

## PROPERTIES OF SPIRAL LOADED CAVITIES

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### Abstract

Spiral loaded cavities are compact rf- structures with a high efficiency in a frequency range between 27MHz and 200 MHz and particle Energies from approx. 10 KeV/A to about 50 MeV/A. We have built a number of cavities for applications as post accelerator structures, debuncher and rebuncher cavities. Typical shuntimpedances are between 20 and 60 M $\Omega$ /m. Spiral structures have been operated with accelerating fields up to 2.5 MV/m. Properties of spiral loaded cavities will be described and applications will be discussed.

### Introduction

Spiral resonators are short and compact rf-structures with a high voltage gain for a wide range of ion species and energies. Spiral resonators are used as rebuncher and debuncher cavities and they are also used as postaccelerator sections. First proposed at Cal-Tech<sup>1</sup> and also in operation at Heidelberg<sup>2</sup> for the postaccelerator there. Recently, spiral loaded resonators have also been built in Frankfurt for upgrading the 7MV CN Van de Graaff generator<sup>3</sup>. Formulae for the determination of the resonance Frequency and efficiency of spiral loaded cavities are derived and applications will be described.

#### Single spiral loaded cavity

The single spiral structure consists of a  $\lambda/4$  spiral element which is formed of two radii<sup>3</sup>. One end is connected with the compact cylindrical outer tank and the free end is equipped with a drift tube (fig. 1). The tank is terminated by end plates, each carrying a drift tube and thus forming the two acceleration gaps.

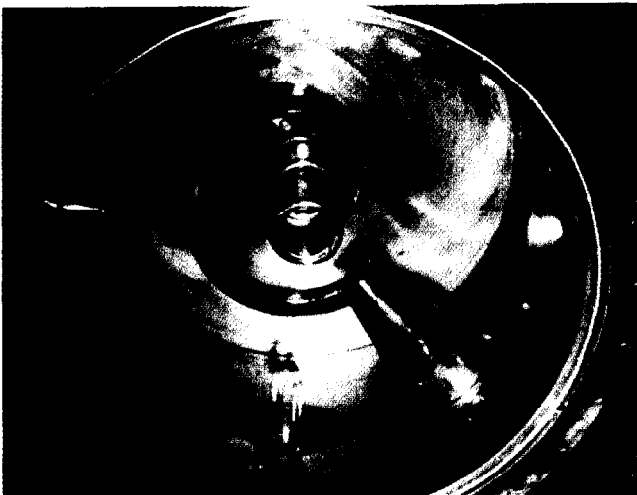


Fig. 1. Single spiral resonator

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In order to calculate the resonance frequency we start with the equivalent circuit of the resonator (fig. 2). C is the capacity between the spiral arm and the tank walls and the capacity of one acceleration gap. L is the inductance of the spiral arm.

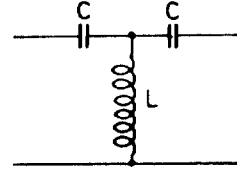


Fig. 2. Equivalent circuit of a single spiral resonator

The lumped circuit for the spiral without the drift tubes can be obtained by considering the structure as a capacitively loaded inhomogeneous  $\lambda/4$ -line. Compared to the geometrical length  $l$ , the electrical length  $l^*$  is lengthened by the capacitive load. To calculate the capacitance of the drift tube, a semiempirical formula is used<sup>4</sup>:

$$C_D = k \cdot \epsilon_0 \left\{ \frac{\pi}{g} [r_a^2 - r_{ap}^2] + r_a \left[ \ln \left( \frac{16\pi r_a}{g} \right) + 1 \right] + \left( 1 + \frac{L_D}{2g} \right) \cdot \ln \left( 1 + \frac{L_D}{2g} \right) - \frac{L_D}{2g} \cdot \ln \left( \frac{L_D}{2g} \right) \right\} \quad (1)$$

$r_a$  : outer radius of the drift tube  
 $r_{ap}$  : radius of beam aperture  
 $g$  : gap width  
 $L_D$  : length of the drift tube  
 $k$  : geometrical factor (value = 0.77).

The capacitive lengthening by the drift tubes is determined by

$$\frac{\Delta l}{2l^*} = \frac{1}{\pi} \cdot \arctan \left( \frac{\pi}{4} \cdot \frac{1}{l^*} \cdot \frac{C_D}{C} \right). \quad (2)$$

The concentrated elements of the spiral (with drift tubes) can be obtained from the formulae of a capacitively loaded  $\lambda/4$ -line:

$$C = \frac{\pi}{4} \int_0^l C' \cdot \sin \left( \frac{\pi \cdot x}{2l^*} \right) dx; \quad L = \frac{4}{\pi} \int_0^l L' \cdot \sin \left( \frac{\pi \cdot x}{2l^*} \right) dx \quad (3.4)$$

According to the approximate validity of the free space wave propagation we determine the inductive layer  $L$  for a known value of  $C$  from

$$L' = \frac{1}{c_0^2 \cdot C'} \quad , \quad c_0 = \text{speed of light}. \quad (5)$$

For the capacitive layer  $C'$  a configuration of a line with a length of  $\Delta x$  between three conducting walls is used (fig. 3).

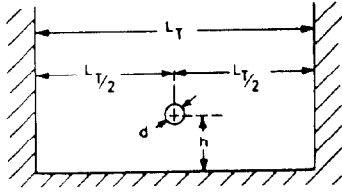


Fig. 3. Line between 3 conducting walls.

The equivalent capacitive layer  $C'$  of this configuration is given by

$$C' = \frac{1}{2 \cdot \ln \left[ \frac{4 \cdot L_T \cdot \tanh \left( \frac{h \cdot \pi}{L_T} \right)}{\pi \cdot d} \right]} \quad (6)$$

#### Double spiral loaded cavity

The double spiral loaded cavity consists of two coupled spirals in one cylindrical tank (fig 4). When excited in  $\pi$ -mode, the current on the two spiral arms oscillate with a  $180^\circ$  phase difference. The resulting accelerating voltage of this three gap structure is for the same rf-power about two times that of a single spiral loaded resonator<sup>5</sup>.

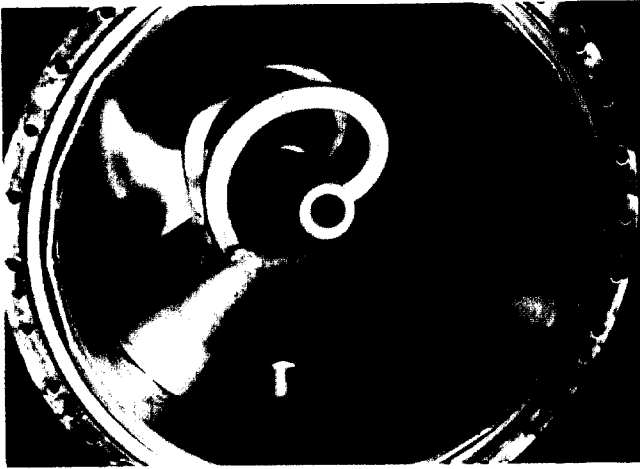


Fig. 4. Double spiral loaded cavity

The equivalent circuit of this double spiral resonator is shown in fig.5. In  $C$  the capacity between one spiral arm and the tank walls and the capacity of an outer acceleration gap are summarized.  $D$  comprises the capacity of the middle gap and the capacitance between the two spiral arms.  $L$  relates the inductance of one spiral arm, and  $\gamma_m$  represents the mutual inductance between the two spirals<sup>4,6</sup>.

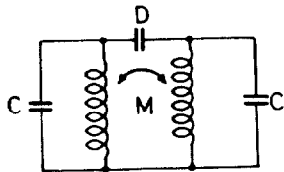


Fig. 5. Equivalent circuit of a double spiral resonator

The resonance frequency of this equivalent circuit is

$$\omega_\pi^2 = \frac{1}{L(1 - \gamma_m) \cdot (C + 2 \cdot D)} \quad (\pi\text{-mode, operating mode}). \quad (7)$$

In case of the  $\pi$ -mode corresponding arm segments have opposite potentials. This capacitive influence is taken into account by using a configuration of two parallel lines with a length of  $\Delta x$  over a conducting wall (fig. 6)<sup>6,7</sup>.

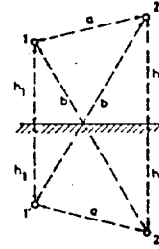


Fig. 6. Two parallel lines over a conducting wall

The capacitive layer  $C'$  of this configuration is given by

$$C' = 2\pi\epsilon_0 \frac{\ln \frac{4h_2}{d} + \ln \frac{b}{a}}{\ln \frac{4h_1}{d} \cdot \ln \frac{4h_2}{d} - \ln^2 \frac{b}{a}} \quad (8)$$

#### Efficiency of spiral loaded cavities

The efficiency of a linac structure can be characterized by the  $R_p$ -value or the shuntimpedance  $\eta_0$ :

$$R_p = \frac{U^2}{P} \quad \text{and} \quad \eta_0 = \frac{U^2}{P \cdot L} = \frac{R_p}{L} \quad (9)$$

$U$ : maximum accelerating voltage;  $P$ : rf-power and  $L$ : tank length.

