# RF Characteristics of Vacuum Chambers with Arbitrary Cross Section and Material Distribution

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**Abstract:** For accelerator vacuum chambers especially in cooling rings the rf characteristic is of high importance. Within a certain rf frequency range for example a certain wave attenuation is required to decouple cooling antennas and kickers from each other. A general purpose 2d-wave analysis program is presented which can calculate wave propagation characteristics in arbitrary shaped chambers. The inside of the chamber may have an arbitrary distribution of permittive and/or permeable material. Losses are treated by solving Maxwell's equations with truly complex field vectors.

## I. Introduction

The presented new CAD program calculates the rf characteristics of waveguides. Thereby the only constraint on the waveguide is that it has to be homogeneous in the direction of propagation. It may have an arbitrarily shaped cross section and may be filled with arbitrary, lossy, linear materials. Thus this program is a very useful tool for designing vacuum chambers, for instance with all kinds of coatings, or for developing new possibilities for particle acceleration. The program represents a two dimensional wave analysis computer code and solves Maxwell's equation by the Finite Integration Algorithm and is part of the MAFIA group of codes [1].

## II. Field Calculations

In order to evaluate the $\vec{E}$-field and the $\vec{H}$-field inside a given structure, the integral form of Maxwell's equations are discretized

$$j\omega\vec{E}\int_A \varepsilon d\vec{A} = \sum_{i=1}^{4}\frac{\vec{B}_i}{\mu_i}\cdot d\vec{s}_i \tag{1}$$

$$-j\omega\vec{B}\int_A \mu d\vec{A} = \sum_{i=1}^{4}\vec{E}_i\cdot d\vec{s}_i \tag{2}$$

From now on $z$ is chosen as direction of propagation. The next step is to change the 3d problem into a 2d problem by using some additional information. For an arbitrary waveguide which is homogeneous in z direction the dependence of the electromagnetic field components on this direction of propagation is known

$$\vec{E} = \vec{E}(x,y)\cdot e^{-jk_z z} \tag{3}$$

where $E(\vec{x},y)$ represents the transverse field and $k_z$ is the wave number. Discretization of equation (3) yields:

$$\vec{E} = \vec{E}(x,y)\cdot e^{-jk_z\Delta z} \approx \vec{E}(x,y)\cdot(1-jk_z\Delta z) \tag{4}$$

for small $\Delta z$. Relation (4) holds exactly, if $\Delta z$ goes to zero. Using the fact that inside the waveguide

$$div\vec{D} = 0 \tag{5}$$

is true, the $E_z$ component can be eliminated and we obtain the two dimensional form of the wave equation

$$curlcurl\,\vec{E}_{tr} = -k_z^2\vec{E}_{tr} \tag{6}$$

in matrix form. Here $\vec{E}_{tr}$ represents the transverse $\vec{E}$-components $\vec{E}_{tr}$ is also an eigenvector of the matrix which describes the curl-curl operator and $-k_z^2$ is the eigenvalue. This eigenvalue problem is solved by an iteration algorithm. In general the matrix is complex and so are its eigenvectors and eigenvalues. In order to handle this truly complex problem a Lanczos algorithm was chosen.

## III. The Lanczos Algorithm

### Lanczos - methods for the eigenvalue computations.

The eigenvalue computation for loss-free waveguides is usually done by a polynomial iteration method. For lossy materials, the eigenvalue spectrum becomes complex, and the task of finding a suitable polynomial is far beyond reach, so that polynomial iteration is very inefficient. Now it has long been realized that single vector Lanczos procedures are at least as efficient as polynomial iteration with an optimal polynomial for exact arithmetic, but react in a somewhat chaotic way to round off errors, so that they have only rarely been used. Recent results showed how these problems can be avoided [5] [4] or isolated [2] [3] with moderate losses in efficiency. The algorithm used here is an adaption from [3] to the special situation of complex harmonic Maxwell's equations.

