MATCHING OF ION SOURCES TO CYCLOTRON INFLECTORS

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Abstract

In general, cyclotron inflectors strongly couple the two transverse subspaces. This leads to a growth in emittance projections for a beam with no initial correlation between the two transverse subspaces. Only in the case of the Müller (hyperboloid) inflector is there no emittance growth. We have made calculations using an optimization routine to match a given beam through the axial magnetic field and inflector of a cyclotron axial injection system. We find that in the limit where all the emittance is due to a beam's axial angular momentum (for example, from an ECR source), matching with no emittance growth is possible even in the case of mirror or spiral inflectors. Moreover, any one of the two transverse emittances circulating in the cyclotron can be made smaller than the source emittance while maintaining the sum of the emittances constant. This is achieved by rotating the matching quadrupoles with respect to the inflector and retuning.

Introduction

Various inflectors have been invented for injecting a beam through an axial transport line and into a cyclotron. For a beam with no initial correlation between the two transverse subspaces, the Müller (hyperboloid) inflector[1] is the best choice from the point of view of optics alone. Spiral and mirror inflectors, however, each have their own advantages[2] and so are often chosen despite the fact that they worsen the beam emittance. A point which is often overlooked is that not all types of sources have uncorrelated transverse subspaces. For example, an axially symmetric source where charge exchange takes place inside a solenoidal field will have an emittance (phase space area $\div \pi$) in one transverse direction given by[3]

$$\epsilon_i^2 = \epsilon_0^2 + \left(\frac{r^2}{2\rho_s}\right)^2 . \tag{1}$$

where ϵ_0 is the intrinsic emittance (due to ion temperature), r is the beam radius at the solenoid exit and ρ_s is the beam rigidity divided by the solenoid field. In this case, $r_{14} = -r_{23} \neq 0 (r_{ij} \equiv \sigma_{ij}/\sqrt{\sigma_{ii}\sigma_{jj}}$, and $\sigma_{11} = \overline{x^2}$, $\sigma_{14} \equiv \overline{xy'}$, etc.) and neglecting this fact leads to an over-estimation of the emittance growth through the inflector.

We have investigated cyclotron matching in both extremes: namely where (I) $\epsilon_i = \epsilon_0$, $r_{14} = r_{23} = 0$, and where (II) $\epsilon_0 = 0$, $r_{14} = \pm 1$, $r_{23} = \mp 1$ (the sign depends upon whether the source solenoid field is parallel or antiparallel to the beam direction). Calculations were performed using a computer code based upon TRANSOPTR.[4] Like TRANSPORT, this code is based upon the 6-dimensional σ-matrix formalism, but in this application, TRANSOPTR has two advantages over TRANSPORT. (1) The transport system is defined in a FORTRAN subroutine. This allows much greater flexibility in defining both the transport system and the fitting constraints. (2) TRANSOPTR has the capability of using the infinitesimal transfer matrix approach.[5] This was originally introduced to efficiently deal with the 3-dimensional space charge forces of bunched beams; in our application it allows one to treat optical elements, such as a varying axial magnetic field and a spiral inflector, where it is not possible to write down an analytic transfer matrix. The infinitesimal transfer matrix, $\mathbf{F}(s)$, can be defined as $(\mathbf{M} - \mathbf{I})/ds$ where $\mathbf{M}$ is the transfer matrix from s to $s + ds$ and $\mathbf{I}$ is the identity matrix. The σ-matrix elements are calculated by numerically solving

$$d\sigma/ds = \mathbf{F}\sigma - \sigma\mathbf{F}^T .$$

We have incorporated the varying axial field and the simple (unslanted electrodes, constant magnetic field) spiral inflector into TRANSOPTR. For these two cases $\mathbf{F}$ is,

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & K'/2 & K & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -K'/2 & -K & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} , \quad \text{(varying axial field)}$$

$$\begin{pmatrix} 0 & 1 & -CK & 0 & 0 & 0 \\ -S^2K^2 & 0 & -SK/A & 0 & 0 & SK \\ CK & 0 & 0 & 1 & 0 & 0 \\ -SK/A & 0 & 0 & 0 & 0 & 2/A \\ -SK & 0 & -1/A & 0 & 0 & 1 \\ -CK/A & 0 & 0 & -1/A & 0 & 0 \end{pmatrix} , \quad \text{(spiral inflector)}$$

where $K = 1/\rho$, A is the inflector height, $S = \sin(s/A)$, $C = \cos(s/A)$, s is the independent variable and is set to zero at the inflector entrance. For the spiral inflector, the two transfer matrices

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & K & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -K & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1/A & 0 & 0 & 1 \end{pmatrix} , \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1/A & 0 & 0 & 1 \end{pmatrix} ,$$

must be used at the entrance and exit respectively to transform to the proper variables. The $\mathbf{F}$ matrices are derived from the well-known equations of motion.[2] The mirror inflector has been incorporated into TRANSOPTR using the transfer matrix of Ref. 6.

We have confined our attention to matching in the 4-dimensional subspace (x, x', y, y'). In this case it is convenient to describe the cyclotron as a dipole with a field index $n = \nu_y^2$. This yields $\nu_x = \sqrt{1-n}$. Matching is achieved by varying the strengths and orientation of quadrupoles placed before the entrance into the cyclotron's axial magnetic field while trying to minimize $r_{12}, r_{34}, x_1, x_2, y_1$, and y_2. The latter 4 parameters are, respectively, horizontal beam sizes at locations in the cyclotron separated by $90°/\nu_x$ of azimuth and vertical beam sizes at locations separated by $90°/\nu_y$. Minimization was performed by the least-squares subroutine BCLSF from the IMSL library.

Circulating emittances in the cyclotron are calculated from

$$\epsilon_x = \nu_x x_{\max}^2/\rho$$

(where $x_{\max}$ is the maximum beam size over one betatron oscillation) and similarly for ϵ_y. The reason for defining emittances in this way is that in cyclotrons where turns are separated, betatron phases always become mixed and so the matched ellipse becomes filled.

Results

In our calculations we used an injected beam energy of 25 keV and cyclotron parameters $\rho = 19$ mm and $\nu_y = 0.2$. These values correspond, more or less, to the 30 MeV H^- cyclotron being designed at TRIUMF.[7] Case I with $\epsilon_x = \epsilon_y$ and no $x-y$ correlations is summarized in Table I. The source emittance is 50 mm-mrad or, normalized, is 0.365 mm-mrad. Since the calculation is linear, however, this emittance is simply a scaling parameter: only the ratio of circulating emittance to source emittance is relevant. Both the mirror and the spiral inflector are characterized by a parameter A which is the height of the entrance above the median plane. We note that different inflectors require different values of θ: the angle between the incoming beam (i.e., the matching quadrupoles) and the inflector electrodes. Emittance growth is largest for the mirror inflector and for both inflectors diminishes as inflector size decreases.

Table I. Normalized cyclotron emittances and quadrupole orientation for optimum inflector matching with an uncorrelated input beam. Normalized input emittances: $\epsilon_x = \epsilon_y = 0.365$ mm-mrad.

Inflector type	height (A) (cm)	electric field (kV/cm)	normalized emittances (mm-mrad) radial	vertical	quad orientation (θ)
Spiral	1.0	50.0	0.47	0.47	-50°
	2.0	25.0	0.68	0.63	-23°
	3.0	16.7	0.77	0.81	0°
Mirror	1.0	36.2	0.93	0.84	25°
	1.5	24.9	0.97	1.18	28°
	2.0	19.6	1.37	1.47	30°

Case II, namely, where the emittance is due to the beam's angular momentum was simulated by allowing a divergenceless beam of radius r to exit a solenoid with magnetic field B_s such that

$$\frac{r^2}{2\rho_s} = 50 \text{ mm-mrad}, \quad \rho_s \equiv \frac{(B\rho)}{B_s},$$

Results are summarized in Table II and Fig. 1. For both the mirror and the spiral inflector, the following conclusions can be drawn.

1. It is possible to match to the cyclotron with no growth in the sum of the transverse emittances.
2. It is possible to trade emittance in the cyclotron between the radial and vertical directions while maintaining the sum of the two emittances at twice the input emittance. For example, for the 2 cm spiral inflector (Fig. 1), the vertical emittance can be continuously varied between 8% and 150% of the source emittance. In the case of the mirror inflector (Table II), the emittances can be varied also, though not over as broad a range as in the case of the spiral inflector.
3. Conclusions (1) and (2) remain true whether the source solenoid is aligned parallel or anti-parallel to the cyclotron magnetic field. However, the quadrupole tune is quite different for the two cases. For the 1 cm mirror inflector (Table II), emittance can be swapped between the radial and vertical directions by switching the direction of the source magnetic field.

Table II. Matched cyclotron emittances as a function of quadrupole orientation angle (θ) for the 1.0 cm mirror inflector with an input beam whose emittance (0.365 mm-mrad normalized) is due solely to angular momentum.

Orientation of source solenoid w.r.t. cyclotron	θ	Normalized emittances (mm-mrad) radial	vertical
−	-40°	0.64	0.09
	-19°	0.68	0.05
	4°	0.64	0.09
	26°	0.62	0.11
+	-5°	0.16	0.57
	10°	0.15	0.58
	30°	0.09	0.64
	55°	0.16	0.57

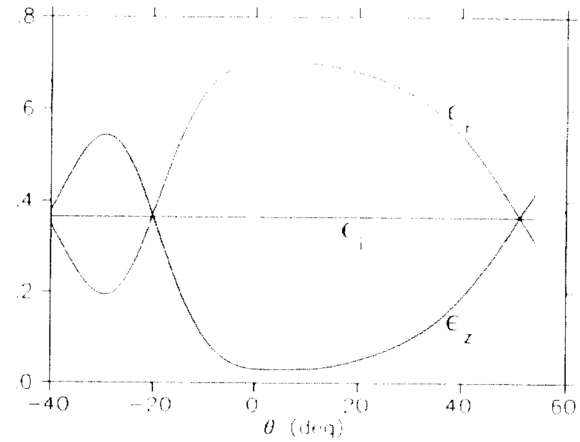

Fig. 1. Matched cyclotron emittances, ϵ_r and ϵ_z, as a function of the orientation angle of the matching quadrupoles for the case of the 2.0 cm spiral inflector with an input beam whose emittance, ϵ_i, is due solely to angular momentum.

It must be emphasized that the 4-dimensional phase space volume is zero for the case summarized in Table II and Fig. 1. Real solenoidal sources will contain emittance contributions from both terms in formula (1). Nevertheless, it is clear that neglecting the $x-y$ correlation will lead to an overestimation of the emittance growth which occurs when injecting through other than a hyperboloid inflector. Moreover, the (usually dominant) emittance contribution due to the source's solenoidal field can be traded between vertical and radial directions in the cyclotron. This kind of flexibility is not possible with the hyperboloid inflector and may have practical uses.

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