{12} \) for the reference pipe and for the cavity are reported in Fig. 4.

On the curve relative to the cavity (dashed) we can see some sharp discontinuities due to the resonances of the system. An accurate analysis of the values of these resonances allows us to recognize the expected TEM modes as well as the TM (Transverse Magnetic) ones. These phenomena are relevant to the high frequency range, after 1 GHz.

In Fig. 5 the longitudinal impedance, obtained from eq. (6), is reported. It has to be pointed out that, besides the expected peaks due to the cavity resonances, there are some peaks due to little shifts between the two \( S_{12} \) curves that can give big differences on expected result.
Fig.4 - Measured scattering parameter, $S_{12}$, for the cavity (dashed line) and for the reference pipe (solid line).

Fig.5 - The longitudinal impedance, $Z(\omega)$, obtained from the experimental data.

Fig.6 - Time domain transform of the scattering matrix reflection coefficient, $S_{11}$, for the reference pipe.

Conclusions:

As conclusion the following comments are in order:

a) in the mentioned theoretical model the reactance due to the step pipe-cavity has to be taken into account. It becomes particularly significant as the frequency increases;

b) the transverse magnetic modes must be included in the model calculations. They can clarify the presence of many different resonances in the cavity responses, $S_{11}$ and $S_{12}$;

c) a resolution improvement of the time domain analysis needs. This can be obtained increasing the frequency acquisition range;

d) a better design of the matching cable-pipe sections is required in order to avoid the unwanted reflections of the measured signal and consequently the unreal maxima on the longitudinal impedance curve.

References:


