TAPERED UNDULATORS FOR FEL

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Abstract

Tapered undulators can improve energy extraction efficiency from an e-beam into laser radiation (FEL amplifier). We present the results of a numerical simulation which utilizes a self-consistent model of electron and field dynamics. The efficiency improvement of an untapered section and a tapered one, relevant to the optimization of a FEL amplifier dedicated to plasma heating in a tokamak is presented and discussed.

1. Theoretical Model

High power, high efficiency FELs can be realized by properly tapered undulators which increase energy extraction from the e-beam.

We report the preliminary results of work in progress aimed at designing a FEL amplifier for the generation of radiation of 1 mm wavelength for ECH (electron cyclotron heating) experiment in Tokamak plasmas.

At present our simulation model is 1-dimensional and is based on a self-consistent system of differential equations for particle motion in the Kroll et al. [1] formulation with γ, ψ as conjugate canonical variables and z as independent variable.

$$\frac{d\gamma}{dz} = -\frac{e_s b_U}{K_U \gamma} \sin\psi \tag{1}$$

$$\frac{d\psi}{dz} = K_U - \frac{K_s}{2\gamma^2}\left|1 + \left(\frac{b_U}{K_U}\right)^2 - \frac{2e_s b_U}{K_U K_s}\cos\psi + \left(\frac{e_s}{K_s}\right)^2\right| + \frac{d\phi}{dz}$$

where:

- K_s laser wavenumber
- K_U undulator wavenumber
- $e_s = eE_s/\sqrt{2}\,mc^2$ normalized electric field
- $b_U = eB_U/\sqrt{2}\,mc$ normalized undulator field
- ϕ electric field phase
- $\psi = (K_U + K_s)z - \omega_s t + \phi$ electron phase

and Maxwell equations in paraxial approximation [2]

$$\frac{de_s}{dz} = \frac{Z_0 b_U}{2mc^2 K_U} \cdot |J| \cdot \left\langle \frac{\sin\psi}{\gamma} \right\rangle - \alpha e_s \tag{2}$$

$$\frac{d\phi}{dz} = \frac{Z_0 b_U}{2mc^2 K_U} \cdot \frac{|J|}{e_s} \cdot \left\langle \frac{\cos\psi}{\gamma} \right\rangle$$

where

- Z_0 vacuum impedance
- J current density
- α phenomenological loss coefficient

and $\langle\ \rangle$ indicates particle average.

Following [2] we design the undulator profile by making the so-called resonant particle approximation [1]. We define the synchronous energy and phase by requiring that the resonant particle has no phase variation during its motion. Under this assumption resonant particle equations become

$$\frac{d\gamma_r}{dz} = -\frac{e_s b_U}{K_U \gamma_r} \sin\psi_r$$

$$\frac{d\psi_r}{dz} = K_U - \frac{K_s}{2\gamma_r^2}\left|1 + \left(\frac{b_U}{K_U}\right)^2 - \frac{2e_s b_U}{K_U K_s}\cos\psi_r + \left(\frac{e_s}{K_s}\right)^2\right| + \frac{d\phi}{dz} \tag{3}$$

$$\frac{de_s}{dz} = \frac{Z_0 b_U}{2mc^2 K_U} \cdot |J| \cdot \frac{\langle\sin\psi\rangle}{\gamma_r} - \alpha e_s$$

$$\frac{d\phi}{dz} = \frac{Z_0 b_U}{2mc^2 K_U} \cdot \frac{|J|}{e_s} \cdot \frac{\langle\cos\psi\rangle}{\gamma_r}$$

We impose two additional constraints to close the set of equations

$$\frac{d\lambda_U}{dz} = 0 \quad \text{(constant undulator period)}$$

$$\frac{d\psi_r}{dz} = 0 \quad \text{(constant synchronous phase).}$$

The undulator we consider is divided into an untapered section and a tapered section. In the former section radiation grows exponentially [3] and the e-beam becomes bunched, so we avoid the problem of the decrease of efficiency due to the unbunched electrons. The tapered section is then designed to achieve a large energy extraction efficiency from the previously bunched e-beam.

2. Numerical Procedure and Results

Our code first simulates, according to the equations already quoted, the evolution of an initially unbunched e-beam in the constant parameters undulator.

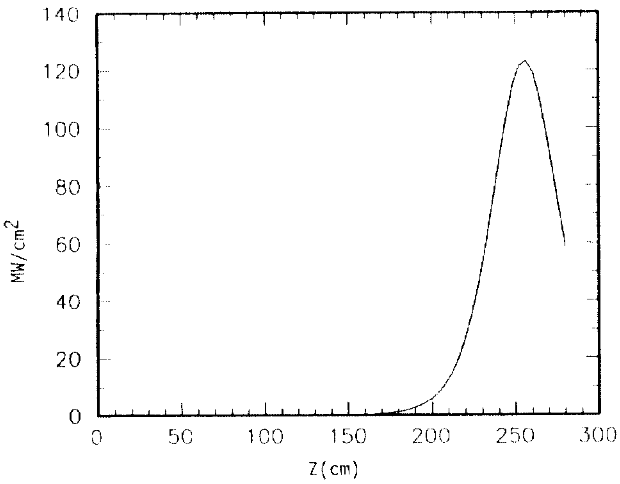

Fig. 1 - Laser intensity predicted for test case I

Phase space electron distribution is assumed to be initially known (uniform or gaussian). The evolution equations for 1024 test electrons and the field variables are integrated by an Adams-Bashforth-Moulton predictor-corrector method (at present space charge effects are neglected so that particle simulation codes are not necessary).

As a test case I, we choose, $\gamma = 16$, $\lambda_s = 1$ mm, $\lambda_U = 8$ cm, $\Delta\gamma = 0.2$, $J = 3.2\times10^2$ A/cm² and an input radiation flux of 16 W/cm². A good agreement, with the exponential gain formula [3] is found within few percent, as shown in Fig. 1.

The last part of our code consists of an iterative procedure which fixes the magnetic field profile for an efficient capture and deceleration of electrons in the ponderomotive potential, simulates the e-beam behaviour in the undulator field previously evaluated, and updates the laser field value. The initial e-beam phase space distribution function is determined from simulation in the constant parameter undulator. The code then chooses one of several options (continuity of B field in two sections, maximum tapering efficiency, maximum output power) to determine the initial parameter (γ_r, ψ_r, $b_U(0)$, $e_s(0)$) of the tapered section. The main core consists of two parts: a design procedure, which at each z step evaluates the magnetic field according to the resonant particle equation; and a simulation procedure of the advancing of the e-beam in the designed magnetic field.

As test case II, in order to gain confidence in the code,whe choose the same parameters as in reference [2], and we reproduce the design and the simulation results in good agreement with Ref. [2] (see Figs 2,3).

The problem we are now faced with is the proper matching of the untapered and tapered sections and the best choice of their lengths. Work is in progress to achieve this goal and the results, together with a 2-D design and 2-D particle simulation code, will be reported elsewhere.

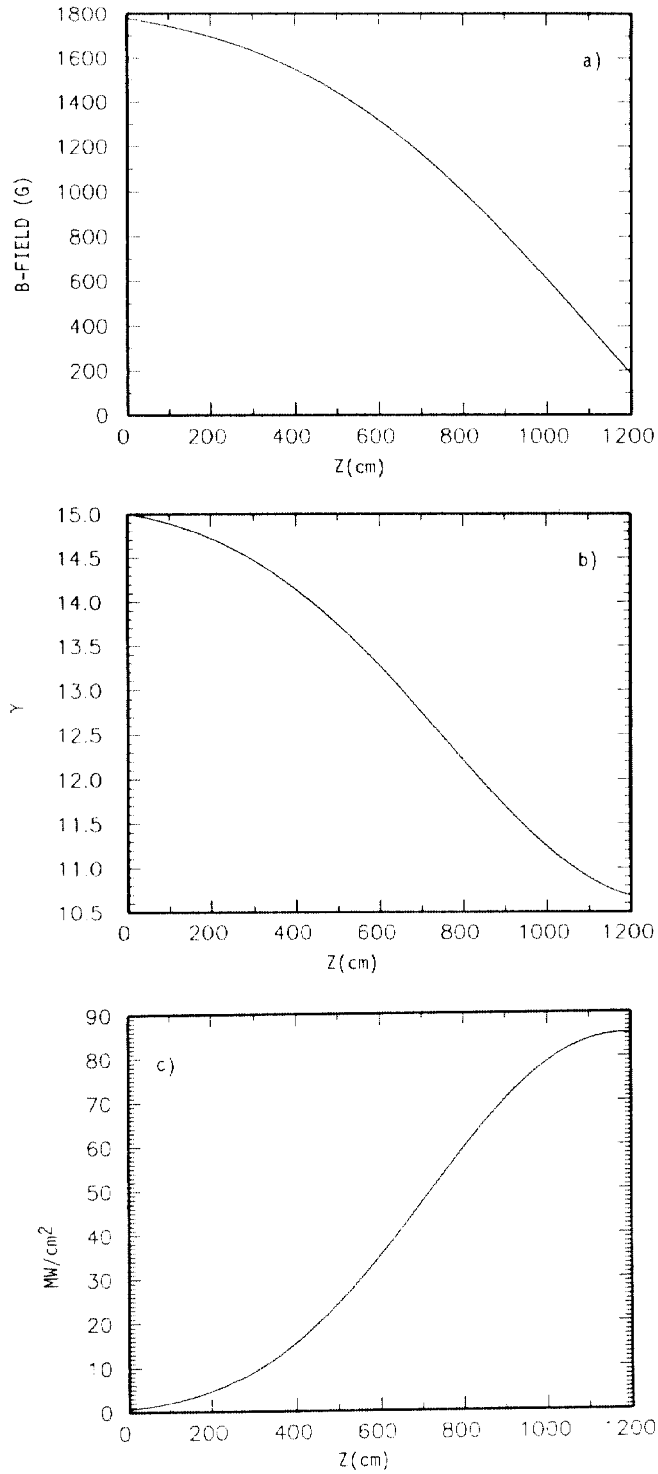

Fig. 2 - Design code predictions for test case II: a) magnetic field profile; b) resonant energy; c) laser intensity

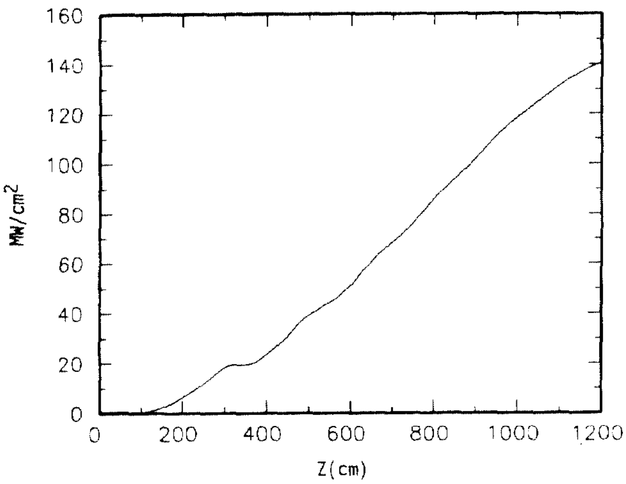

Fig. 3 - Laser intensity predicted by the simulation code for test case II

References

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