The basic procedure in [3] is the following: Let $A$ be a matrix and $v_1$ a vector ( usually randomly chosen ). Set $w_1 = v_1$, $v_0 = w_0 \equiv 0, \beta_1 = 0$ and compute the sequences $v_i, w_i, \alpha_i, \beta_i$ defined by

$$\beta_{i+1}v_{i+1} = Av_i - \alpha_i v_i - \beta_i v_{i-1} \equiv r_{i+1} \tag{7}$$

$$\beta_{i+1}w_{i+1} = A^T w_i - \alpha_i w_i - \beta_i w_{i-1} \equiv t_{i+1} \tag{8}$$

$$2\alpha_i = w_i^T(Av_i - \beta_i v_{i-1}) + v_i^T(Aw_i - \beta_i w_{i-1}) \tag{9}$$

$$\beta_{i+1}^2 = (r_{i+1}^T t_{i+1}) \tag{10}$$

With $V_m = (v_1,\ldots,v_m), W_m = (w_1^T,\ldots,v_m^T)$, the matrix

$$T_m^1 = \begin{pmatrix} \alpha_1 & \beta_2 & & 0 \\ \beta_2 & \ddots & \ddots & \\ & \ddots & \ddots & \beta_m \\ 0 & & \beta_m & \alpha_m \end{pmatrix} = W_m A V_m \tag{11}$$

is the the m-dimensional Lanczos matrix to A and $v_1$. In exact arithmetic, the vectors $v_1, \ldots, v_m$ and $w_1, \ldots, w_m$ are biorthogonal. With incresing m,the eigenvalues of $T_m$ will converge against the eigenvalues of $A$ . If $x$ is an eigenvector of $T_m^1$ to a converged eigenvalue, then $y = x^T V_m^1$ is an approximations to the eigenvector of $A$ to the same eigenvalue. The quality of the approximation can be estimated reliably from the size of the last component of the $T$ eigenvector.

The algorithm requires memory for storing $A$ and 6 vectors, and the $v_i$ have to be stored on disk. The storage requirements still may be more severe than the cpu requirements.

In finite precision arithmetic, the biorthogonality of the vectors is lost, so the matrices $T_m^1$ have 'spurious' eigenvalues that have no correspondence to eigenvalues of $A$ , and 'numerically multiple' eigenvalues that are multiple copies of eigenvalues of $A$. In [2] and [3] ways of recognising spurious eigenvalues of $T_m^1$ and of treating multiple eigenvalues of $T$ are proposed.

## The Implementation

*Testing for convergence:* For certain $m$ convergence is tested. An eigenvalue $\lambda$ of $T_m^1$ has converged if it is either multiple or the last component of the corresponding eigenvector is very small. Eigenvalues of $T_m^1$ that have not converged are considered spurious if they are also eigenvalues of $T_m^2$ ( see [2] ), which is a strong indication that they are generated from round off errors. An equivalent criterion is the size of the first component of the corresponding eigenvector. As the calculation of the eigenvalues of $T_m^2$ is about as costly as the calculation of 20 eigenvectors by inverse iteration, this is the more efficient test in our case. Eigenvalues that are neither converged nor spurious are still good values, they are some kind of averages of a few $A$ eigenvalues. The Lanczos iteration may stop if all good values in the desired range are converged.

*Treating multiple eigenvalues:* Normally, $A$ should not have multiple eigenvalues. The most common reason for multiple eigenvalues is a symmetry of the problem that is not exploited in the formulation, and this should be corrected by the user. Multiple eigenvalues not caused by symmetry are possible, but rare.

For $m$ large enough, $T_m^1$ almost certainly contains a few multiple eigenvalues. Further copies of eigenvalues will be built from round off. If the eigenvalue is a simple $A$ eigenvalue, the reconstructed $A$ vectors will be linearily dependent, and except for one of them may be numerical approximations to the null vector. If the $A$ eigenvalue is multiple, the reconstructed vectors will span the full eigenspace except for the extremely unlikely case that the round off is linearily dependent of a subspace of the eigenspace.

With our implementation, as soon as a $T_m$ eigenvalue has converged, it is frozen, that is for increasing $m$, only zero components are added. If this eigenvalue gets a higher multiplicity, the next $T$ eigenvector is made orthogonal to the already existing one. This guarantees that the first reconstructed vector is a good one, while the follwing ones usually are not. If the eigenvalue is $A$ - multiple, the leading reconstructed vectors will have fairly large angles, though they will not be as good as could be.