For a single spiral loaded cavity the  $R_p$ -value is given by

$$R_p = 8 \cdot \frac{Q_0}{\omega C} \cdot \frac{\sin^2 \Phi}{1 - \frac{1}{2\pi} \cdot \sin(2\Phi) + \frac{C_D}{C} \cdot \sin^2 \Phi}$$

and for a double spiral loaded resonator the  $R_p$ -value is as twice as for a single spiral cavity.

$C_D$  is the capacity of the drift tubes

$$\Phi = \frac{\pi}{2} \cdot \frac{\lambda_0}{\lambda^*} \quad (\lambda^* \text{ is the wavelength of the structure with the additional capacitive load of the drift tubes}) \quad (10)$$

$$Q_0 = \frac{\pi^2 \cdot \omega L}{8 \cdot R' \cdot I} \quad ; \quad R' = \frac{1}{d} \cdot \sqrt{\frac{\omega \cdot \mu_0}{2 \cdot \sigma \cdot \pi^2}} \quad (11,12)$$

### Spiral loaded cavities for 108 MHz.

Two single spiral and double spiral loaded cavities have been built for the UNILAC at GSI in Darmstadt. Two double spiral resonators and one single spiral resonator are working as rebunchers for the final time focus on the target. The cavities are optimized for particle energies of 8 MeV/A. For the longitudinal matching of the UNILAC beam, another single spiral resonator is located between two Alvarez tanks. This rebuncher cavity is designed for a particle energy of 4.5 MeV/A. With an  $R_p$ -value of 12 M $\Omega$  and a required resonator voltage of 1 MV, the total power consumption of the double spiral resonator is about 80 KW. The single spirals have an  $R_p$ -value of 6 M $\Omega$  and for a required resonator voltage of 500 KV, the total power consumption is only 40 KW.

### 27 MHz single spiral loaded cavity

For the new UNILAC high current injector a short rebuncher cavity is required for longitudinal matching between the Wideroe I and Wideroe II at a particle energy of 216 KeV/amu<sup>8</sup>. In this area where the gas stripper will be located, the longitudinal matching can be done with a 27 MHz single spiral loaded cavity with a resonator voltage of 500 kV. To investigate the mechanical stability of the structure which is operated at the very low frequency of 27 MHz and high field levels, a prototype of a spiral resonator has been built, using a cavity with an outer diameter of only 500 mm. Due to the small diameter of the cavity, the spiral consists of 3.5 turns. The spacing of two adjacent turns is 20 mm and the spiral tube diameter is 25 mm (fig. 7). The outer diameter of the final rebuncher cavity will be 700 mm and the spacing of two adjacent turns will be 25 mm. This will increase the  $R_p$ -value significantly.

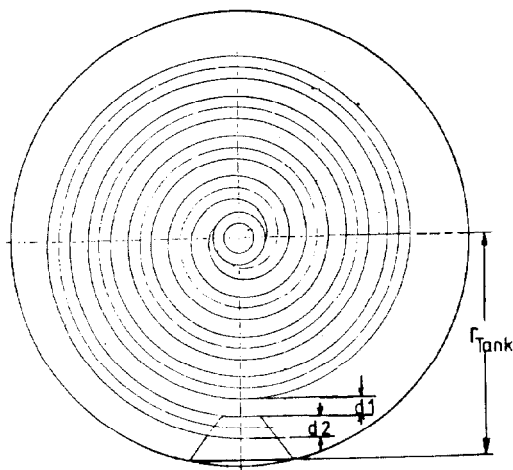


Fig. 7. Schematic drawing of the 27 MHz resonator

The measurements on the prototype have shown, that the resonator has an  $R_p$ -value of about 6 M $\Omega$ . With this  $R_p$ -value the total power consumption for the the required resonator voltage of 500 kV will be 45 KW. With a gap width of 40 mm the maximum electric field in the gap is only 6 MV/m. The prototype resonator has been tested at an rf-power of 65 KW with a duty cycle of 1 up to 50 % and a variation of the pulse frequency between 0.1 Hz and 100 Hz without showing any ponderomotive effects. It could be operated without any problems.

### 202 MHz single spiral resonator for HERA

For the HERA project at DESY in Hamburg a single spiral loaded cavity has been built as a debuncher between the Alvarez tanks and the synchrotron. This compact resonator will decrease the energy spread of the 50 MeV  $H^-$  beam behind the Alvarez tanks. For a required resonator voltage of 500 KV and an  $R_p$ -value of 5.6 M $\Omega$ , the total power consumption is only 45 KW. Due to the high resonance frequency of 202 MHz and the particle energy of 50 MeV/A, the resonator length is 400 mm and the outer diameter of the tank is 350 mm. Fig. 8 shows the effective  $R_p$ -value versus the particle energy.<sup>9</sup>

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