It may be - though there is so far no case known - that a multiple $A$ eigenvalue will pass unnoticed. This may be avoided by starting a second Lanzcos procedure with a starting vector that is orthogonal to all eigenvectors to $A^T$ belonging to eigenvalues calculated so far. This is not done at the moment and the problem of multiplicity is left to the user. If there is any doubt, it is advaisable to increase the number of iterations. If the reconstructed vectors to a multiple $T$ eigenvalue are almost linear dependent, the $A$ multiplicity of this value can safely be taken as the number of independent vectors.

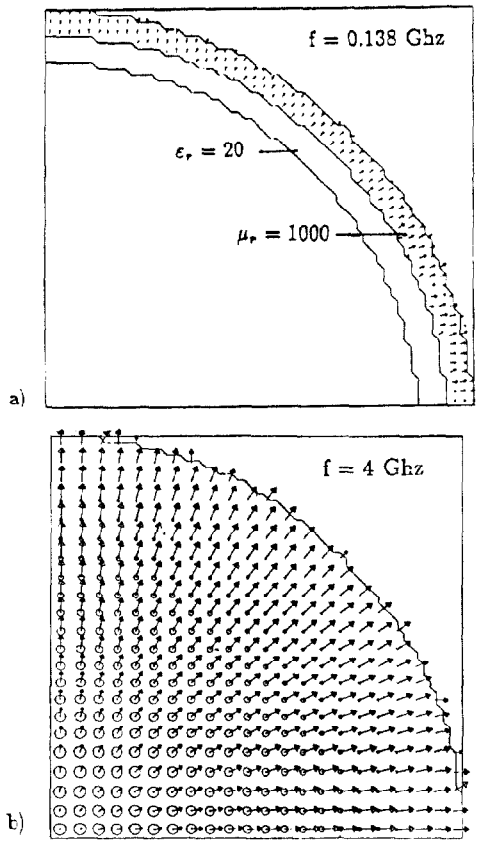



Figure 1: $\vec{E}$-Field of the lowest mode; $k_z = 78\ /m$
a) empty vacuum chamber; b) coated vacuum chamber.

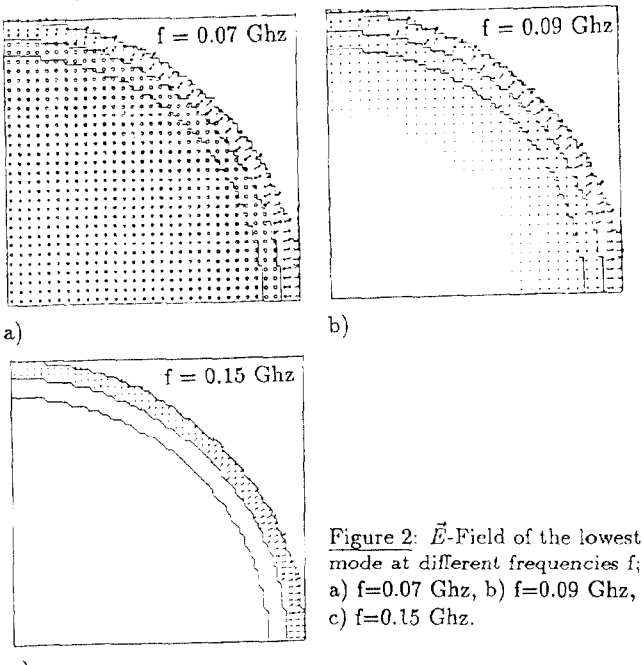
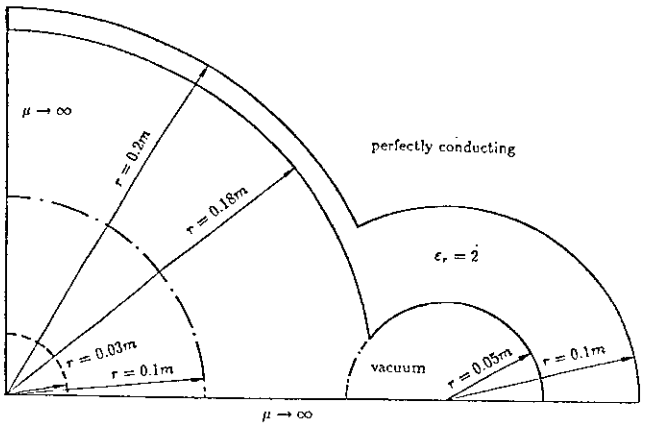
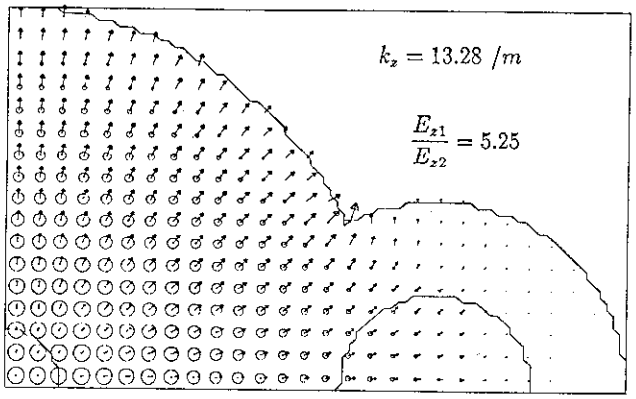


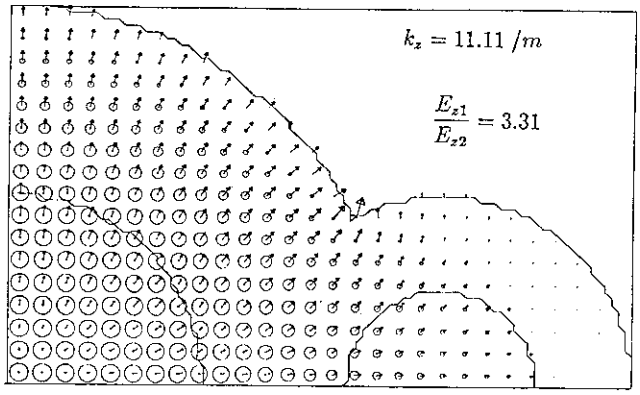

Figure 2: $\vec{E}$-Field of the lowest mode at different frequencies f; a) f=0.07 Ghz, b) f=0.09 Ghz, c) f=0.15 Ghz.



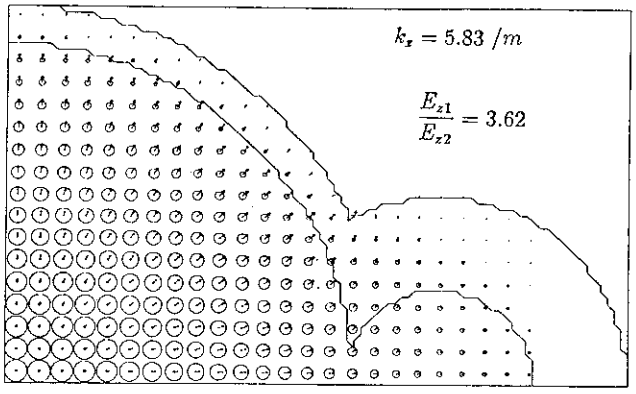

geometry of the transformer




a)




b)




c)

Figure 3: $\vec{E}$-Field of the lowest mode, f = 0.6 Ghz; inner radius r of the coating of the big tube is a) r=0.03 m, b) r=0.1 m, c) r=0.18 m.

## IV. Applications

The first example demonstrates the effects of coating a vacuum chamber [6]. The inside of an empty chamber was coated with a layer of a permittive material with $\mu_r = 1000$ and a second layer of a dielectric material with $\varepsilon_r = 20$. Figure 1 shows the lowest monopole mode in the empty and in the coated chamber whereby the wave number respectively the phase velocity was kept equal. We see that the frequency goes down due to the coating and the mode tends to concentrate its energy in the permittive material. This effect grows with frequency as can be seen in Figure 2.

As another example a dielectric structure was examined with respect to its properties as an accelerating structure. Figure 3 shows the ratio of the parallel $\vec{E}_z$-field components at the centers of the big tube 1 and the small tube 2 [7].

For lossy materials there is at the moment no graphic representation of the fields, the program returns only wave numbers.

## V. Conclusion

A new program for waveguide analysis was presented that solves a truly complex problem by use of the Lanczos algorithm in order to take into account losses in materials. Since only two dimensions are needed to describe a waveguide which is homogeneous in the direction of propagation the number of discretization points and according the dimension of the matrix stays comparably small and so does the running time of the program. Finally two examples demonstrated the applicability of the code.

## References

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